STUDIES OF THE FLOW CHARACTERISTICS OF A SHARP-EDGED CIRCULAR ORIFICE

BY

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CHAPTER THERE

ABSTRACT

An equipment was designed and constructed to study the flow characteristics of a fluid through a sharp-edged circular orifice. These characteristics include the coefficients of discharge for different values of Reynolds number and the coefficient of velocity. The equipment comprises a measuring tank, weir tank, weighing container, a sump and a simple beam balance. The components of the equipment were adopted for the study of error phenomenon in physical measurements. The error parameters considered were the mean, standard deviation, standard error, percentage error and the levels of accuracy.

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Indeed, I treasure very highly the patience, understanding and unequal love of my amiable spouse during the period of study.

CERTIFICATION BY SUPERVISOR

I certify that this work has been carried out by T. Ewetumo in the Department of Physics, The Federal University of Technology, Akure.

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DEDICATION

This work is dedicated to my loving mother **Mrs. E. N Adeniji**

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CHAPTER ONE

1.0 INTRODUCTION

1.1.0 The Characteristics of Fluids.

A fluid is a substance which may flow; its constituent particles continuously change their position relative to one another. Moreover, it offers no lasting resistance to the displacement, however great, of one layer over another. This means that, if the fluid is at rest, no shear force can exist in it. A solid, on the other hand, can resist a shear force while at rest. The shear force may cause some displacement of one layer over another, but the material does not continue to move indefinitely. In a fluid, however, shear forces are possible only while relative movement between layers is actually taking place. A fluid is further distinguished from a solid in that a given amount of it owes its shape at any particular time to that of the vessel containing it, or to forces which in some way restrain its movement.

The distinction between solid and fluid is usually clear, but there are some substances which are not easily classified. Some fluids, do not flow easily. For example, thick tar may atimes appear to behave like a solid. If a block of such a substance is placed on the ground, although its flow takes place very slowly, over a period of time - perhaps several days- it would spread over the ground by the action of gravity. The essential difference between solids and fluid remains. Any fluid, no matter how viscous, begins to flow, even if imperceptibly, under the action of the slightest net shear force and the fluid continues to flow as long as such a force is applied. A solid, however, no matter how plastic it is, does not flow unless the net shear force on it exceeds a certain value.

In a fluid the forces opposing the movement of one layer over another exist only while the movement is taking place, and so static equilibrium between applied force and resistance to shear force never occurs. Deformation of the fluid takes place continuously so long as a shear force is applied. If this applied force is removed the shearing movement subsides and, as there are then no forces tending to return the particles of fluid to their original relative positions, the fluid keeps its new shape. Therefore, a fluid is an aggregation of particles which yield to the slightest effort made to separate them from each other.

A fluid may be sub-divided into liquids and gases. A fixed amount of a liquid has a definite volume which varies only slightly with temperature and pressure. If the capacity of the containing vessel is greater than this definite volume, the liquid occupies only part of the container, and it forms an interface separating it from its own vapour, the atmosphere or any other gas present.

On the other hand, a fixed amount of a gas, by itself in a container, will always expand until its volume equals that of the container. Only then can it be in equilibrium. In the analysis of the behaviour of fluids, the most important difference between liquid and gases is that, whereas under ordinary conditions, liquids are so difficult to compress that they may for most purposes be regarded as incompressible, gases may be compressed much more readily. Where conditions are such that an amount of gas undergoes a negligible change of volume, its behaviour is similar to that of a liquid and it may then be regarded as incompressible. If, however, the change in volume is not negligible, the compressibility of the gas must be taken into account in examining its behaviour.

1.1.1 MOLECULAR STRUCTURE

The different characteristics of solids, liquids and gases result from differences in their molecular structure. A substance consists of a vast number of molecules separated by empty space. The molecules have an attraction for one other, but when the distance between them becomes very small (of the order of the diameter of a molecule) there is a force of repulsion between them which prevents them all gathering together as a solid lump.

The molecules are in continual movement and when two molecules come very close to one another the force of repulsion pushes them vigorously apart, first as though they had collided like two billiard balls. In solids and liquids the molecules are much closer together than in a gas. A given volume of a solid or a liquid therefore contains a much larger number of molecules than an equal volume of a gas, and so solids and liquids have a greater density (i.e. mass per unit volume) then a gas.

In a solid, the movement of individual molecules is slight, just a vibration of small amplitude. Therefore, the force of attraction between the molecules is so great that they do not readily move relative to one another. In a liquid, the movement of the molecules is greater, but they continually attract and repel one another so that they move in curved, wavy paths rather than in straight lines. The force of attraction between the molecules is sufficient to keep the liquid together in a definite volume, although, because the molecules can move past one another, the substance is not rigid. In a gas the molecular movement is very much greater than in a liquid. The number of molecules in a given space is much less and so any molecule travels a much greater distance before meeting another. The force of attraction between molecules is inversely proportional to the square of the distance between them.

The activity of the molecules increases as the temperature of the substance is raised. Indeed, the temperature of a substance may be regarded as a measure of the average kinetic energy of the molecules.

1.1.2 THE DEVELOPMENT OF MECHANICS OF FLUIDS

The Mechanics of Fluids is the study in which the fundamental principles of general mechanics are applied to liquids and gases. The principles are those of the conservation of matter, the conservation of energy and Newton's laws of motion. To the study of compressible fluids we also bring some of the laws of thermodynamics. By the use of these principles we are able to explain and predict, at least approximately, the behaviour of fluids under a set of specified conditions.

Ever since his first attempt to move water from one place to another without filling a container and carrying it, man has been interested in mechanics of fluids. For centuries, however, his knowledge of the subject was gained solely from simple observations and the tedious processes of trial and error. Thus from the time of the ancient Greeks and Romans, the largely empirical subject of *hydraulics* grew up. Against such a background, the recognition of fundamental principles governing the observed phenomena was not easy.

Since about 1750, mathematicians and mathematical physicists have attempted to obtain solutions to many of the problems of fluid motion , solutions not relying on the results of experimental measurement. Most of these attempts have been made possible, however, only by the introduction of major simplifying assumptions, and so the results have sometimes borne little relation to practical problems. The solutions, which refer to an 'ideal' fluid, not to be found in the physical world, constitute a body of knowledge now known as

Classical Hydrodynamics. Although this ideal fluid lacks some of the attributes of real fluids, in particular, viscosity, the descriptions of its behaviour given by classical hydrodynamics are not entirely without value in those circumstances where the neglected properties have only a small effect on the behaviour of a real fluid. The mathematicians' outlook, moreover, was the praiseworthy one of attempting to find solutions of as wide an application as possible rather than just the formulation of rules obtained from a limited range of observed conditions and therefore strictly applicable to those conditions.

It is not always easy to decide whether a particular simplifying assumption is justifiable, and ultimately experimental verification of theoretical results is always required. Conversely, experimental results themselves sometimes suggest assumptions which may profitably be made in further theoretical work. Modern mechanics of fluids, which is one of the sciences on which many important branches of engineering are founded, combines both theoretical and experimental approaches to problems.

Fluids mechanics can be discussed under two topics: hydrodynamics and gas dynamics. Hydrodynamics deals primarily with the flow of fluids for which there is virtually no density change, such as the flow of a liquid or the flow of a gas at low speeds. Hydraulics, for example - the study of liquid flow in pipes, open channel or orifices falls within this category. Gas dynamics, on the other hand, deals with fluids that undergo significant density changes. High-speed flow of a gas through a nozzle, the flow of low-speed aircraft and the movement of a body through the low density air of the upper atmosphere fall within the general category of gas dynamics.

1.2. Properties of Fluids

Specific Weight

The specific or unit weight (w) is defined as the weight per unit volume. It is the product of acceleration due to gravity g and density thus

$$w = \frac{\text{mass x g}}{\text{Volume}}$$

$$w = \frac{\text{weight}}{\text{volume}} \quad \text{.........................} (1)$$

The unit is newtons per unit cubic metre (N/m^3)

Density

Density, ρ, is defined as mass per unit volume and is significant in all flow problems where acceleration is important. It is obtained by dividing the specific weight w by the acceleration due to gravity g

Thus

$$\rho = \frac{\text{mass}}{\text{volume}} \quad \text{..................................} (1.2)$$

The unit is kg/m^3.

The specific gravity of water is the ratio of its density at some temperature to that of pure water at 20°C.

Compressibility

All matter is to some extent compressible, that is, a change in the compressive stress applied to a certain amount of a substance always produces some change in its volume. Although the compressibilities of different substances vary widely, the proportionate change in volume of a particular material which does no change its phase (e.g. from liquid to solid) during the compression is directly related to the change in the compressive stress. The degree of compressibility of a substance is characterized by the bulk modulus of elascity, K, which is defined by the equation

$$K = \frac{\Delta p}{\Delta v/v} \text{..................(1.3)}$$

Here ΔP represents a small increase in pressure applied to the material and ΔV the corresponding small increase in the original volume V. The unit is newton per unit square metre (Nm^{-2}).

Viscosity

Viscosity, μ, of a fluid, also called the coefficient of viscosity, absolute viscosity, or dynamic viscosity, is a measure of its resistance to flow. It is expressed as the ratio of the tangential shearing stress between flow layers to the rate of change of velocity with depth.

$$\mu = \frac{E}{dv/dy} \text{ .... (1.4)}$$

where E is shear stress (N/m^2), and dv/dy is the velocity gradient. Velocity decreases as temperature increases.

Kinematic viscosity

Kinematic viscosity v is defined as viscosity μ divided by density ρ. The unit is square metre per second. Kinematic viscosity deals with the study of motion without regard to the causes of the motion, and is concerned with lengths and time intervals only, not with masses. That is why the name kinematic viscosity is now in universal use.

$$\nu = \frac{\mu}{\rho} \quad \text{.......................(1.5)}$$

Surface tension

The surface tension, γ, of a liquid, sometimes called the coefficient of surface tension, is defined as the force per unit length acting in the surface at right angles to one side of a line drawn in the surface of liquid.

$$\gamma = \frac{\text{Force}}{\text{length}} \qquad (\text{N/m})$$

Capillarity

Capillarity is due to both the cohesive and adhesive forces between liquid molecules. It shows up as differences in liquid surface elevations between the inside and outside of a small tube that has one end submerged in the liquid. Since the adhesive forces of water molecules are greater than the cohesive forces between water molecules, water wets a surface and rises in a small tube, as shown Figure 1.1.

1.3 Review of Previous Work

One of earliest experimenters on sharp-edged orifices was Hamilton Smith, Jr [1886]. His values of the coefficient of discharge, C_d, for round and square orifices are given Table 1-1. There have been many subsequent investigations of circular orifices, not all of which are in agreement. Investigations by Medaugh and Johnson [1940] check Smith's coefficients for orifices larger than 6.37mm in diameter within 12.70mm of 1 percent. Values of the coefficient of discharge for a 25.40mm orifice as determined by various investigators and plotted by Medaugh and Johnson are shown in Fig. 1.2. The difference between the values are undoubtedly not entirely due to experimental errors. Many other factors such as the ratio of the orifice diameter to the dimensions of the tank wall, the

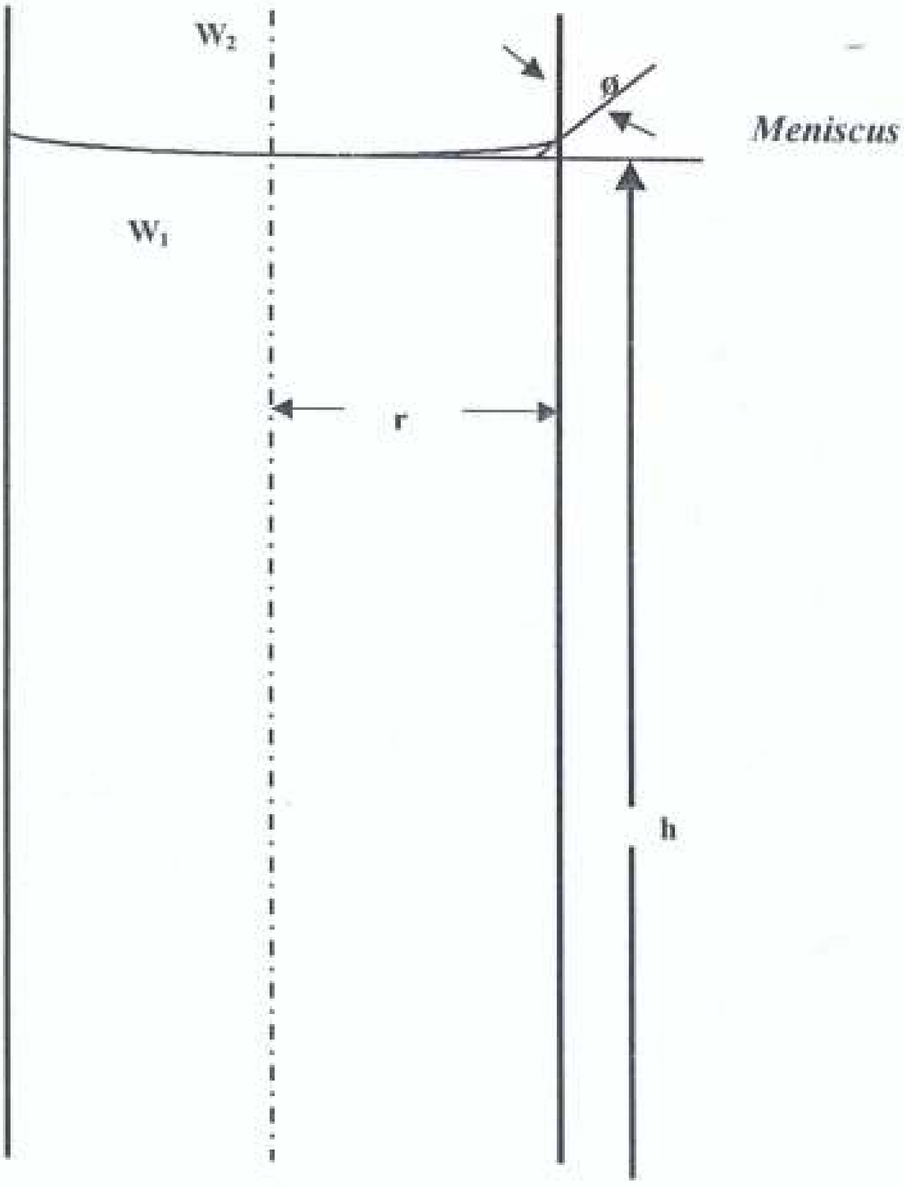


Fig.1.1 CAPILLARY ACTION. WATER RISES IN A SMALL TUBE

sharpness of the edge of the orifice, the roughness of inner surface, the orifice plate, and temperature of the water may also be considered. The effect of having the tank wall approach the orifice is to suppress contraction and therefore to make C_d approach the value of coefficient of velocity, C_v.

Smith and Walker [1923] found values of C_v to vary from 0.954 to 0.991 for orifices varying in diameter from 19.01mm to 73.50mm, respectively. They also found a small variation with head, the above values being averages for heads varying from 304.80mm to 18288.00mm.

Values of coefficient of contraction, C_c for circular sharp-edged orifices were found to vary from approximately 0.67 for 19.01mm orifice to 0.614 for 73.50mm orifices when the head is 609.60mm or more. Values are slightly larger for lower heads.

The effect of the water temperature is to change the viscosity and density taken in conjunction with the variation of C with the velocity and orifice diameter shown in Table 1.1. It becomes clear that here again the Reynolds number is likely to be the coordinating factor. This was shown to be the case by Lea [1942], who plotted more than one hundred experimental values of coefficient of discharge against Reynolds number. The author's curve, derived from Lea's plotted points, is shown in Fig 1.3. The fluids used in the tests were water, various mixtures of water and glycerin, and different oils. The flow is laminar for Reynolds number less than 12 and fully turbulent for Re greater than 10,000, intervening values corresponding to a transition region. Except in the transition range, all points, plotted by Lea showed that a spread of approximately 15 percent occurred in the vicinity Re = 1,000. The range of Reynolds number covered by the tests of Medaugh and Johnson as shown in Fig 1.3 by dashed line AB.

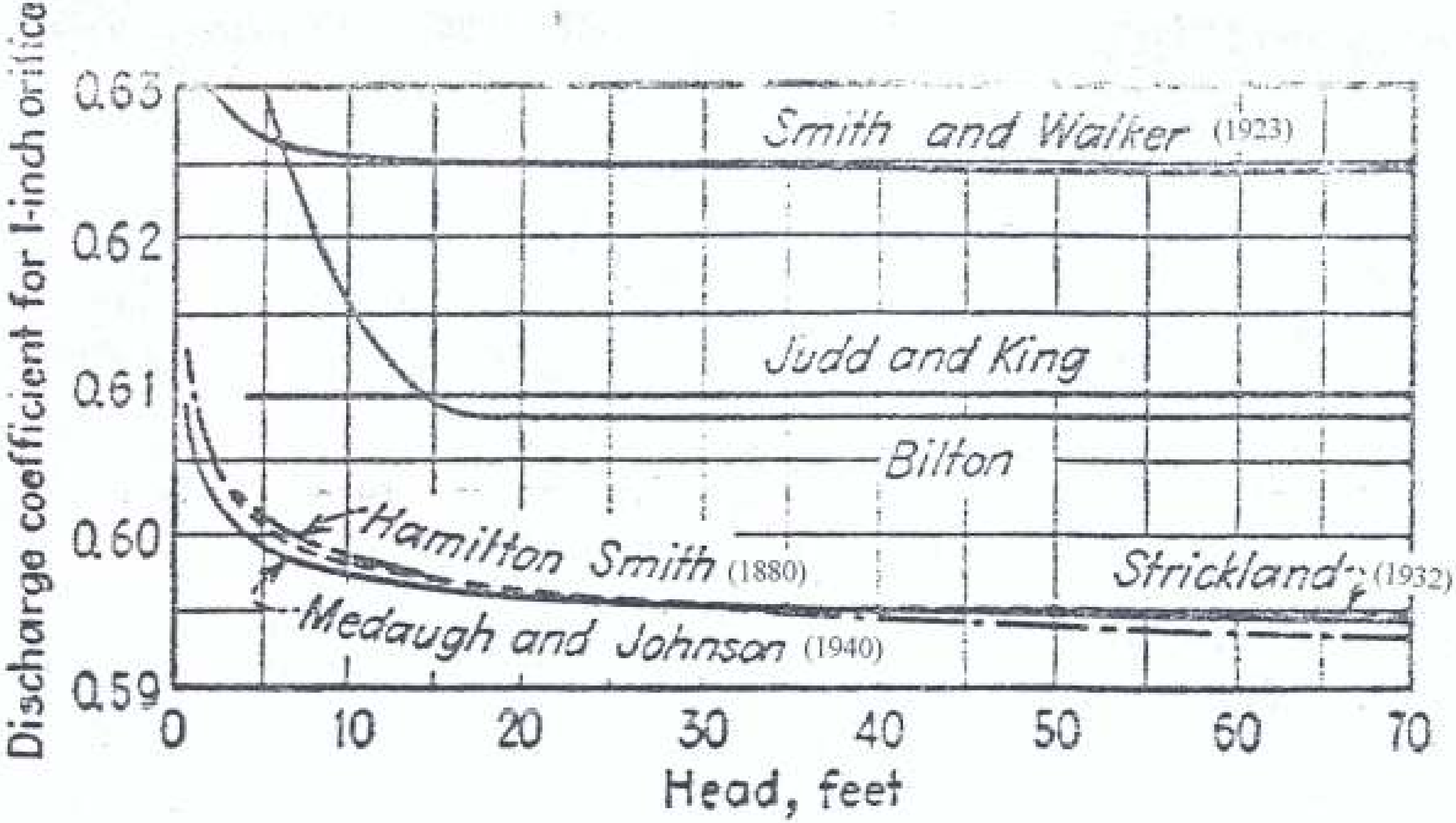
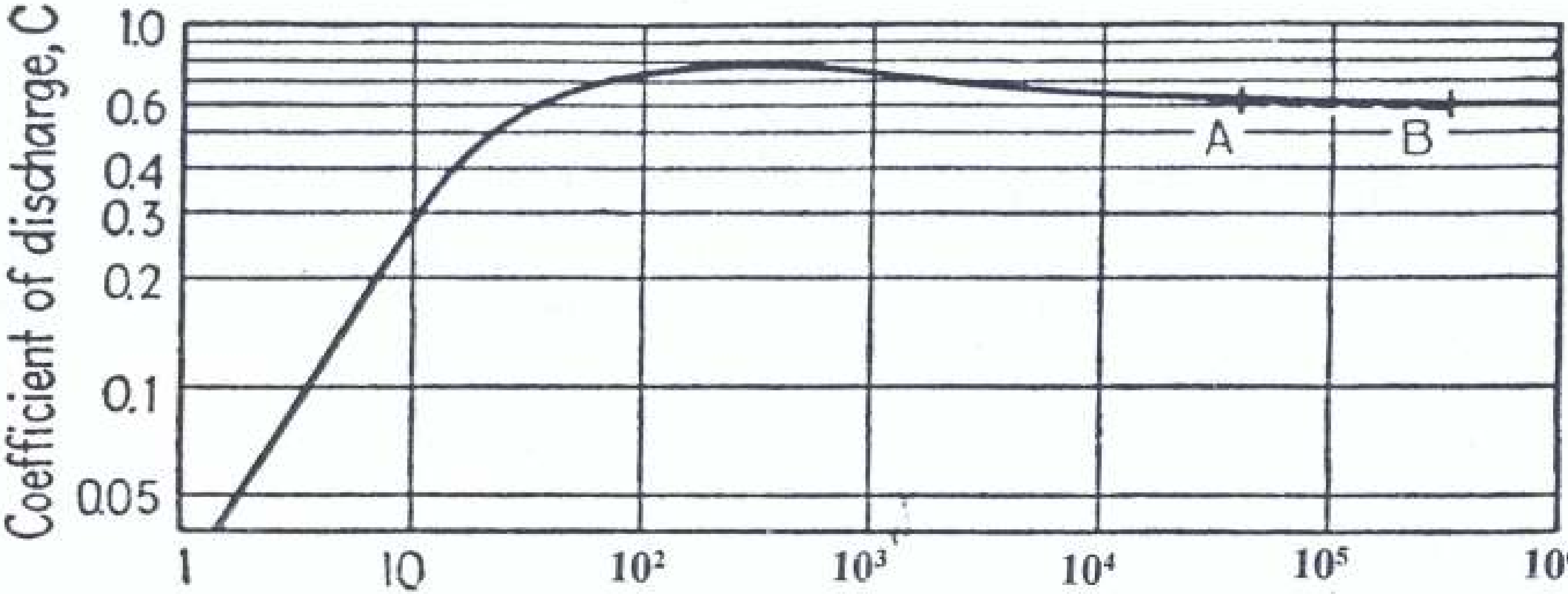


Fig. 1.2: ORIFICE COEFFICIENTS



$$\frac{d\sqrt{gh\rho}}{\mu}$$

Fig. 1.3: CHARACTERISTIC CURVE OF CIRCULAR SHARP-EDGED ORIFICE ACCORDING TO LEA (1842-1912)

Table 1.1: Smith's Coefficients of Discharge for Circular and Square Orifices with Full Contraction (1886)

Diameter of circular orifices, feet							Head feet	Side of square orifices, feet						
0.02	0.04	0.07	0.1	0.2	0.6	1.0		0.02	0.04	0.07	0.1	0.2	0.6	1.0
	0.637	0.624	0.618				0.4		0.643	0.628	0.621			
0.635	0.630	0.618	0.613	0.601	0.593		0.6	0.660	0.636	0.623	0.617	0.605	0.598	
0.648	0.626	0.615	0.610	0.601	0.594	0.590	0.8	0.652	0.631	0.620	0.615	0.605	0.600	0.597
0.644	0.623	0.612	0.608	0.600	0.595	0.591	1	0.648	0.628	0.618	0.613	0.605	0.601	0.590
0.637	0.618	0.608	0.605	0.600	0.596	0.593	1.5	0.641	0.622	0.614	0.610	0.605	0.602	0.601
0.632	0.614	0.606	0.604	0.599	0.597	0.595	2	0.637	0.619	0.612	0.608	0.605	0.604	0.602
0.629	0.612	0.605	0.603	0.599	0.598	0.596	2.5	0.634	0.617	0.610	0.607	0.605	0.604	0.602
0.627	0.611	0.604	0.603	0.599	0.598	0.597	3	0.632	0.616	0.609	0.607	0.605	0.694	0.603
0.623	0.609	0.603	0.602	0.599	0.597	0.598	4	0.628	0.614	0.608	0.606	0.605	0.603	0.602
0.618	0.607	0.602	0.600	0.598	0.597	0.596	6	0.623	0.612	0.607	0.605	0.604	0.603	0.602
0.614	0.605	0.604	0.600	0.598	0.596	0.596	8	0.619	0.610	0.606	0.605	0.604	0.603	0.602
0.611	0.603	0.599	0.598	0.597	0.596	0.595	10	0.616	0.608	0.605	0.604	0.603	0.602	0.601
0.601	0.599	0.597	0.596	0.596	0.596	0.594	20	0.606	0.604	0.602	0.602	0.602	0.601	0.600
0.596	0.595	0.594	0.594	0.594	0.594	0.593	50	0.602	0.601	0.601	0.600	0.600	0.599	0.599
0.593	0.592	0.592	0.592	0.592	0.592	0.592	100	0.599	0.598	0.598	0.598	0.598	0.598	0.598

1.4 SCOPE OF THIS PRESENT WORK

The present work pertains to the design and construction of equipment to study the flow through a sharp-edged circular orifice. The equipment was used to collect data for the establishment of a relationship between the coefficient of discharge C_d and Reynolds number R_e.

CHAPTER TWO

2.0 RELEVANT THEORY

2.1.0 RATIO OF FORCES ARISING IN DYNAMIC SIMILARITY

The forces controlling the behaviour of fluids arise in a number of ways. Not every kind of force enters every problem. Such forces include

* Inertia forces
* Pressure forces
* Forces resulting from the action of viscosity
* Forces acting from outside the fluid-gravity
* Forces due to surface tension
* Elastic forces, i.e. those due to the compressibility of the fluid.

2.1.1 DYNAMIC SIMILARITY OF FLOW WITH VISCOUS FORCES ACTING

There are many instances of flow which are affected only by viscous, pressure and inertia forces. If the fluid is in a full, completely closed conduit, gravity cannot affect the pattern; surface tension has no effect since there is no free surface, and if the velocity is well below the speed of sound in the fluid the compressibility is of no consequence.

Now, for dynamic similarity between two systems, the magnitude ratio of any two forces must be the same at corresponding points of the two systems (and, if the flow is unsteady, at corresponding times also). There are three possible pairs of forces of different kinds but, by convention, the ratio of |inertia force | to |viscous force | is chosen to be the same in each case (see Figure 2.1).

The inertia force acting on a particle of fluid is equal in magnitude to the mass of the particle multiplied by its acceleration. The mass is equal to the density ρ times the volume (and the latter may be taken as proportional to the cube of some length, l which is characteristic of the

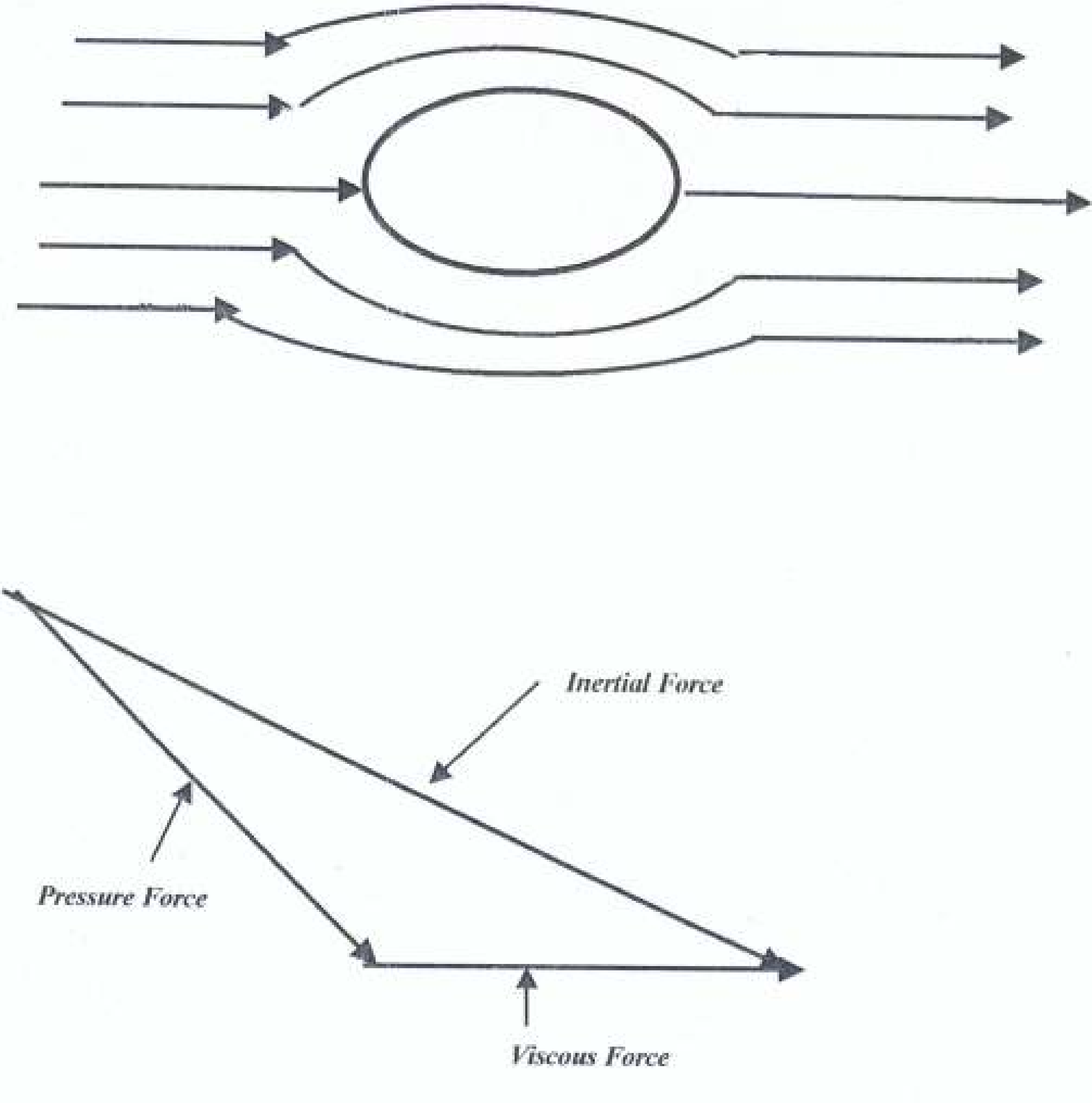


Fig. 2.1 FORCE VECTOR

geometry of the system). The mass is proportional to ρl^3. The acceleration of the particle is the rate at which its

velocity in that direction changes with time and so is proportional in magnitude to some particular velocity divided by some particular interval of time, that is u/t. The time interval, however, may be taken as proportional to chosen characteristic length l divided by the characteristic velocity, so that finally the acceleration may be set proportional to $u \div l/u = u^2/l$. The magnitude of the inertia force is thus proportional to $\rho l^3 u^2/l = \rho l^2 u^2$.

The shear stress resulting from viscosity is given by $\mu\ \Delta u/\Delta y$ which is proportional to $\mu u/l$. The magnitude of the area over which the stress acts is proportional to l^2 and thus the magnitude of the viscous force is proportional to $\mu u/l \times l^2 \quad = \quad \rho u l$.

Consequently, the ratio

$$\frac{|inertiaforce|}{|viscousforce|} isproportionalto \frac{\rho l^2 u^2}{\mu u l} = \frac{\rho l i}{\mu} = \frac{lu}{v} \text{.......................}(2.1)$$

The ratio $\rho l u/\mu$ is known as the Reynolds number R_e.

Reynolds number is of great importance in the theory of hydrodynamic stability and the origin of turbulence Reynolds stresses.

3.1.1.1 SIGNIFICANCE OF REYNOLDS NUMBER

The length l and velocity u appearing in Reynolds number are, as it is written above, quantities chosen as representative ones. Which length and velocity are selected for the purpose naturally affect the numerical value R_e but they do not affect its fundamental significance. In many applications, convention has standardized the length and velocity to be considered; for flow in a circular sharp-edged orifice, for example, the representative length measurement is the diameter and the representative velocity is the mean velocity (that is, rate of flow divided by cross-sectional area); for flow pass a single plane boundary the length is usually measured from the forward edged in the direction of flow, in more unusual applications the length and velocity to be used must be specified.

Reynolds number R_e is essentially a means of comparing one flow with another, and provided the corresponding lengths and velocities are compared in the two flows, the particular choice of length and velocity does not matter. For turbulent flow, the velocity considered is inevitably an average velocity. In such flow the instantaneous velocity at a point in continual fluctuation, but if the flow is "steady", an average of the velocity at a particular point, taken over a sufficient time interval, is constant in magnitude and direction. These average velocities are those which are characteristic of the given pattern of flow and they are readily measurable.

The essential condition of geometry similarity must not be forgotten. It would be quite wrong, for example, to take a value R_e which refers to the onset of turbulence in a circular pipe and to apply this value to predict the onset of turbulence in the flow past a circular sharp edge orifice.

It can be considered that the Reynolds number concerns only the forces due to viscosity and inertia. Inertia forces are present even when the flow as a whole is steady and not changing direction. Individual particles of fluid may not move entirely in straight paths, if the path-line followed by a particle has even the slightest degree of curvature, the particle must undergo an acceleration. When forces arising from other causes (those due, for example, to gravity or the compressibility of the fluid) play an important part in the flow, its general characters may be determined by other criteria. In situations where viscous and inertia forces are the most significant, however, provided that the necessary condition of geometry similarity is met, Reynolds number is the parameter which may be used to compare experimental observations and to assemble even apparently unrelated data into comprehensive laws.

It is evident from the form expression $\rho lu/\mu$ that high values of ρ, l or μ, or a small value of μ, gives a high value of R_e. Conversely, a low value of R_e is brought about by high viscosity, or low-density, low velocity or small size. A high value of R_e indicates that inertia forces dominate the flow while viscous forces play only a small part; when R_e is small in value, the viscous forces have the upper hand and inertia forces take second place. However, for a Reynolds number less than 2100, the flow is *laminar*. When the Reynolds number is greater than 2100, laminar flow is unstable, causing the flow to become *turbulent*.

2.1.2 DYNAMIC SIMILARITY OF FLOW WITH GRAVITY FORCES ACTING

Now consider a flow in which the significant forces are gravity forces, pressure force and inertia forces. A motion of this type is found when a free surface is present or when there is an interface between two immiscible fluids. One example is the flow of liquid in an open channel; another is the wave motion caused by the passage of ship through water, other instances are the flow over weirs and spillway, the flow of jets from orifice into the atmosphere and other outlet structures, or in which relatively large surface wave play an important part with surface ship and beach and habour structures.

The condition for dynamic similarity of flows of this type is that the magnitude ratio of inertia to gravity forces should be the same at corresponding points in the systems being compared. The pressure forces, as in the previous case where viscous forces were involved, are taken care of by the requirement of zero resultant force. The magnitude of the inertia force on a fluid particle is as discussed above, proportional to $\rho l^2 u^2$ where ρ represents the density of the fluid, l a characteristic length and u a characteristic velocity. The gravity force on the particle is its weight, that is $\rho l' g$ which is proportional to $\rho l^3 g$ where g represents the weight per unit mass and V is the volume. Consequently the ratio

$$\frac{|inertia force|}{|gravity force|} is proportional to \frac{\rho l^3 u^2}{\rho l^2 g} = \frac{u^2}{lg} \qquad (2.2)$$

In practice, it is often more convenient to use the square root of this ratio so as to have the first power of the velocity. The ratio $u\,(lg)^{1/2}$ is known as the Froude Number after William Froude (1810 -79), a pioneer in the study of naval architecture, who first introduced it. Some writers have termed the square of this as Froude number, but the definition Froude number $F = u/(lg)^{1/2}$.

The Froude number is thus a significant parameter in determining that part of a ship's resistance which is due to the formation of surface waves.

2.2. FLUID FLOWS CONCEPTS AND MEASUREMENT

The kinematics of fluid deals with space time relationships for fluids in motion. In the Lagrangian method of describing the fluid motion, one is concerned to trace the path of the individual fluid particles (elements) with the passage of time. The co-ordinates of a particle A (x,y,z) at time t_1 (Fig 2.2) are dependent on its initial co-ordinates (a,b,c) at the instant t_0 and can be written as functions of a, b, c and t, i.e

$$x = \phi_1 (a,b,c,t)$$

$$y = \phi_2 (a,b,c,t) \qquad (2.3)$$

$$z = \phi_3 (a,b,c,t)$$

The path traced by the particles over a period of time is known as the path-line. In Lagrangian method, it is difficult to describe the motion of individual particles of a flow field with time. More appropriate for describing the fluid motion is to know the flow characteristics such as velocity and pressure, of a particle at a chosen point in the flow field at any particular time. Such a description of fluid flow is known as Eulerian method.

In any flow field, the velocity is the most important characteristic to be identified at any point. The velocity vector at a point in the flow field is a function of space (s) and time (t) and can be resolved into U, V and W components, representing velocities in x, y, and z directions respectively. These components are functions of x, y, z and t and are written as

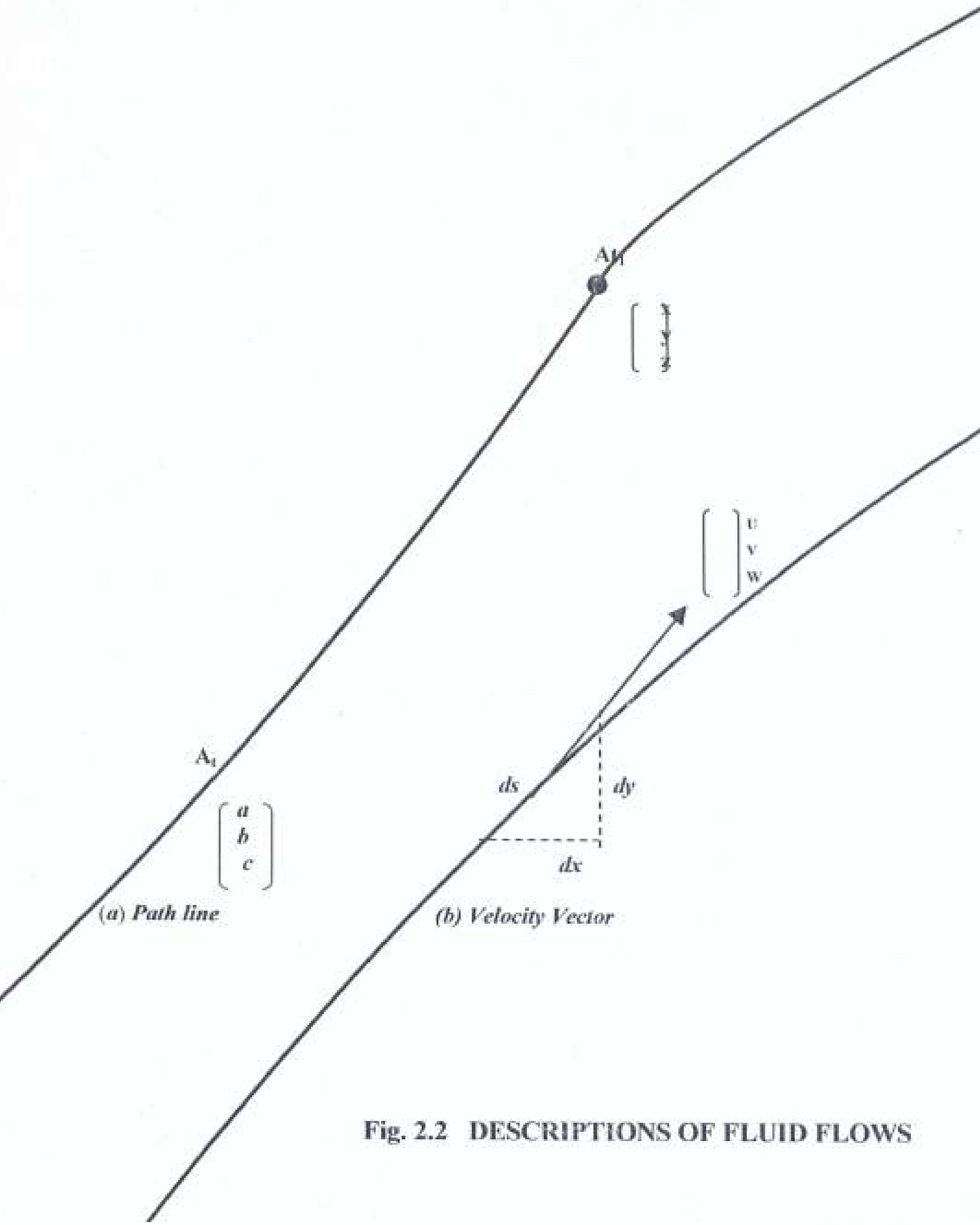


Fig. 2.2 DESCRIPTIONS OF FLUID FLOWS

$$U = f_1(x, y, z, t)$$

$$V = f_2(x, y, z, t) \qquad (2.4)$$

$$W = f_3(x, y, z, t)$$

defining the vector v at each point in space, at any instant t.

Lagrangian (or material coordinate) is a system of coordinates in which fluid parcels are identified for all time by assigning their coordinates which do not vary with time. Examples of such coordinates are:

- The value of any properties of fluid conserved in the motion or
- More generally the position in space of the parcels at some arbitrary selected moment.

Subsequent position in space of the parcels are then the dependent variables f (t, and the Lagrangian coordinate).

Eulerian coordinate system is any system of coordinates in which properties of a fluid are assigned to points in space at each given time without attempt to identify individual fluid from one time to the next.

2.2.1 ACCELERATION OF A FLUID PARTICLE

In general, the velocity of a fluid particle is a function of both position and time. As the particle moves from, say point A to point B, its velocity changes for two reasons. One is that a particle at B has a different velocity from the particle at A, even at the same instant of time. The other reason is that during the time the given particle moves from A to B the velocity at B changes. If B is at only a small distance s from A the total increase in velocity due to the passing of a time interval ∂t:

$$\partial U = \frac{\partial U}{\partial s}\partial s + \frac{\partial U}{\partial t}\partial t$$

and so, in the limit, as $\partial t \rightarrow 0$, the acceleration, a_s in the direction of flow is given by:

$$a_s = \frac{dU}{dt} = \frac{\partial U}{\partial s}\frac{ds}{dt} + \frac{\partial U}{\partial t}$$

or, since $\frac{ds}{dt} = U$,

$$a_s = \frac{dU}{dt} = U\frac{\partial U}{\partial s} + \frac{\partial U}{\partial t} \qquad (2.5)$$

The total rate of increase dU/dt for a given particle is often termed the substantial acceleration. The term $\partial U/\partial t$ represents only the local or temporal acceleration, i.e. the rate of increase of velocity with respect to time at a particular point in the flow. The term $U(\partial U/\partial s)$ is known as the addictive acceleration, i.e. the rate of increase of velocity due to the particle's change of position. Although in steady flow, $\partial U/\partial t$ is zero, the addictive acceleration is not necessarily zero and so the substantial acceleration is not necessarily zero.

2.2.2 STREAMLINES AND FLOW PATTERNS

Often it is desired to construct lines in the flow field to indicate the speed and direction of flow. Such a construction is called a ***flow pattern***, and the lines, called ***streamlines***, are continuous curves traced tangentially to the velocity vector at each point in flow field. Consequently, a tangent to the curve at any point along the streamline gives us the direction of the velocity vector at that point in flow field. For example, consider a flow of water from a slot in the side of tank as shown in Fig 2.3.

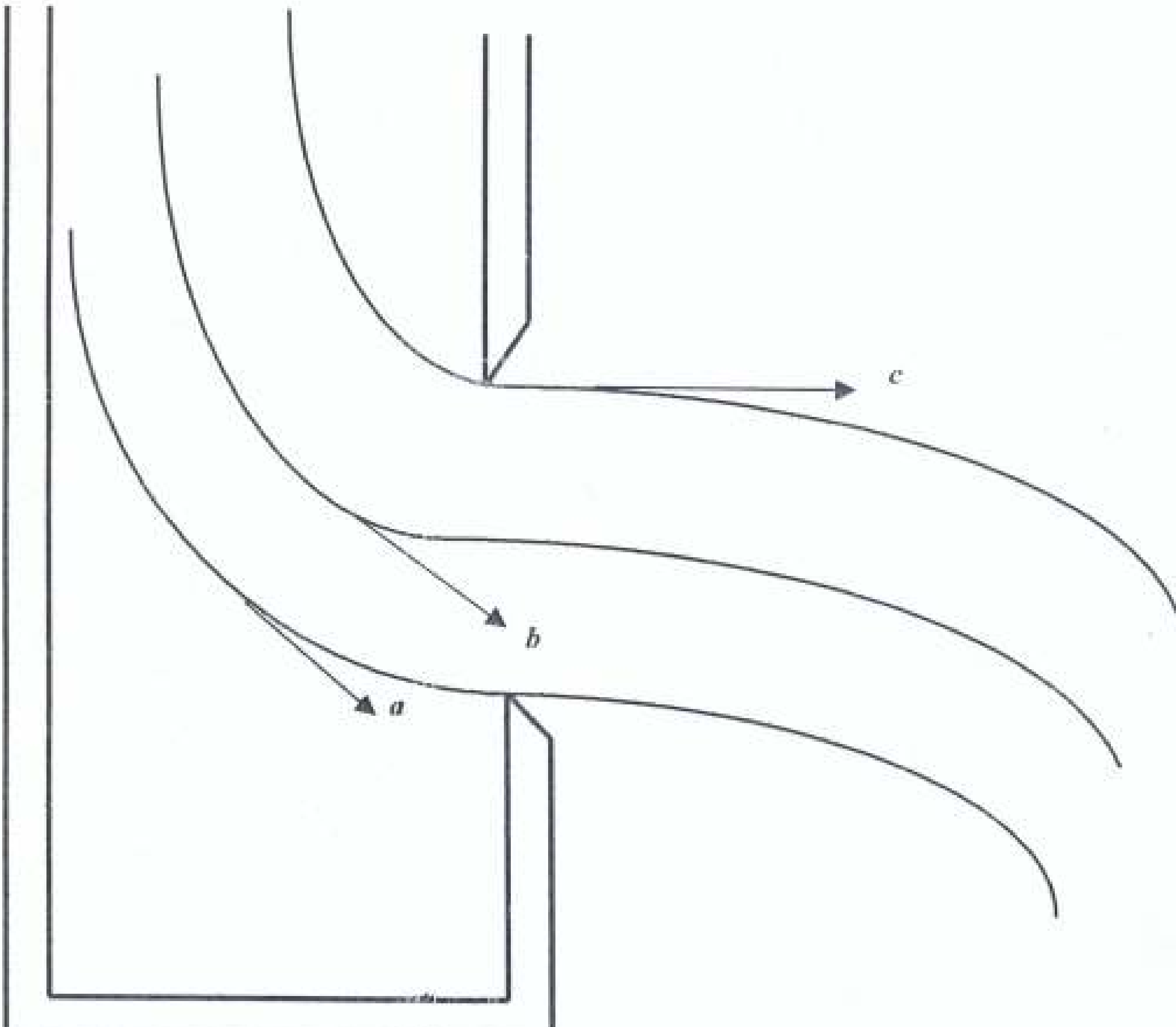


Fig. 2.3 FLOW FROM A SLOT

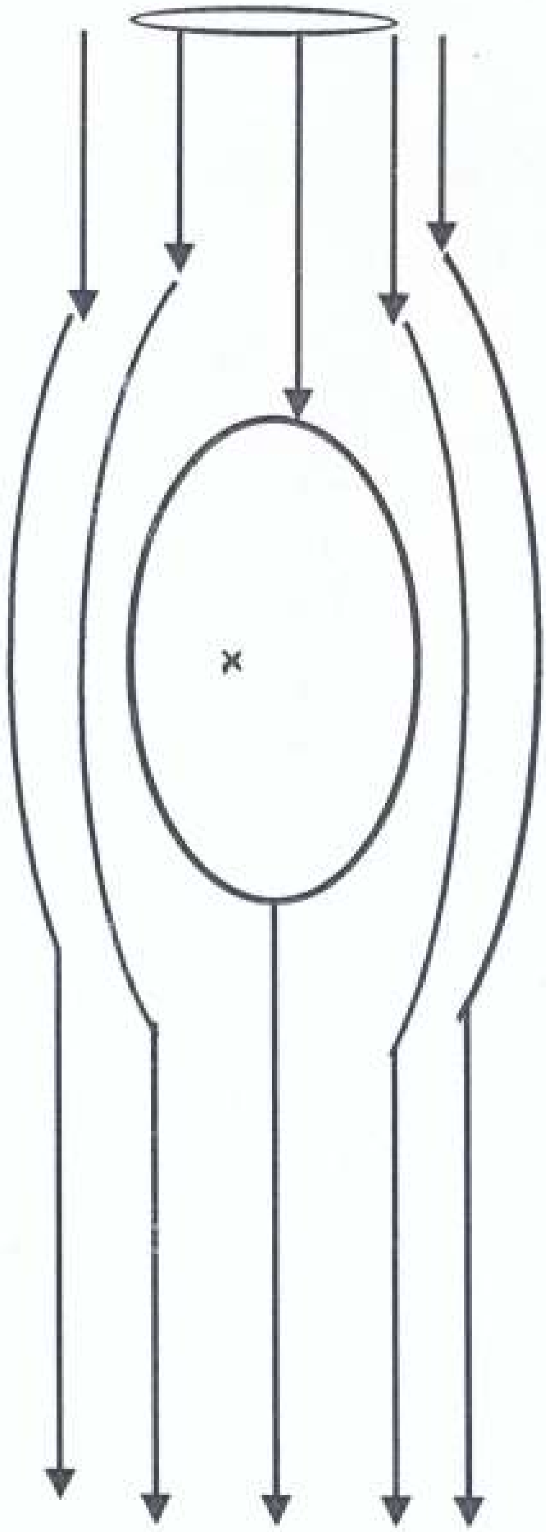


Fig. 2.4 THE STAGNATION POINT

The velocity vectors have been sketched at three different positions, a, b, and c. One can see that the flow pattern is a very effective way of illustrating the geometry of fluid flow. In Fig 2.3. it may be noted that two outer streamlines that bound the free jet also follow the walls inside the tank. It can be observed that all velocity vector of the flow adjacent to the boundaries must be parallel to the boundary. Therefore, all streamlines directly adjacent to the wall are also parallel and actually follow the contour of the wall.

Whenever a fluid flow occurs around a body, part of it will go to one side and part to the other side. The streamline that follows the flow division (that divides on the up-streamside of the body and joins again on the downstream side) is called the dividing streamline. Also at the point of division the velocity will be zero. This point X is called the Stagnation Point (see Fig 2.4)

2.3. TYPES OF FLUID FLOW

In general, the parameters such as velocity, pressure and density, which describe the behaviour of a fluid, are not constant in a particular set of circumstances. They may vary from one point to another, from one instant to another, or they may vary with both position and time.

2.3.1 STEADY AND UNSTEADY FLOW

A steady flow is defined as that in which various parameters at any point do not change with time. A flow in which changes with time do occur is termed unsteady or non-steady. These conditions of steady or non-steady flow are expressed mathematically as follows.

The acceleration of the flow (from equation 2.5) is given as

$$a_s = \frac{dU}{dt} = U\frac{\partial U}{\partial s} + \frac{\partial U}{\partial t}$$

Therefore, if local acceleration is zero, its steady flow

$$\frac{\partial U}{\partial t} = 0$$

and, if local acceleration is not zero, its unsteady flow.

$$\frac{\partial U}{\partial t} \neq 0$$

where U is the total velocity at a given point on a streamline.

2.3.2 UNIFORM AND NON-UNIFORM FLOW

If at a particular instant the various quantities do not change from point to point over a specified region, then the flow is said to be uniform over that region. If however, changes do occur from one point to another, the flow is said to be non-uniform. These changes with position may be found in the direction of the flow or in directions perpendicular to it. This latter kind of non-uniformity is always encountered near solid boundaries past which the fluid is flowing. This is because all fluids possess viscosity which reduces the relative velocity to zero at a solid boundary. These conditions of uniform and non-uniform flows are expressed as follows

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0..............(non-uniformflow)$$

When ρ is constant.

$$\nabla . \underline{U} = 0 \quad \text{(uniform flow)}$$

Here the density ρ of the fluid changes with time. U is the total velocity at a given point on a streamline.

2.3.3. ROTATIONAL AND IRROTATIONAL FLOWS

If the fluid particles within a flow rotates about any axis, the flow is called rotational and if they do not suffer rotation, the flow is irrotational. The non-uniform velocity distribution of real fluid close to a boundary causes particles to deform with a small degree of rotation whereas, the flow is irrotational if the velocity distribution is uniform across a section of the flow field. Therefore, if a velocity potential exists the motion is said to be irrotational and when a velocity potential does not exist, the motion is called rotational. It can be expressed as

$$\text{Curl } \underline{U} = \nabla \times \underline{U} = 0 \quad \text{(irrotational flow)}$$

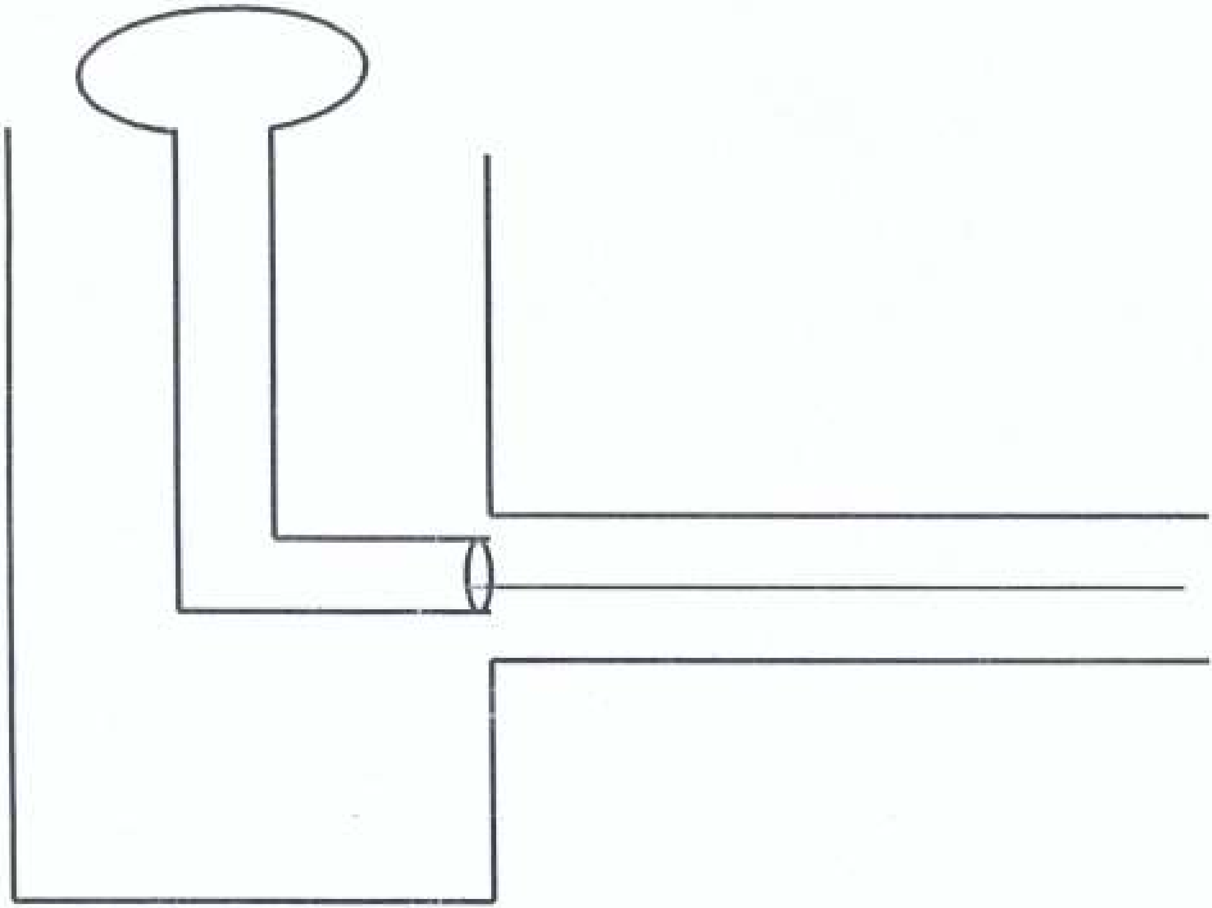
and

$$\text{Curl } \underline{U} = \nabla \times \underline{U} \neq 0 \quad \text{(rotational flow)}$$

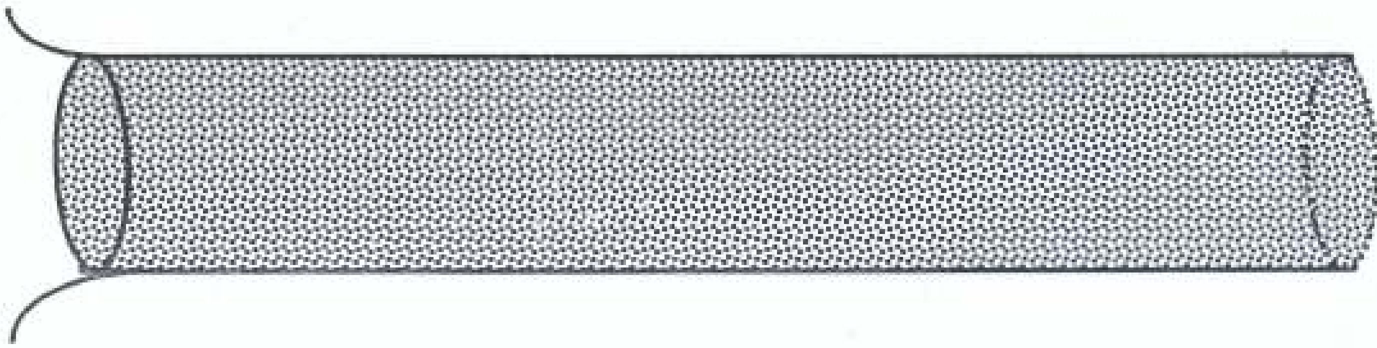
2.3.4 LAMINAR AND TURBULENT FLOW

In a laminar or viscous flow, fluid particles move in parallel layers in one direction or with no cross-currents. This means that in steady motion the particles are travelling in lines, or "filaments", which are straight lines or curves. Osborne Reynolds (1883) did some pioneer work associated with the steady or "streamlined" flow of liquids. The Fig. 2.5 shows the simple piece of apparatus used.

A thin filament of coloured liquid is introduced into a quantity of water flowing through a smooth glass tube. When the velocity of water in the tube is low, it will be seen that the coloured liquid travels in a straight line and appears to be quite separated from the water in the tube. Therefore, for highly viscous liquid, liquid flowing slowly and liquid flowing through in a narrow tube satisfied the necessary conditions for the laminar flow.



(a) *Laminar Flow*



(b) *Turbulent Flow*

Fig. 2.5 OSBHORNE REYNOLD'S EXPERIMENT(1883)

A turbulent flow is characterized by pulsatory cross current velocities. As shown in Fig 2.5, as the velocity of water is gradually increased, a state will be reached when coloured filament at some distance from the jet will break up, and mix with the surrounding water. It can be observed that the more uniform velocity found in turbulent flow is brought about by the interchange of momentum between fast-moving particles near the centre and slower ones nearer the walls. One of the most important practical difference between laminar and turbulent flow is that the dissipation of energy in turbulent flow consequently occurs at a greater rate than in laminar flow, even at the same mean velocity.

2.4.1 STREAMTUBE FLOWRATE OR DISCHARGE, MEAN VELOCITY AND CONTINUITY EQUATION.

A bundle of neighbouring streamline may be imagined which forms a passage through which the fluid flows Fig 2.6, and this passage (not necessarily circular) is known as a stream-tube. A stream-line with a cross-section small enough for the variation of velocity over it to be negligible is sometimes termed a stream-line filament since the stream-line is bounded on all sides by streamlines and since, by definition, there can be no velocity across a streamline, no fluid may enter or leave across a streamline, no fluid may enter or leave a stream-tube through its ends. The entire flow may be imagined to be composed of stream-tubes arranged in some arbitrary pattern. This concept of stream-tube is very useful in deriving the continuity equation.

In considering an elemental stream-tube of the flow (Fig 2.6) we can state that

mass entering the tube per unit time = mass leaving the tube per unit time

Since there is no mass flow across the tube

$$\rho_1 v_1 dA_1 \quad = \quad \rho_2 v_2 dA_2$$

where v_1 and v_2 are the steady average velocities and exist of the elementary stream-tube of cross-sectional area dA_1 and dA_2 and ρ_1 and ρ_2 are corresponding densities of entering and leaving fluid.

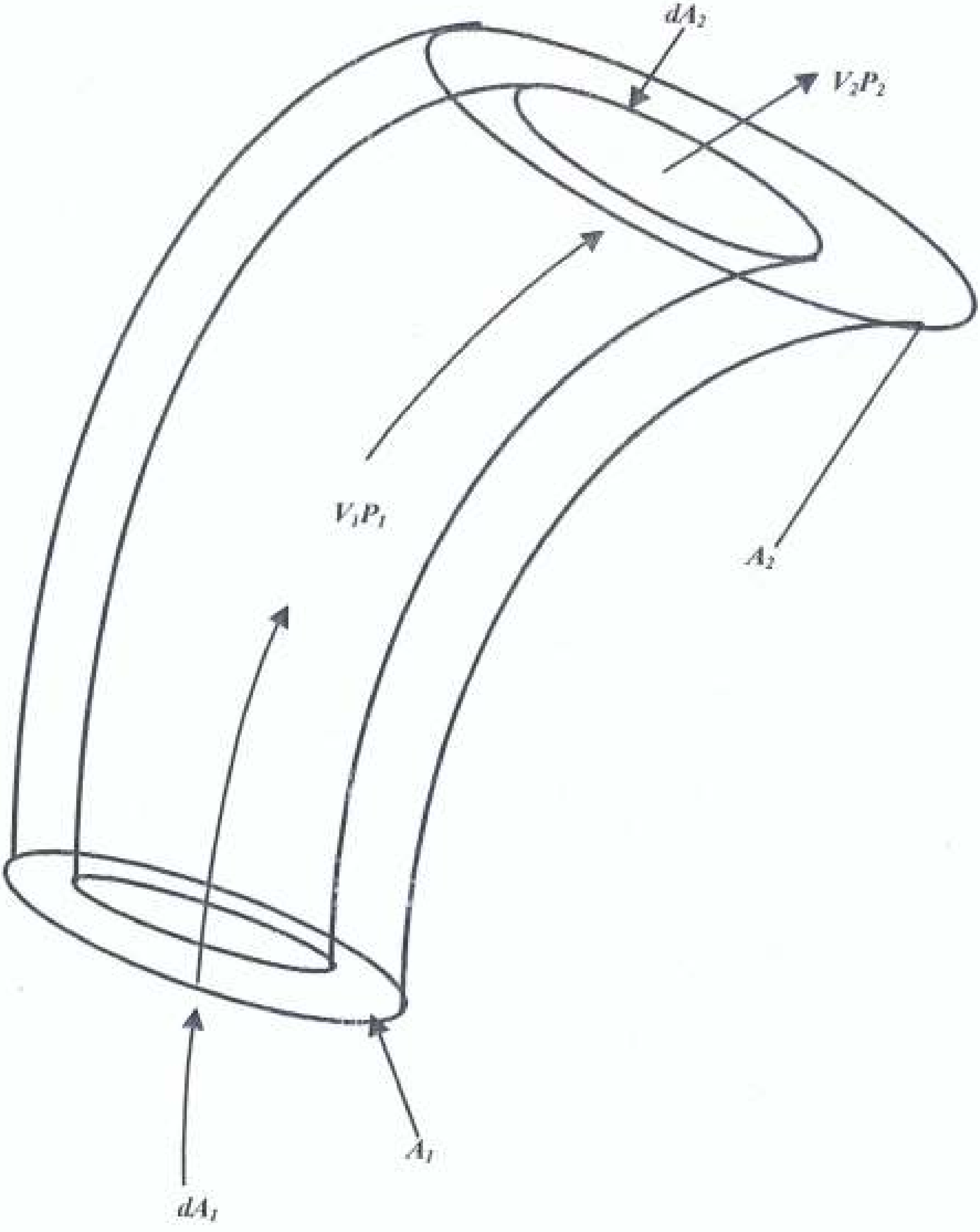


Fig. 2.6 STREAM-TUBE

Therefore, for a collection of such stream-tubes along the flow:

$$\rho_1 v_1 A_1 = \rho_2 v_2 dA_2$$

where ρ_1 and ρ_2 are the average densities of fluid at the entrance and exit and v_1 and v_2 are average velocities over the entire entrance and exit sections of areas A_1 and A_2 of the flow tube.

For incompressible steady flow, equation above reduces to the one dimensional continuity equation

$$A_1 v_1 = A_2 v_2 = q \qquad (2.10)$$

and q is the rate of flow called discharge, expressed as the product of the area and velocity of flow.

The flow rate can further be expressed in terms of density ρ of the fluid, and mass rate of flow through a section is given by

$$q = m/\rho T \qquad (2.11)$$

2.4.2 ENERGY EQUATION FOR AN IDEAL FLUID FLOW

Let us consider a small element of the fluid over which the changes of velocity and pressure are very small. Though small, the element consists of a large number of molecules, so that the characteristic property of a fluid continuum is retained. It is so chosen that it occupies part of a stream-tube of small cross-section (Fig 2.7). The ends of the element are plane and perpendicular to the central streamline, but may be of any geometrical shape.

The forces under investigation are those due to the pressure of the fluid all round the element, and to gravity. Other forces, such as those due to viscosity and surface tension are assumed negligible. Viscosity is less restrictive than it may at first seem. The fluids that are more frequently encountered have small values of viscosity, and except eddies are present, viscous forces are significant only very close to solid boundaries. The behaviour of an actual

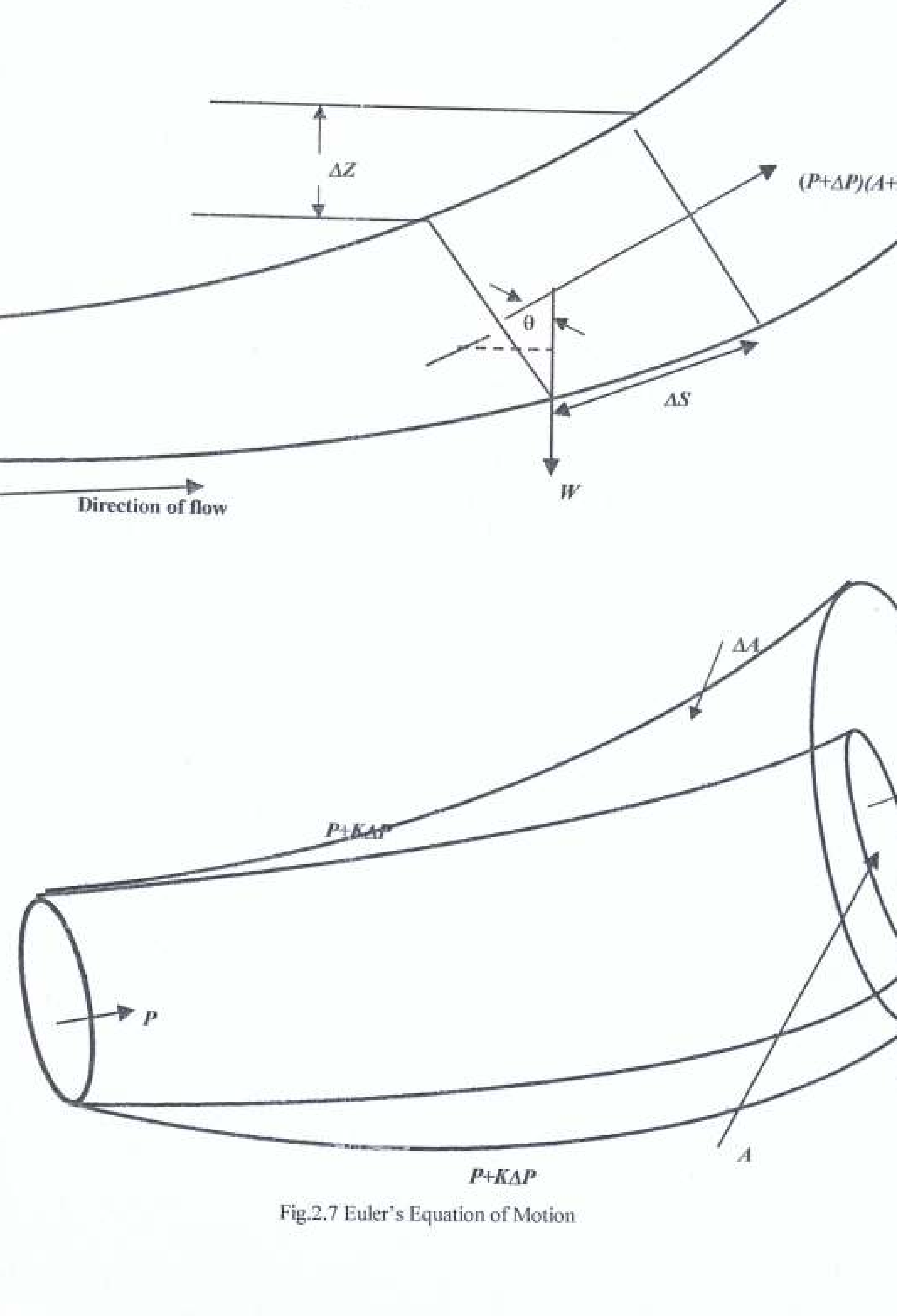


Fig.2.7 Euler's Equation of Motion

fluid is thus often remarkably similar to that of an ideal, inviscid one. In the absence of shear forces, any force acting on a surface is perpendicular to it, whether the surface is that of a solid boundary or that of an adjacent element of fluid. It is also assumed that the flow is steady.

The element is of length δs where s represents the distance measured along the stream-tube in the direction of flow. The length δs is so small that curvature of the streamlines over this distance may be neglected.

The pressure, velocity and so on will, in general, vary with s, but, as the flow is steady, quantities at a particular point do not change with time and so, for the stream-tube considered, each variable may be regarded as a function of s only.

At the upstream end of the element the pressure is P_1 and at the downstream end $P + \delta P$ (which may, of course, be negative). At the sides of the element the pressure varies along he length, but a mean value of $P + k\delta P$ may be assumed where k is a fraction less than unity. The pressure at the upstream end (where cross-sectional area A) results in a force PA on the element in the direction of flow; the pressure at the downstream end (where the cross-sectional Area is A + δA) causes a force

$(P + k\delta P)(A + \delta A)$ on the element in the opposite direction.

The force due to the pressure at the sides of the element also have a component in the flow direction. Since the force in any direction is given by the product of the pressure and the projected area perpendicular to that direction, the net axial force downstream due to the pressure at the sides of the element is $(P + k\delta P)\,\delta A$ since δA is the net area perpendicular to the flow direction.

The weight of the element W, equals $\delta gA\delta s$ (neglecting second orders small quantities) and its components in the direction of motion of $\rho gA\delta s\mathrm{Cos}\Theta$, where ρ represents the density of the fluid and Θ the angle between the vertical and the direction of motion. This in the absence of other forces, such as those due to viscosity, the total force acting on the element in the direction of flow is

$PA - (P + \delta P)(A + \delta A) + (P + K\delta P)\,\delta A - \rho g A\delta s\cos\Theta$ .... (2.12)

When the second order small quantities is neglected, this reduces to

$$-A\delta P - \rho g A\delta s\cos\Theta$$

Since the mass of the element is constant, this net force must, by Newton's second law, equal the mass multiplied by the acceleration in the direction of the force that is $\rho A\delta s\, dU/dt$.

We may write $dz = \delta s\cos\Theta$ where dz represents height above some convenient horizontal datum plane and is the change in level from one end of the element to the other.

Then

$$-A\delta P - \rho g A\delta z = \rho A\delta s\, dU/dt$$

Dividing by $\rho A\delta P$ and taking the $\delta s \rightarrow 0$ we obtain

$$\frac{1}{\rho}\frac{dp}{ds} + \frac{dU}{dt} + g\frac{dZ}{ds} = 0$$

from equation 2.5

$$\frac{dU}{dt} = U\frac{\partial U}{\partial s} + \frac{\partial U}{\partial t}$$

but for steady flow the local acceleration $\partial U/\partial t = 0$ and so $dU/dt = U\,(dU/ds)$ (the full

$$\frac{1}{\rho}\frac{dP}{ds} + U\frac{dU}{ds} + g\frac{dZ}{ds} = 0 \quad \text{...............}(2.14)$$

differential now taking the place of the partial because it is a function of s only for this stream-tube). We then have as the required equation in differential form. This is often referred to as Euler's equation, after the Swiss mathematician Leonard Euler (1707 -83). It cannot be completely integrated with respect to s unless ρ is either constant or a known function of P. For a fluid of constant density, however, the result of integration is

$$\frac{P}{\rho} + \frac{U^2}{2} + gZ = Constant$$

or, if we divide by g and put ρg = W, the specific weight of the fluid,

$$\frac{P}{W} + \frac{U^2}{2g} + Z = Constant \quad \ldots\ldots\ldots\ldots\ldots (2.15)$$

The Bernoulli equation is essentially a statement of the conservation of energy per unit mass which shows that total sum of kinetic energy, potential energy and work done by pressure forces is constant.

This result is usually known as Bernoulli's equation in honour another Swiss mathematician, Daniel Bernoulli's (1700 - 82) who in 1738 published one of the first books on fluid flow.

For Bernoulli's equation to be applicable the fluid must be frictionless (inviscid) and of constant density; and the flow must be steady, and with constant density. The relation holds, in general, only for a single streamline.

2.4.2 THE SIGNIFICANCE OF THE TERMS IN BERNOULLI'S EQUATION

Equation 2.14 states that the sum of three quantities is constant. Consequently, the separate quantities must be interchangeable and thus of the same kind. The second term $U^2/2g$, represents the kinetic energy of a sma!! element. It has dimensional formula $[L^2/T^2] \div [L/T^2] = [L]$.

The third term, Z, also represents energy/mass and corresponds to the energy given to unit mass of the fluid by raising it from datum level to the height Z. This energy may be regarded as gravitational energy of unit mass of fluid that is the ability of the fluid to do work in falling through the height Z.

Similarly the term P/W must also represent the ability of unit mass of the fluid to do work by virtue of its pressure

If the equation is cast in the form

$$\frac{P_1}{W} + \frac{U_1^2}{2g} + Z_1 = \frac{P_2}{W} + \frac{U_2^2}{2g} + Z_2$$

or

$$\frac{P_1 - P_2}{W} + (z_1 - z_2) = \frac{U_2^2 - U_1^2}{2g}$$

It may be interpreted thus: In the steady flow of an ideal, incompressible fluid between two given position (1 and 2) the increase of kinetic energy per unit mass is equal to the work done on unit mass of the fluid by the "pressure forces" and the gravity forces.

Each of the terms P/W, $U^2/2g$ and Z, as we have seen, has the dimensional formula of [energy/mass] = $[ML^2/T^2] \div [ML/T^2] = [L]$. The quantities are therefore usually referred to respectively as "pressure head" (or "static head") "velocity head" and "gravity head" or "elevation", and their sum as the "total head".

2.5 FLOW MEASUREMENT THROUGH THE ORIFICES

Application of continuity, energy and momentum equations to a given system of fluid makes velocity and volume measurement possible.

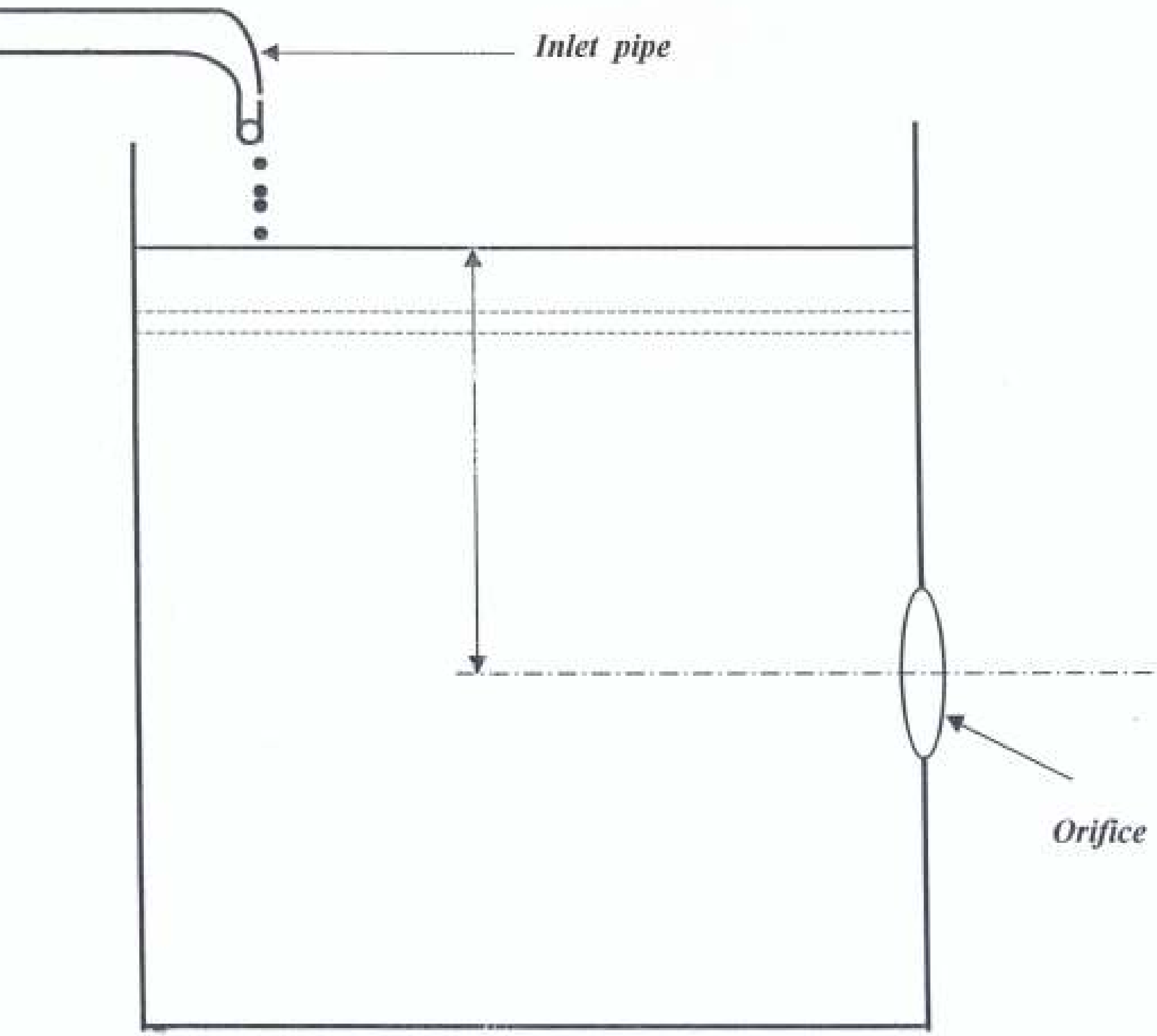


Fig. 2.8 FLOW THROUGH ORIFICE

2.5.1 FLOW THROUGH ORIFICES

An orifice is an opening with a closed perimeter through which fluid flows. An orifice may have any shape, although it is usually round, square, or rectangular.

If a tank is provided with an orifice near to its base, then discharge of liquid from the tank through the orifice will depend upon the velocity of flow through the opening, together with the size of the opening. The velocity of flow through an orifice will be much greater when the tank is full than when it is nearly empty. In fact, the velocity is very closely connected with the head of liquid above the level of the orifice-centre.

Fig 2.8 shows a tank containing water which is maintained at a constant level by means of an inlet pipe. The level of water is h metres above the level of the orifice. With reference to the orifice, water at the free surface possesses potential energy, but, since it is reasonably still, it has no kinetic energy. On the other hand, the water passing through the orifice has no potential energy, but, due to its velocity, it possesses kinetic energy.

energy at free surface = energy at orifice

i.e

potential energy at free surface = kinetic energy a orifice

consider m kg of water Potential energy at surface = mhg

Kinetic energy at orifice = $\frac{1}{2}mU^2$

where u is velocity of water at orifice

Equating these two expression

$$mhg = m\frac{U^2}{2}$$

$$\frac{U^2}{2} = hg$$

$$U = (2hg)^{\frac{1}{2}} \quad \text{...(2.15)}$$

If the orifice is small in comparison with h, the velocity of the jet is constant across the Vena Contracta. Evangelista Torricelli(1608-47) a pupil of Galileo, demonstrated experimentally in 1643 that the velocity with which the jet of a liquid escapes from a small orifice in proportional to the square root of the head above the orifice, and so equation 2.15 is often known as Torricelli's formula. The equation refers to the velocity at the vena contracta: in the plane of the orifice itself neither the pressure nor the velocity is uniform and the average velocity is less than that at the vena centracta.

In the above analysis, friction and surface tension forces have been neglected and so equation 2.15 represents the ideal velocity. The velocity actually attained at the vena contracta is slightly less, and a coefficient of velocity C_v is defined as the

$$\frac{\text{actual (mean) velocity}}{\text{the ideal velocity}}$$

Therefore,

actual (mean) velocity = C_v x the ideal velocity

$$= C_v (2hg)^{1/2} \quad \text{..................... (2.16)}$$

2.5.2 CONTRACTION IN AREA OF JET

It was observed that at a short distance from the orifice, the jet has a minimum diameter. There is a characteristic contraction in jet diameter, and consequently in area of the jet. This contracted diameter which is referred to as the Vena Contracta is shown in Fig 2.9.

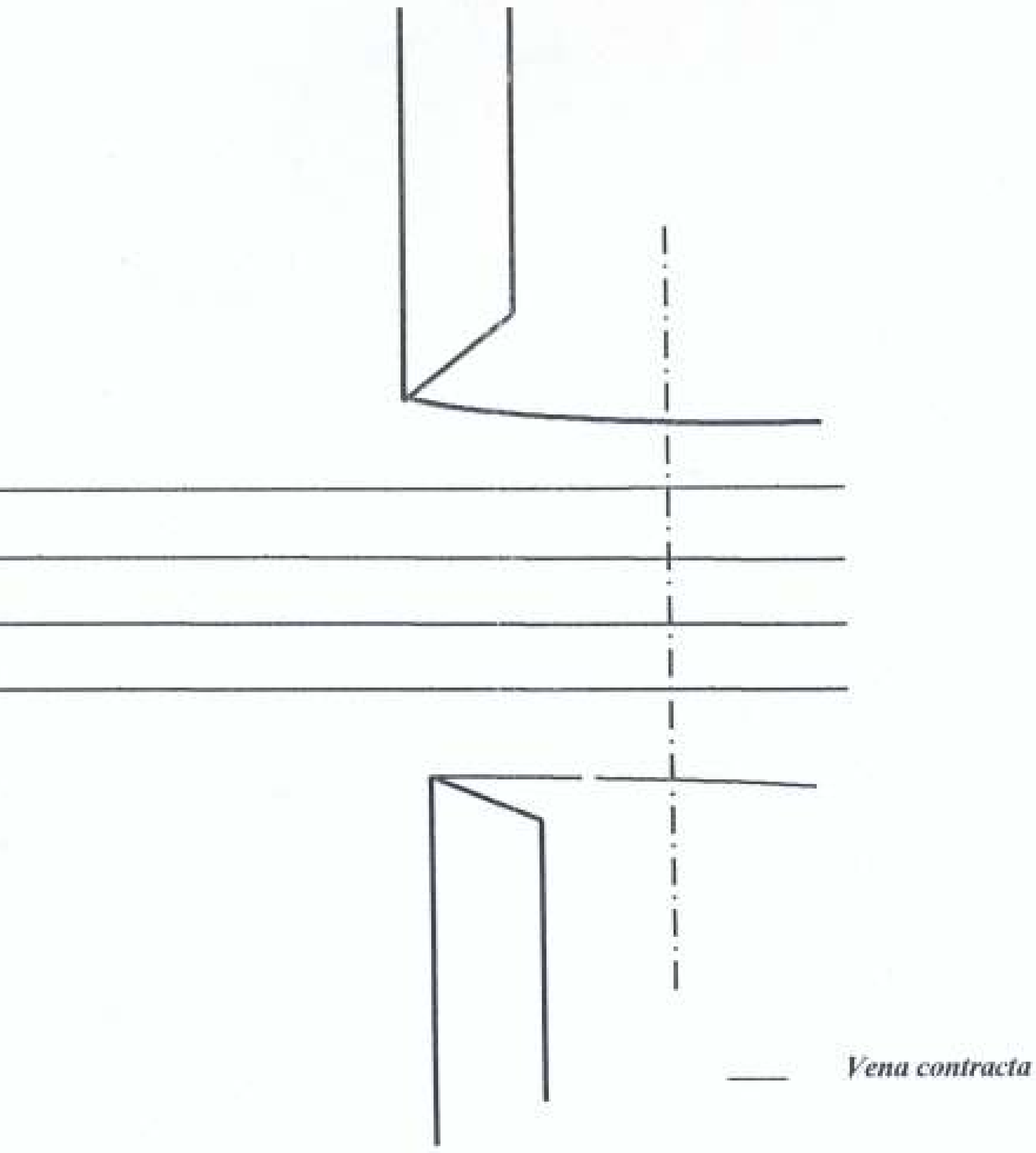


Fig. 2.9 VENA CONTRACTA

$$\text{Coefficient of Contraction, } C_c = \frac{\text{Area of vena contracta}}{\text{area of orifice}}$$

Area of vena contracta $= C_c$ x area of orifice

At the vena-contracta, the velocity is normal to the cross-section of the jet and hence the discharge can be written as

$$\text{Discharge, } q = C_v(2gh)^{1/2} \times C_c \times \text{area of orifice}$$

$$= C_v C_c a_O (2gh)^{1/2}$$

$$= C_d a_O (2gh)^{1/2} \quad \text{.............................. (2.17)}$$

where C_d is the Coefficient of discharge, and is equal to the product of C_v and C_c

Another factor that may contributed to the reduction from the total theoretical value is the friction at orifice.

2.5.3 DETERMINATION OF COEFFICIENT OF DISCHARGE AND COEFFICIENT OF VELOCITY

The value of the coefficient of discharge, C_d, can be obtained in a very simple experimental manner. The actual discharge from the tank can be measured by means of a tank and weighting tank. The weight of water discharged in a given interval of time can be reduced to newtons per second, and then to cubic metre per second.

Theoretical discharge is given by $(2gh)^{1/2}$ x area of orifice; the coefficient of this discharge is given by

$$C_d = \frac{\text{actual (mean) discharge}}{\text{Theoretical discharge}}$$

In order to determine the actual velocity with which the jet leaves the orifice, we measure the vertical distance through which the jet falls in a known horizontal distance (Fig

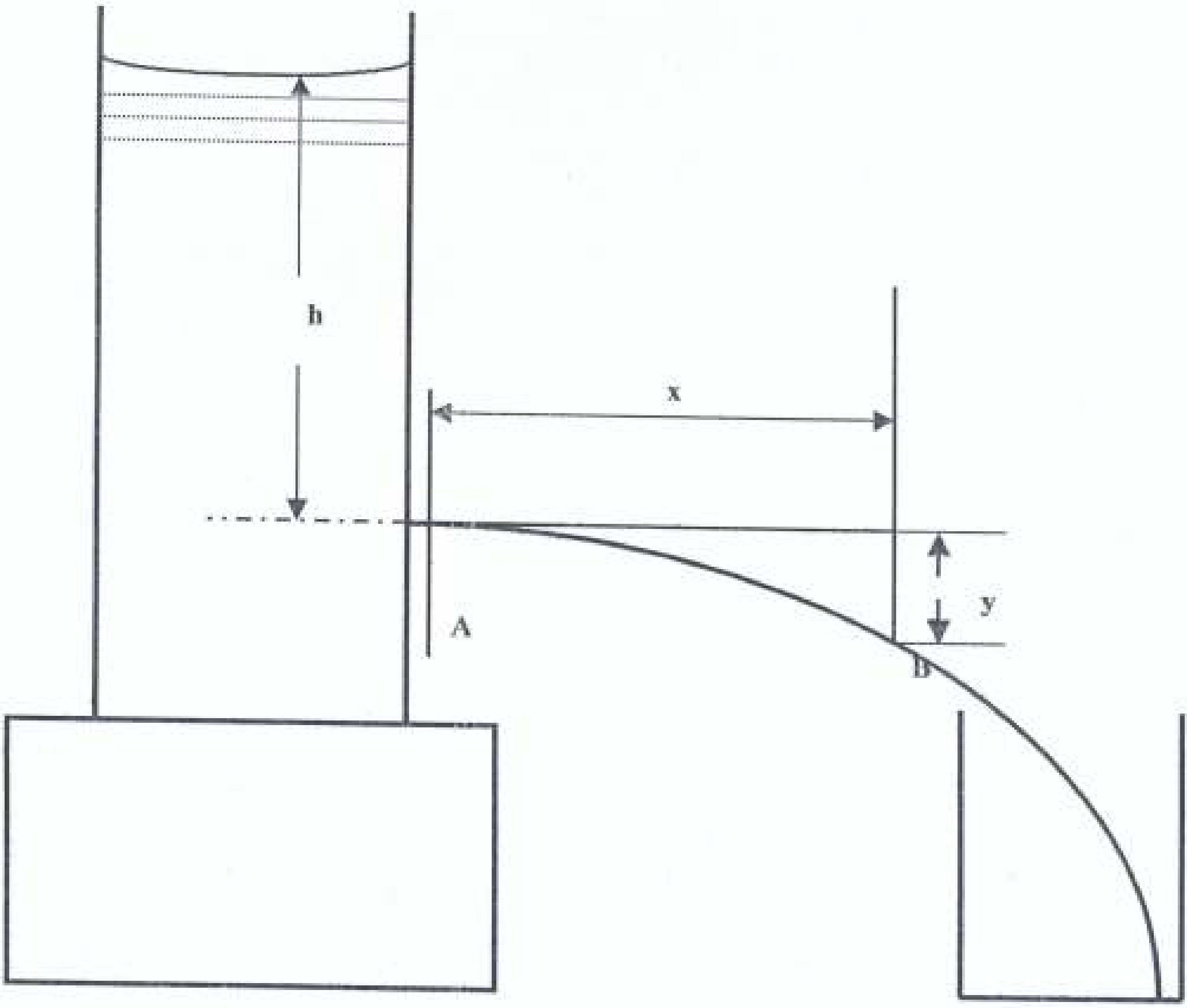


Fig. 2.10 DETERMINATION COEFFICIENT OF DISCHARGE

2.10). Let t seconds be the time the jet takes to move from A to B, U m/s be the actual velocity with which the jet leaves the orifices, x and y the ordinates of the point B taking A as the origin.

Horizontal Motion

Moving with forward velocity U_1 the jet moves a horizontal distance if x metres in t seconds.

$$t = \frac{x}{U}$$

Vertical Motion

Starting with zero initial vertical velocity, the jet moves through a vertical distance y in t seconds, with an acceleration g m/s^2. Using the expression

$S = Ut - ½gt^2$

$S = -y$(negative, since s increases downward)

$U = 0$

$t = x/U$

$-y = 0 - ½gt^2$

$U = x\,(g/2y)^{1/2}$ m/s

Actual velocity $= x(g/2y)^{1/2}$ .... (2.18)

Theoretical velocity $= (2gh)^{1/2}$ .... (2.19)

Coefficient of velocity, $C_v = \dfrac{\text{actual velocity}}{\text{theoretical velocity}}$

$$= \frac{x\sqrt{\dfrac{g}{2y}}}{\sqrt{2gh}} = \frac{x}{2\sqrt{yh}} \quad \text{.................... (2.20)}$$

Hence, if x, y and h are known, the coefficient of velocity can be determined.

2.6. PHYSICAL MEASUREMENT AND ERROR ANALYSIS

The quantity and time method of flow measurement is absolute in the sense of being free from assumptions about the relationship between an observed pressure drop and the mean velocity or effective flow area to which it refers.

Also, once measurements have been made, they must be organized, evaluated and interpreted. Because of human and instrumental limitations, no measurement is absolutely accurate or exact. A measurement or experimental result is of little value if nothing is known about its accuracy.

Errors are not mistakes or blunders since mistakes can be avoided and errors cannot. Errors arise because of limitations and imperfections of apparatus, imperfection of personal judgement, scale limitations of environmental conditions. Since such errors are inevitable, every experimental report should be accompanied by some indication of its reliability.

The International Organization for Legal Metrology (OTML, 1984) defines accuracy as "the closeness of agreement between the result of a measurement and the (conventional) true value of the measurand". We see from the definition that when we measure some physical quantity with an instrument and obtain a numerical value, usually we are concerned with how close this value may be to the "true" value or "best" value. It is first necessary to understand that this so-called true value or best value is, in general unknown and cannot be known, since a perfectly exact definition of the physical quantities to be measured are impossible, the term "true value" or "best value", then, refers to a value that would be obtained if the quantity under consideration were measured by an exemplar method, that is, a method agreed on by experts as being sufficiently accurate for the purposes to which the data ultimately will be put. It can be noticed that at a single measurement point there are three sources of error. The average of many readings might be offset from true value (bias error or systematic error), the reading may be randomly scattered about the offset (random error or precision error), and one reading might fall well outside the majority of the reading (illegitimate error). It is the combination of the first two that establishes the accuracy or quality of the instrument.

Other important terms that one need to consider is high quality instrument for measurement are: Precision which is the degree of resolution or number of significant figures an instrument will give. Reliability is defined as the degree of consistency with which an instrument performs. The accuracy and precision are independent of one another but equally essential.

2.6.1 MEAN, STANDARD DEVIATION, STANDARD ERROR AND PERCENTAGE ERROR

2.6.1.1 MEAN

The best known and most useful "average" is the arithmetic mean usually referred to as the mean; it is calculated by adding all observations and dividing the sum by the total number of observations.

$$\text{Sample mean } x = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \ldots\ldots\ldots\ldots(2.21)$$

2.6.1.2 STANDARD DEVIATION AND STANDARD ERROR

With the presence of random errors in a measured quantity, it is not appropriate to estimate the maximum error but we estimate the random error using the standard deviation. The standard deviation is the quantity which measures the amount of random fluctuations in a value which is the function of deviations of the raw scores from the mean. Thus the standard error of arithmetic mean equals the standard deviation of any single measurement divided by the square root of the number of measurements.

Standard deviation of a set or sample

$$\sigma_s = \sqrt{\frac{\sum v^2}{n-1}} \text{ ....(2.22)}$$

where v is deviation of raw scores from mean.

Therefore, standard error of a set or sample is given as

$$\sigma_m = \sqrt{\frac{\sum v^2}{n(n-1)}} = \frac{\sigma_s}{\sqrt{n}} \ (a) \ \text{or} \ \sigma_m = \frac{x_{max} - x_{min}}{\sqrt{n}} \ (b) \text{ ......(2.23)}$$

2.6.1.3 SIGNIFICANT OF STANDARD DEVIATION AND STANDARD ERROR

Assume that one set of measurements has been made. The arithmetic mean (x) of the several values is calculated, the value best descriptive of the sample. The standard deviation (σ_s) of the sample is also found, which is truly descriptive of the spread and the distribution of the values in the sample. Thus σ_s is descriptive of where the individual values fall within the sample, and it is descriptive of "normality" of their spread.

But to describe the mean value (x) we use the standard error (σ_m). This relies on the value of n also; larger values of n tend to lessen this error of the mean. By carrying the reasoning to its limit, we see that as n approaches infinity, the σ_m approaches zero and the mean of the entire population of such measurements approaches the true value or best value.

Hence the standard error can be seen to be indicative of the nearness of the sample mean to the population mean, thus to true value.

Fig.3.11 shows the areas contained in the various sections of normal distribution curve (probability curve). The abscissa would to σ_s for one sample or set of measurements, but σ_m for a very large sample or for a group of several samples or sets of measurements.

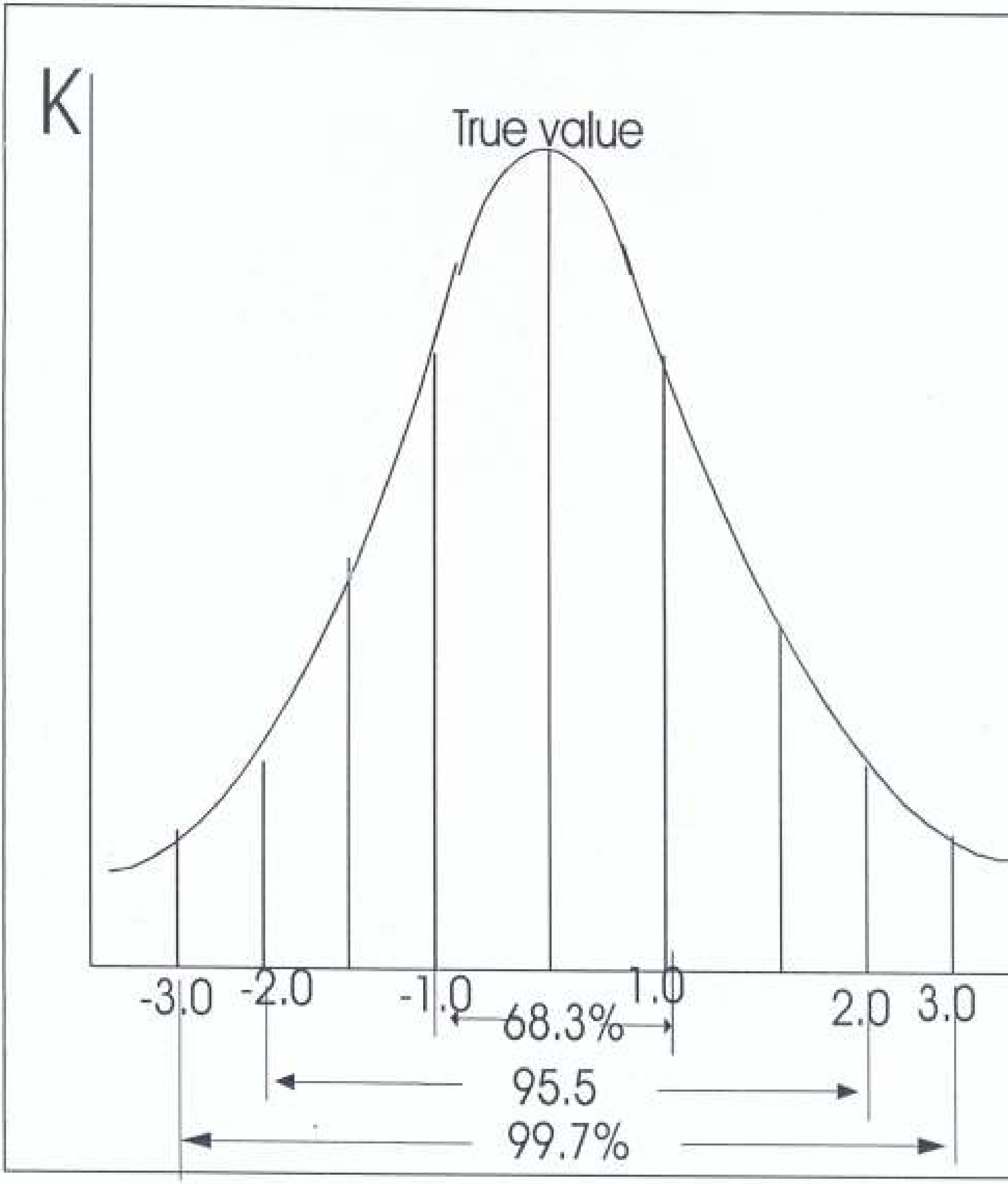


Fig. 2.11 CHARACTERISTIC OF THE NORMAL DISTRIBUTION CURVE

In any event, the significant point is that our level of confidence that the sample mean lies within $\pm\sigma$ of the population mean is 68.3% (only 1 chance in 3 or $^{2}/_{3}$ of lying beyond); within $\pm 2\sigma$, 95.5% (only 1 chance in 20 or $^{19}/_{20}$ of lying beyond); within $\pm 3\sigma$, 99.7% (only 1 chance is 370 or $^{369}/_{370}$) of lying beyond; and so on.

2.6.1.4 PERCENTAGE ERROR

Percentage error is derived from fractional error. Fractional error is the ratio of the error to the true value. When this is expressed in percentage, we have percentage error

$$\text{Percentage error} = \frac{\text{Actual Error}}{\text{Standard Value}} \times 100 \text{ .... (2.24)}$$

2.6.2 BIAS ERRORS/SYSTEMATIC ERRORS

Bias errors do not follow the laws of chance, and no confidence level can be derived from them. There are two types of bias errors, variable and constant.

Typical sources are as follows

Sources of Variable Systematic Error

(i) Progressive wear on the linkage of a recorder (a time-dependent wear function)

(ii) Progressive wear of the orifice edge in a dirty system

(iii) An uncorrected zero shift in a measuring device of the system.

Sources of Constant Systematic Error

(i) An unknown systematic error in a reference standard that is used to calibrate secondary devices.

(ii) A scale used to determine mass but not corrected for liquid or air buoyancy.

(iii) Incorrect measurement of an orifice size.

The Principal Sources of Accountable Error in the System of interest would be

(i) the sight-glass used to measure the head h

(ii) the stop-clock used to time the flow into the weight-tank

(iii) the weights used with the balance.

Other systematic errors would arise from a wrongly-assumed value for the density of the water and on incorrect determination of the mechanical advantage of the balance arm.

Unaccountable errors will arise from mirror fluctuations in the head and possibly from irregularity in the operation of the clock mechanism. There will also be personal errors in reading the clock and the head scale and in recognising the instant of equilibrium of the balance.

In the determination of the initial velocity of the jet from its co-ordinates, the largest error will be in locating the probe at the centre of the jet, as shown in Fig 2.12. The insertion of the probe into the jet will interfere with the behaviour of the system, but unless it is very large or close to the orifice, this is unlikely to be significant.

2.7.0 COMBINATION OF COMPONENTS ERRORS IN OVERALL SYSTEM-ACCURACY CALCULATION

When using direct measurement values to compute final results, such as to reach indirect measurements, it is necessary to guard against carrying excessive random error into the result. Thus it is necessary to know the size of the error in the result after arithmetic operation have been performed on any measured value or on the mean of several values.

2.7.1 ADDITION OF VALUES CONTAINING ERRORS

According to the laws of probability when quantities are added, each containing an error their sum contains an error equal to their square sum contains an error equal to their square of the error of the added quantities.

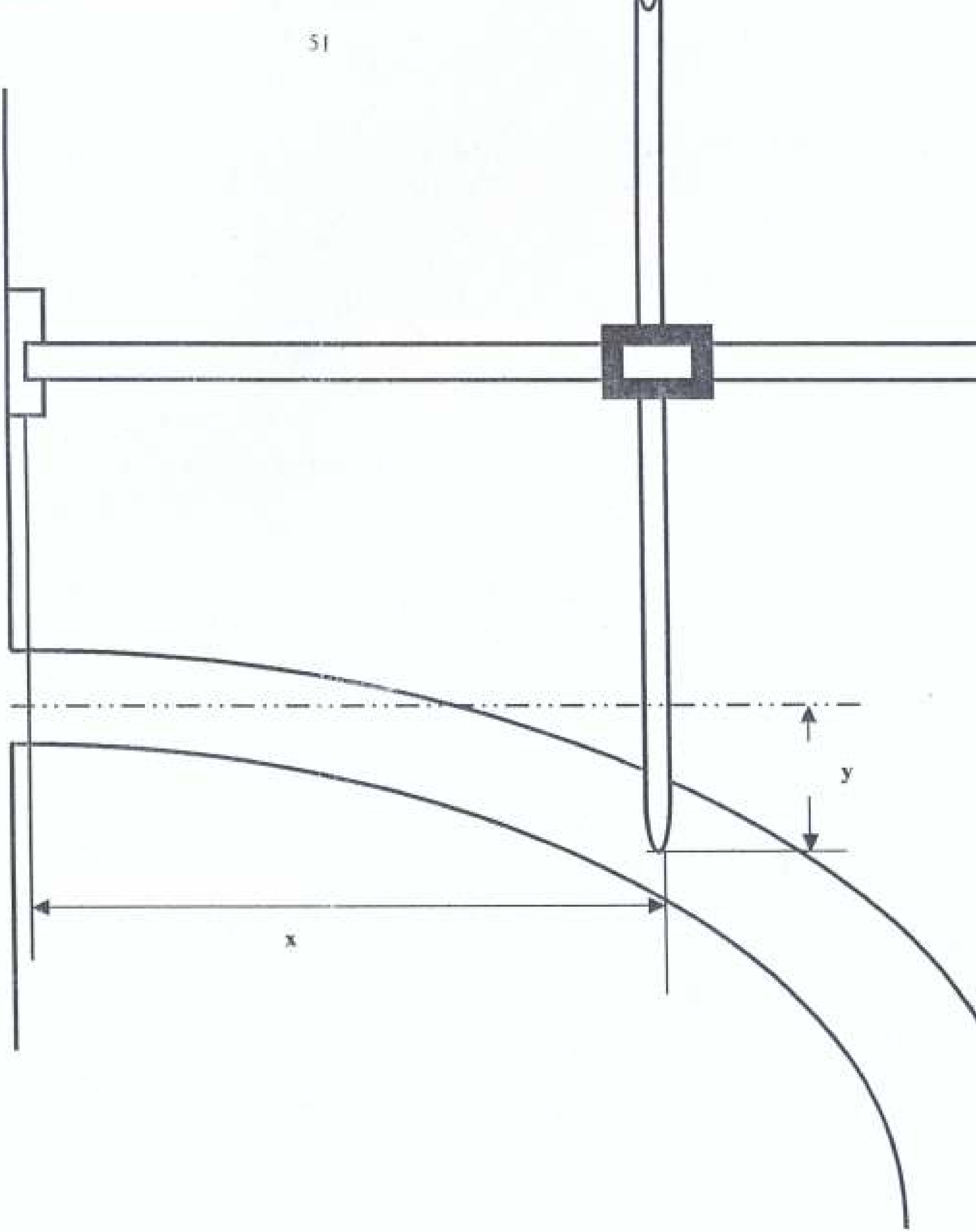


Fig.2.12 PROBE FOR OBTAINING CO-ORDINATES OF JET TRAJECTOR

thus $(A \pm E_A) + (B \pm E_B)$

therefore,

$$E_{SUM} = \sqrt{(E_A^2 + E_B^2)} \;..........(2.26)$$

2.7.2 MULTIPLICATION OF VALUES CONTAINING ERRORS.

In multiplying two quantities that contain errors, this is the general form

$$(A \pm E_A) + (B \pm E_B) = AB \pm E_AB \pm E_BA \pm E_AE_B$$

To neglect the last term is not serious since both factors are extremely small.

Hence, it may be seen (from the pervious rule for addition of errors) that

$$E_{product} = \pm \{(E_A/A)^2 + (E_B/B)^2\}^{1/2} \;........................(2.27)$$

2.7.3 DIVISION OF VALUES CONTAINING ERRORS

In dividing two quantities that contain errors, the general form is;

$$(A \pm E_A) + (B \pm E_B)$$

the error is given as

$$E_{quotient} = \Delta \;\{(E_A/A)^2 + (E_B/B)^2\}^{1/2} \;........................(2.28)$$

CHAPTER THREE

3.1 DESCRIPTION OF APPARATUS

The apparatus which was designed and constructed for the experiment is shown in Fig. 3.1. It consists of a measuring tank, weir tank, weighing container, sump and a simple beam balance. These parts are arranged on a wooden table frame of dimension 146cm x 310cm x 97cm. The beam balance is attached to the frame.

The measuring tank is 15cm x 15cm x 121cm and consists of a constant head device located at the centre of the base of the measuring tank of 2cm diameter and contains a pipe that can be moved up and down to change the level of the water inside the measuring tank. There is an inlet valve for the supply of water into the measuring tank. It is located at 5cm x 5cm at one corner of the base of the measuring tank . The orifice is at one of the faces shown in Fig 3.1. Opposite the orifice there is a glass tube connected to the measuring tank which is graduated for easy measurement of the level of the water inside the measuring-tank. The probe for obtaining the co-ordinates x and y of jet trajectory is 1cm above the orifice position.

The weir tank with dimensions 30cm x 65cm x 15cm has a rectangular opening at the top. This is where the water from the orifice falls into. There is another opening at one of 30cm x 15cm face with a "V" shape through which water flows into the weighing container.

A four litre container is used as weighing container.

An open tank is used as sump where the excess water from the measuring tank via the constant head tube is poured. The weighing container has a direct link with it. After each reading has been taken, the water from it goes into the sump.

There is a simple beam balance which is attached as shown in Fig 3.1. At the shorter arm of the beam, the weighing container is mounted and balance at the other arm of the beam, including the weight hanger and an indicator/balance index. All these attachments made

Fig 3.1a APPARATUS FOR SPECIMEN

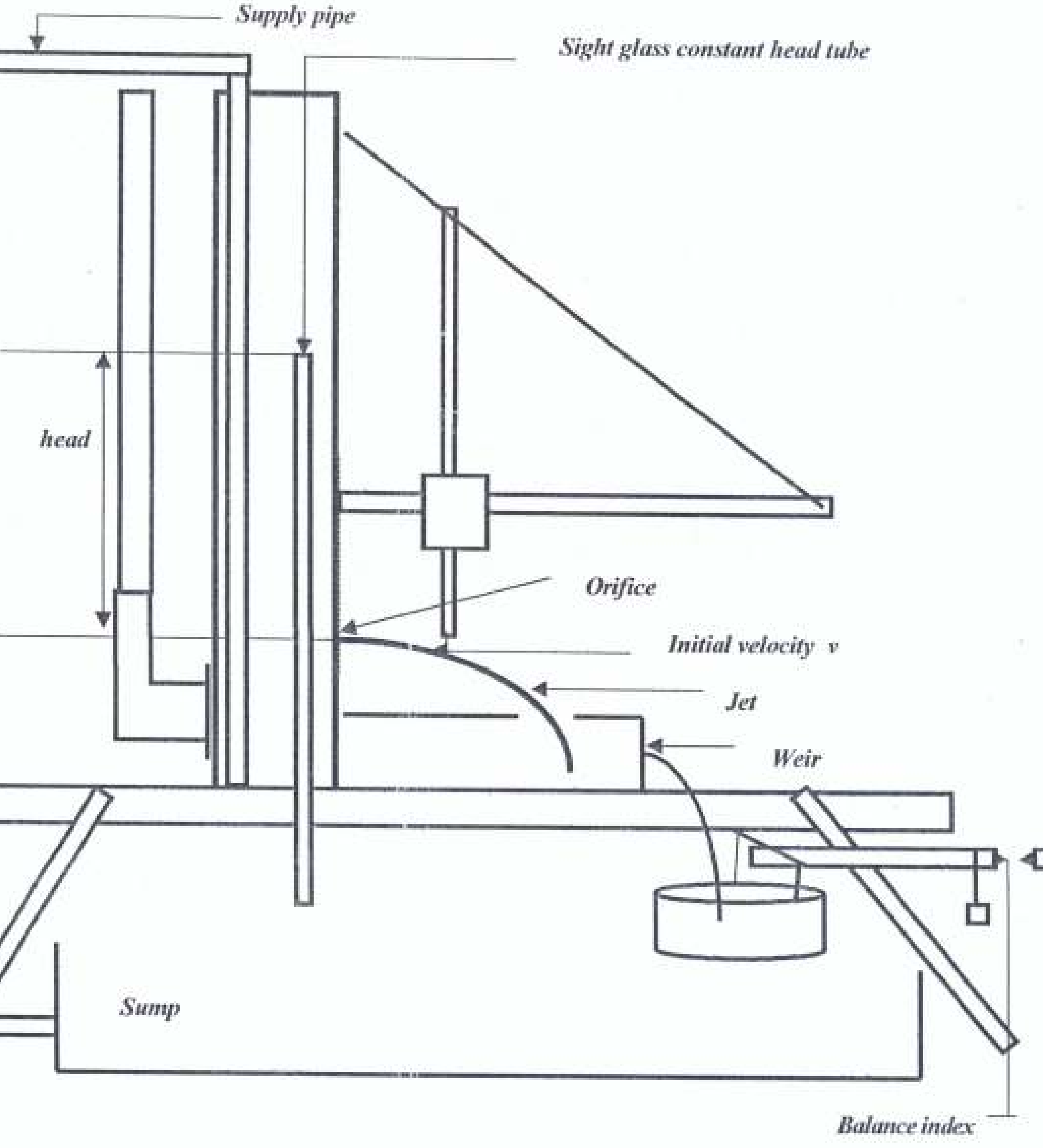


Fig.3.1b APPARATUS FOR SPECIMEN

it more convenient to measure the period of time for the water entering into the weighing container at a fixed or pre-determined quantity of water.

3.2 PHYSICAL MEASUREMENT DEVICES

3.2.1 MEASUREMENT LENGTH

Static calibration devices can be satisfactorily accomplished by using ordinary measuring steel type or standard measuring gauge as the standard. When used directly to measure the length of the transducer, these devices usually are suitable to read to the nearest 0.005mm. If smaller increments are necessary, lever arrangement (about a 100:1 ratio is fairly easy to achieve) or wedge-type mechanism (100:1) can be employed. A unique mechanical gauge of high sensitivity, may be also useful in measuring small length down to a few thousandth of a mm.

3.2.2 MASS MEASUREMENT

An unknown mass of water from orifice can be determined by a simple system - analytical balance or beam balance. In Fig 3.2, beam balance, while simple in principle, requires careful design and operation to realize its maximum performance.

The beam is designed so that the centre of mass is only slightly below the knife-edge pivot and thus barely in stable equilibrium. This makes the beam deflection a very sensitive indicator of inbalance. A known mass, m_o (which is comparable to the mass m) is suspended at a convenient distance, x cm from the central balanced point, O. The unknown mass m_y is then suspended from the other side and its position is varied until the beam once again balance. Let y cm be its distance from O. From the principle of moments, m_y can be calculated. This approach to measure weight is fast but requires that the pointer at one arm must be accurately marked at the balance point. This tends to vary with the load on the balance, because of

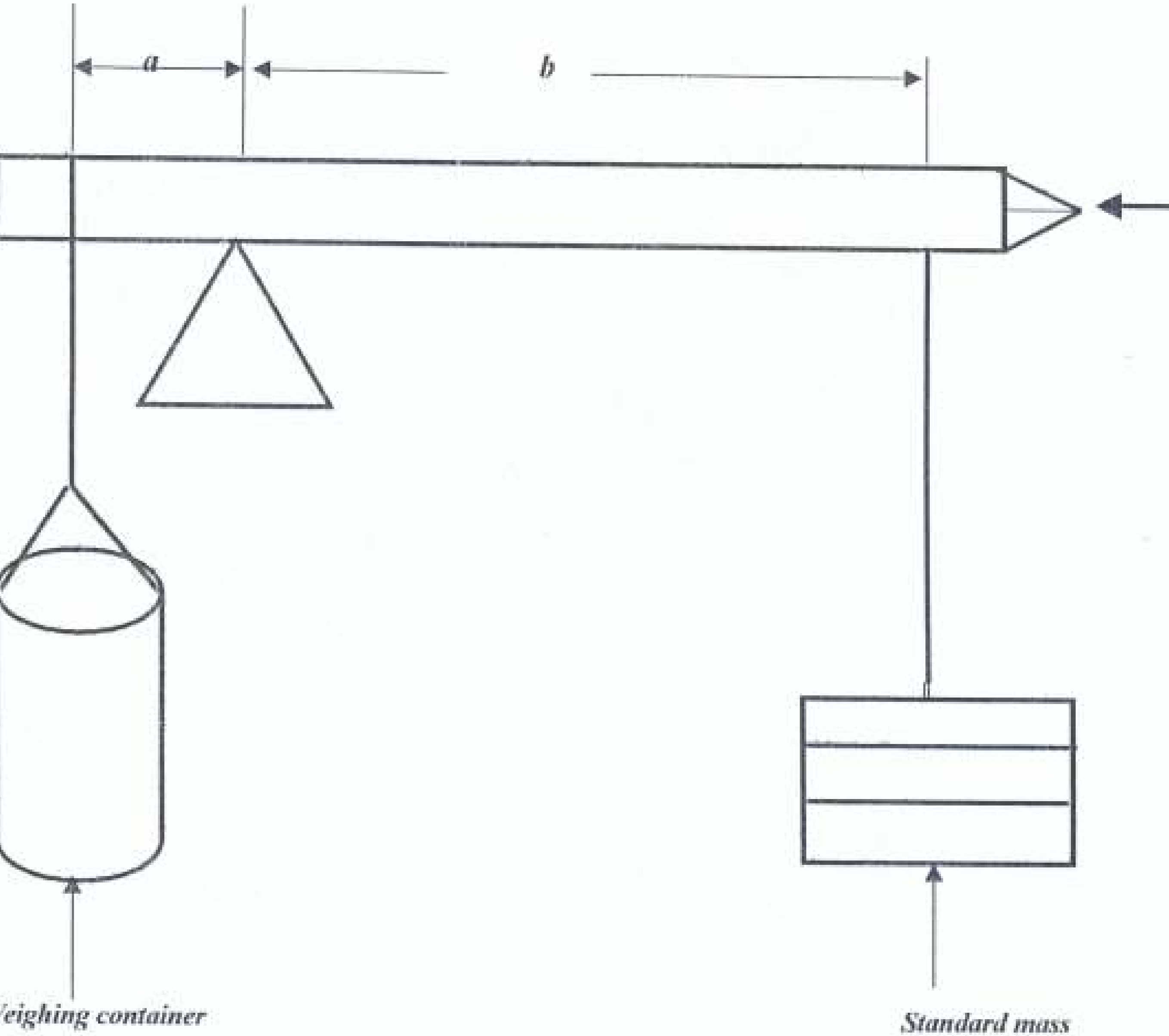


Fig. 3.2 BEAM BALANCE OR ANALYTICAL BALANCE

deformation of knife edges. However, a careful design of the system can keep this at a minimum. For highly accurate measurements the buoyant force due to the immersion of the standard mass in air must be taken into account.

3.2.3 TIME MEASUREMENT

In the laboratory, time is usually measured by a stop watch or stop clock. Some stop watches can measure time of the order of 0.1s, while stop clock are not as accurate, having an accuracy between 0.2s and 0.5s. Nowadays quartz clock and watches give a high degree of accuracy.

3.3 CALIBRATION

It is necessary to calibrate the head scale, the probe, the weights and the weighing mechanism. A standard steel tape of 12cm is carefully paste to the glass tube for easy reading for the height of water inside the measuring tank. No further check was done due to lack of facilities.

A carefully graduated paper was inserted in the tube. This is used for vertical components measuring devices.

A 70cm ruler is used for horizontal measurement.

3.3.1 Weighing and Weighing Mechanism

There were no facilities for calibrating each of the balance weights directly. They were checked for internal consistency with a simple beam balance and digital weighing system, and a set of standard chemical balance weights. The discrimination of balance in these tests is found to be less 2g, so it is concluded that the errors in the balance weights are negligible for the present purposes.

A rough measurement with a metre rule shows the mechanical advantage of weighing tank balance arm, b\a, to be about 3.21. Difficulties in measuring the lengths a and b of the balance arm (see Fig 3.2) dictate a direct check of this property. Employing the weights from the standard set, whose largest weight is 1kg, does this. The following results are obtained with measurements made as the load in the tank increased.

Weight in Container	Weight in Hanger	Mechanical Advantage
660g + 6g	200g	3.330
1660 + 7g	500g	3.327

The discrimination of the balance being of the order of a gramme. The value obtained with the lowest load is not very reliable so as further check is made using a 2-litre graduated cylinder. A large beaker is attached to the hanger and water poured into the tank until balance is just achieved. Exactly 1000ml of water is then poured into the beaker and 3300ml into the tank, bringing it almost into balance. Exact balance is finally achieved by the addition of another 28ml of water from the burette. Hence a further estimate of the mechanical advantage is 3328/1000 = 3.328. Hence it was decided to use the value 3.33 in the experimental calculations.

3.4 THE LEVEL OF ACCURACY IN CONSTANT HEAD h

The principal source of error is expected to be the uncertainty in measuring the head h, so that effectiveness of the constant head device (in Fig 3.3) has to be examined first. When the inlet valve and constant-head tube had been set and the equipment given time to settle, it was noted that the water level in the sight glass shows no significant variation at low heads (of the order of 10 cm) probably due to the steadiness, test-readings are therefore, taken near the middle head of one metre. Several readings at this setting, taken at interval of one minute, appear in the first column Table 3.1 below. The least count of the scale is 1 minute.

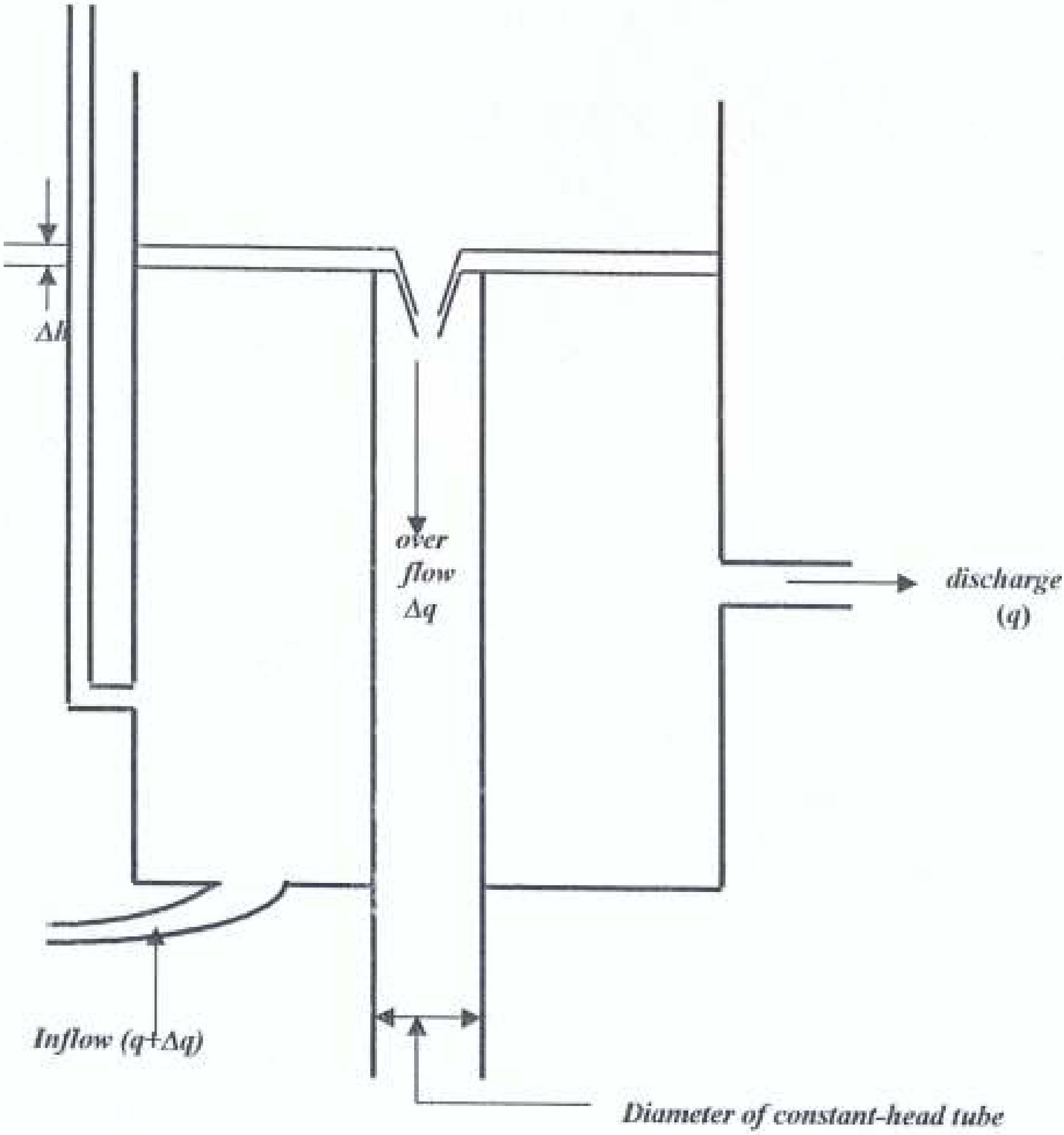


Fig. 3.3 CONSTANT-HEAD TUBE

The manipulation of the results was worked through in detail as shown table 3-1. Suppose that a provisional mean m = 509.0mm is chosen.

Mean Value h = $m + \sum[(h-m)/10]$

$= 509 + (-0.5)/10 = 509 - 0.05$

$= 508.95$mm

Standard deviation S^2 = 10.75

$S^2 = 10.75 - 0.50$

$S = 3.563$

Then the best estimate of the standard deviation of all possible measurements at this setting would, from equation 3.24 be about 3.56. The uncertainty of this estimate is fairly low, since we have obtained it from a sample of ten readings, but we used it to make a rough estimate of how many readings we ought to take at each test to give us a reasonable precision in the mean value h. If we assumed that the true standard deviation of the population σ to be about 0.45% of the mean, we see that the standard deviation of the means of samples 2,3,4 .... readings would be 0.32%, 0.25%, 0.23% etc. From statistical table, we would expect 99% of such means to be within 3 times these values of the true mean. Then, bearing in mind the uncertainty of our estimate of σ, it was decided to make three readings at each test level, which should ensure that the mean values of h were nearly all within about 1% of the true values.

3.5 PLANNING AND PERFORMING THE EXPERIMENT

The principal independent variable is the head h, which will be varied over a range from 10 to 100cm. Equation 2.19 shows that the flow-rate q is expected to be proportional to the square root of h, so that if we are to have a distribution of points, they should correspond to a roughly equal incremental in $h^{1/2}$. The co-ordinates of the jet are obtained at the same time.

TABLE 3.1

VARIATION OF HEAD NEAR A SETTING OF h = 510.0mm

Time interval of 1 minute	h/mm	h -m/mm	$(h-m)^2/mm^2$
1	508.5	-0.5	0.25
2	510.0	+1.0	1.00
3	509.0	0.0	0.00
4	510.0	+1.0	1.00
5	508.5	-0.5	0.25
6	509.0	+0.5	0.25
7	507.0	-1.0	1.00
8	511.0	+2.0	4.00
9	507.0	-2.0	4.00
10	508	-1.0	1.00

Mean Value h $= m + \Sigma(h-m)/10$

$= 509 + -0.5/10 = 509 - 0.05$

$= 508.95mm$

3.5.1 RELATIONSHIP BETWEEN THE NON-DIMENSIONAL GROUPS AND COEFFICIENT OF DISCHARGE

In the relatively simple problem, with only one independent variable, we cannot expect much reduction in effort if the variables are grouped into numerics. However, by expressing the results in non-dimensional form, we can make them much more general, with applications to other orifices and other fluids. If we suppose that the dimensions of the orifice

tank are unimportant and that we may neglect minor forces such as surface tension, the important variables would be

- q the flow rate
- h the head
- d the orifice diameter
- g the gravitational acceleration
- ρ the density of the fluid
- μ the viscosity of the fluid

These six quantities form three independent numerics, between which we expect a functional relationship such as

$$\frac{q}{d^2 g^{1/2} h^{1/2}} = f\left(\frac{h}{d}, \frac{g^{1/2} h^{1/2} d\rho}{\mu}\right) \qquad (3.1)$$

The reasons for choosing these particular groupings will be understood when we recall that $2^{1/2} g^{1/2} h^{1/2}$ is the ideal jet velocity, V, as in equation 3.15.

Then the group

$$\frac{q}{d^2 g^{\frac{1}{2}} h^{\frac{1}{2}}}$$

is seen to be a form of the coefficient of discharge

$$c_d = \frac{4q}{\pi d^2 v}$$

wanting only the numerical instant. The third numeric

$$\frac{g^{\frac{1}{2}} h^{\frac{1}{2}} d\rho}{\mu}$$

is seen to be a form of

$$\frac{vd\rho}{\mu}$$

that is, the Reynolds number of the flow through the orifice, and h/d is simply a shape parameter, though it might be noted in passing that it is a form of

$$\frac{g^{\frac{1}{2}}}{g^{\frac{1}{2}}}(\frac{h}{d})^{\frac{1}{2}} = \frac{v}{g^{\frac{1}{2}} d^{\frac{1}{2}}}$$

which is the Froude number for the flow. Then we expect to be able to present the results in general form

$$C_d = f(\frac{h}{d}, R_e) \ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

Equation 3.2 has much greater utility than the simple relation between q and h which applied to our orifice and our fluid only. However, to obtain the form equation 3.2 by a classical test plan we would have to keep $^h/_d$ constant at several levels whilst we establish the relation between C_d and R_e for each. This means that we would have to change the orifice with each change in h. It has been established by Rouse (1946) that the dependence of C_d on $^h/_d$ is very weak unless $^h/_d$ is quite small, say less than 3. As we are not intending work below $^h/_d = 10$, we can expect to find a single relationship between C_d and R_e, using one orifice size, which will be valid for all others as long as h/d is not less than 3.

3.5.2 DATA COLLECTION

The collection of the necessary data is in three concurrent steps. Three readings of the height, h were taken during each run by adjusting three sets of the coordinates x, and y of the jet trajectory are measured on each run while the rate of flow of water into the weighing tank is also measured. The constant-head tube is adjusted between runs and the water temperature is measured from time to time.

3.5.3 DATA PROCESSING

The raw data of Table 3-2 is now converted into the required quantities. We recall the following information:

the stopwatch is slow by 0.003%

the balance arm ratio is not 3.21, but 3.33

the density of water at 27.5°C relative to air is 0.99636g/cm^3 (Kaye, et.al 1973).

TABLE 3.2: PRIMARY RESULTS FOR SPECIMEN PROBLEM 1cm ORIFICE

Head	Mean Time to collect 4335g of water t (s)	Co-ordinates of x (cm)	y	Temperature °C
8.40		6.00	1.20	
8.40		8.00	2.10	
8.40	69.10	10.00	2.50	27.5
18.50		11.80	3.60	
18.50		16.80	3.80	
18.50	46.30	22.00	6.70	27.5
21.90		14.10	2.60	
22.10		19.10	3.80	
22.00	42.70	30.00	9.50	27.5
30.25		18.00	3.95	
29.90		22.90	4.55	
30.00	36.80	29.30	7.40	27.5
41.30		21.80	2.95	
41.45		28.00	5.00	
41.40	31.70	37.80	9.30	27.5
50.65		23.00	2.80	
50.50		28.00	3.20	
50.40	28.70	33.00	6.80	27.5
60.50		29.00	3.50	
60.70		35.00	5.10	
60.60	26.20	40.00	7.15	27.5
70.60		25.00	2.35	
70.70		32.00	3.25	
70.65	24.40	41.00	5.80	27.5
81.30		19.30	1.20	
81.25		32.00	3.30	
81.20	22.75	42.00	5.70	27.5

Then from equation 3.1,

$$q = \frac{M}{\rho T}$$

$$= \frac{4335}{0.99636} x \frac{3.33}{3.21} x \frac{1}{1.0037T} (cm^3/s)$$

$$q = \frac{4375.38}{T} (cm^3/s) .... 3.3$$

Coefficient of discharge C_d

When q is plotted against $h^{1/2}$, as in Fig 3.4, It is shown that q is not directly proportional

$$R_e = \frac{vd}{\nu}$$

to $h^{1/2}$. The conclusion is that the coefficient of discharge C_d in not a constant, and following the reasoning leading to equation 3.2. We now calculate C_d and Reynolds numbers R_e. The diameter of the orifice is 0.998cm. The kinematic viscosity υ of water at about 27.5°C required in the definition is taken to be 0.008414 cm²/s (Kaye, et. al. 1973). Values of the velocity obtained using equation 2.16 are shown in Table 3.3.

Figure 3.5 shows the resulting plot of Reynolds number, R_e against C_d.

To establish an empirical relationship to represent the data, attempts are made to rectify the curve of Fig 3.5. It is found that plotting the logarithms of both variables fails to eliminate the curvature. The impression that C_d is becoming constant at large R_e suggests plotting the difference between C_d and its asymptotic value. From the Fig 3.5 the asymptotic is taken to be 0.590, plotting logarithms of the quantities yields by table 3-5 a straight line, as in Fig 3.6.

Therefore, the equation of the line is given by:

$$\log_{10}[C_d - 0.590]^{-1} = -0.746102 \log_{10} R_e + 1.84468$$

$$C_d = 0.590 + \frac{69.98}{R_e^{0.746}} \quad \text{..........3.4}$$

$$(14{,}000 < R_e < 48{,}000)$$

These results indicate that the discharge through the orifice by about 40% relative to ideal value given by equation 2.19. Previous experience indicates that this is mainly due to the reduction in area of the jet as shown in Fig 2.9.

Coefficient of Velocity

The data calculated in the trajectory of the jet (table 3-2) to see how much of the reduction might be due to a loss in velocity, represented by the coefficient of velocity, given by equation 2.20 shown on the Table 3-6.

There is no trend in these values, suggesting that they merely show sampling variations about a mean. It is then calculated that

mean C_v = $0.9559 \approx 0.956$

Standard Deviation of C_v = $0.0286 \approx 0.03$

TABLE 3:3

RESULTS DERIVED FROM TABLE 3-2

Test No.	Mean Head h(cm)	Flow Rate q(cm^3/s)	Square Root of h ($cm^{1/2}$)
1	8.40	63.29	2.828
2.	18.50	94.43	4.301
3.	22.00	102.38	4.690
4.	30.05	118.93	5.481
5.	41.39	138.33	6.433
6.	50.53	152.53	7.107
7.	60.60	166.91	7.785
8.	70.65	179.88	8.405
9.	81.25	192.54	9.014

TABLE 3:4

REDUCTION OF DATA TABLE 3-3

Test No	C_d	$R_e \times 10^4$
1	0.6431	1.49
2	0.6309	2.26
3	0.6273	2.46
4	0.6235	2.88
5	0.6179	3.38
6	0.6167	3.68
7	0.6161	4.09
8	0.6141	4.41
9	0.6130	4.73

TABLE 3.5

RESULTS GENERATED FROM TABLE 3-4

Test No	C_d -0.590	Log_{10} (Cd - 0.590)$^{-1}$	R_e x 10^4	Log_{10} R_e
1	0.0531	1.2749	1.49	4.1732
2	0.0409	1.3883	2.26	4.3541
3	0.0374	1.4271	2.46	4.3909
4	0.0335	1.4750	2.88	4.4594
5	0.0274	1.5622	3.38	4.5289
6	0.0267	1.5735	3.68	4.5658
7	0.0261	1.5834	4.09	4.6117
8	0.0241	1.6180	4.41	4.6444
9	0.0230	1.6383	4.73	4.6749

TABLE 3.6

COEFFICIENT OF VELOCITY

Test No	h (cm)	Mean Value of X (cm)	Mean Value of Y (cm)	C_v
1	8.40	8.000	1.933	0.9926
2	18.50	16.867	4.700	0.9044
3	22.00	21.067	5.300	0.9706
4	30.05	23.400	5.300	0.9271
5	41.39	29.000	5.700	0.9506
6	50.53	28.000	4.267	0.9536
7	60.60	34.667	5.250	0.9718
8	70.65	32.667	3.800	0.9969
9	81.25	31.100	3.400	0.9356

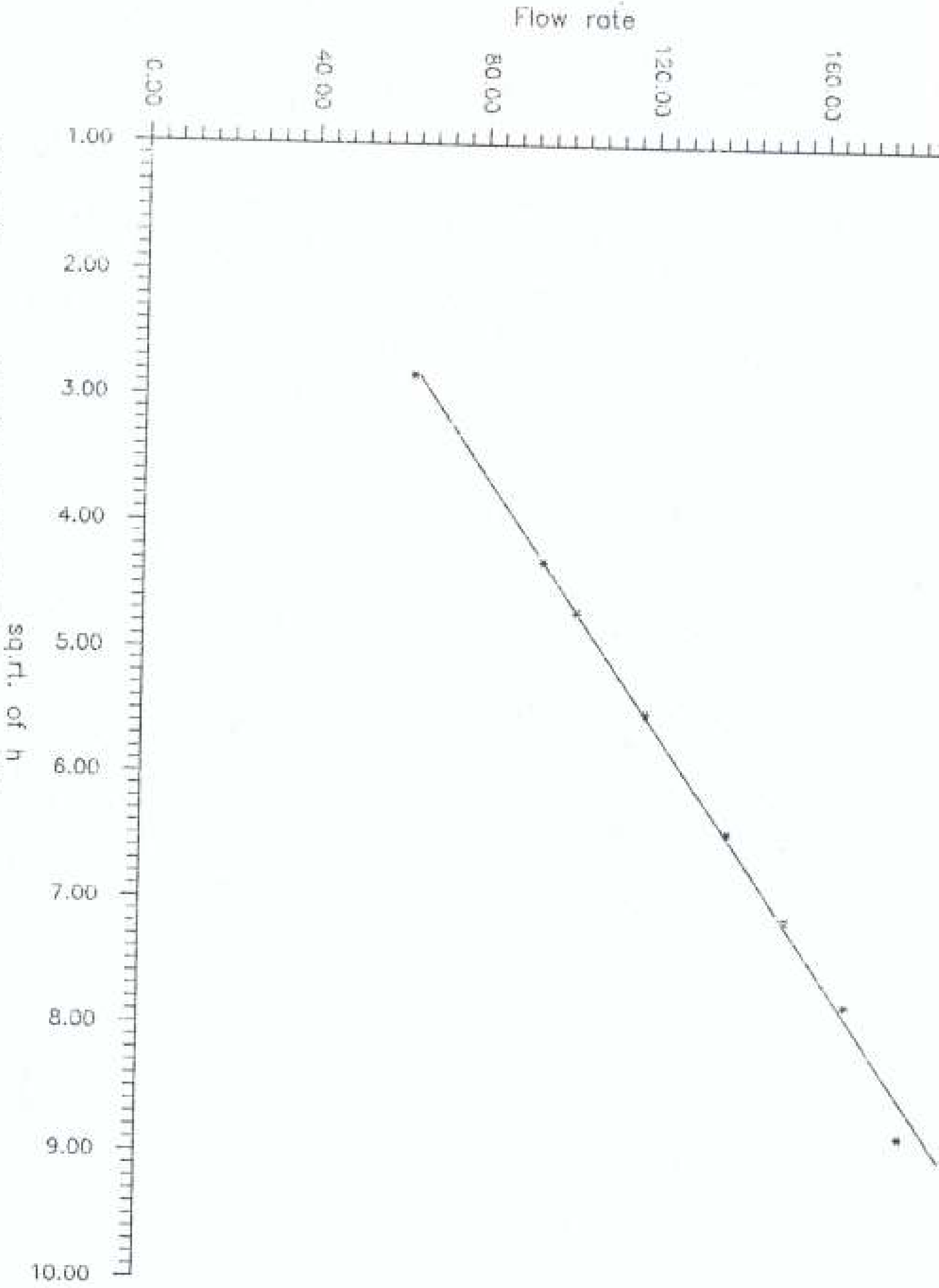


FIG. 3.4: GRAPH OF FLOW RATE Vs $h^{1/2}$

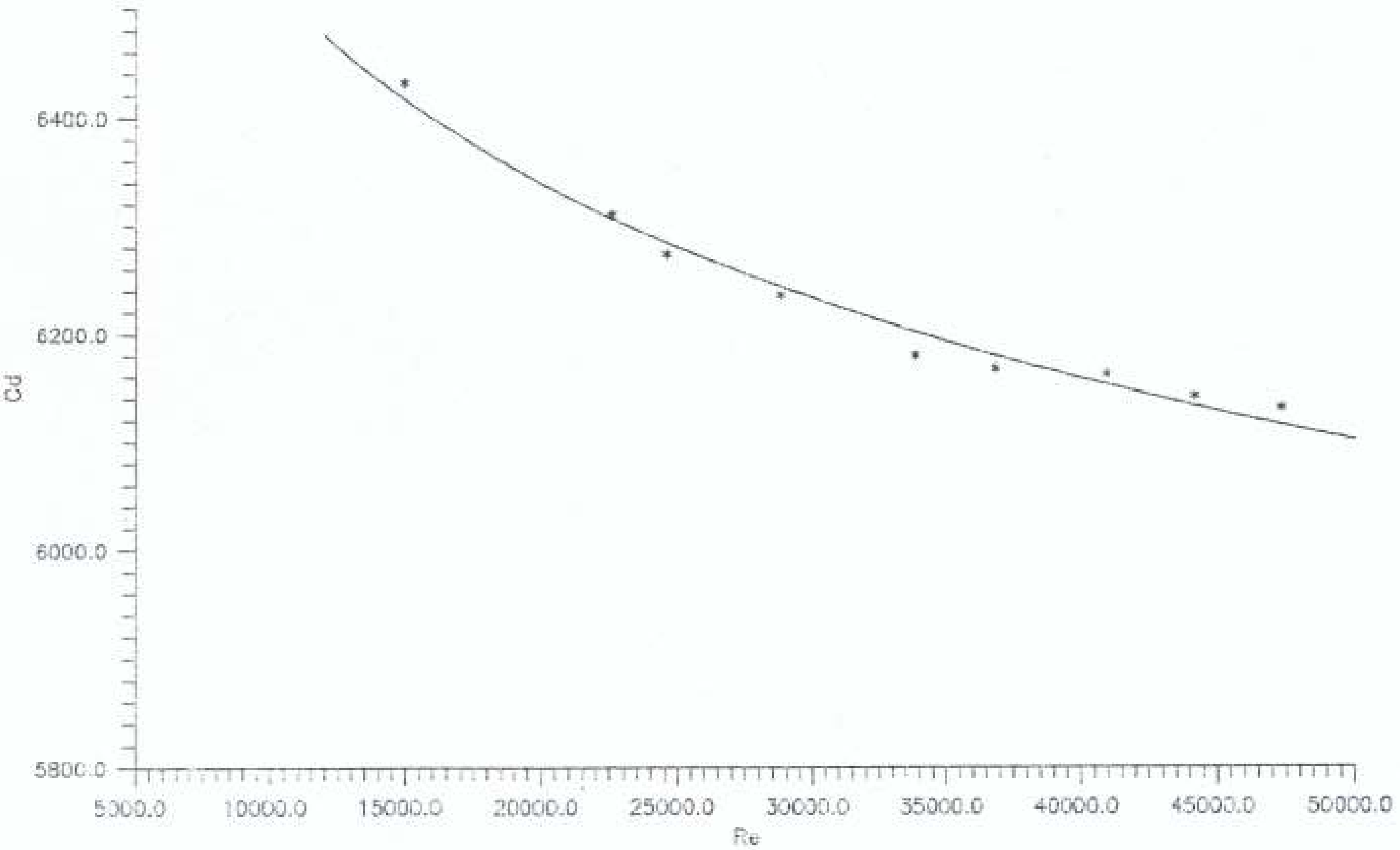
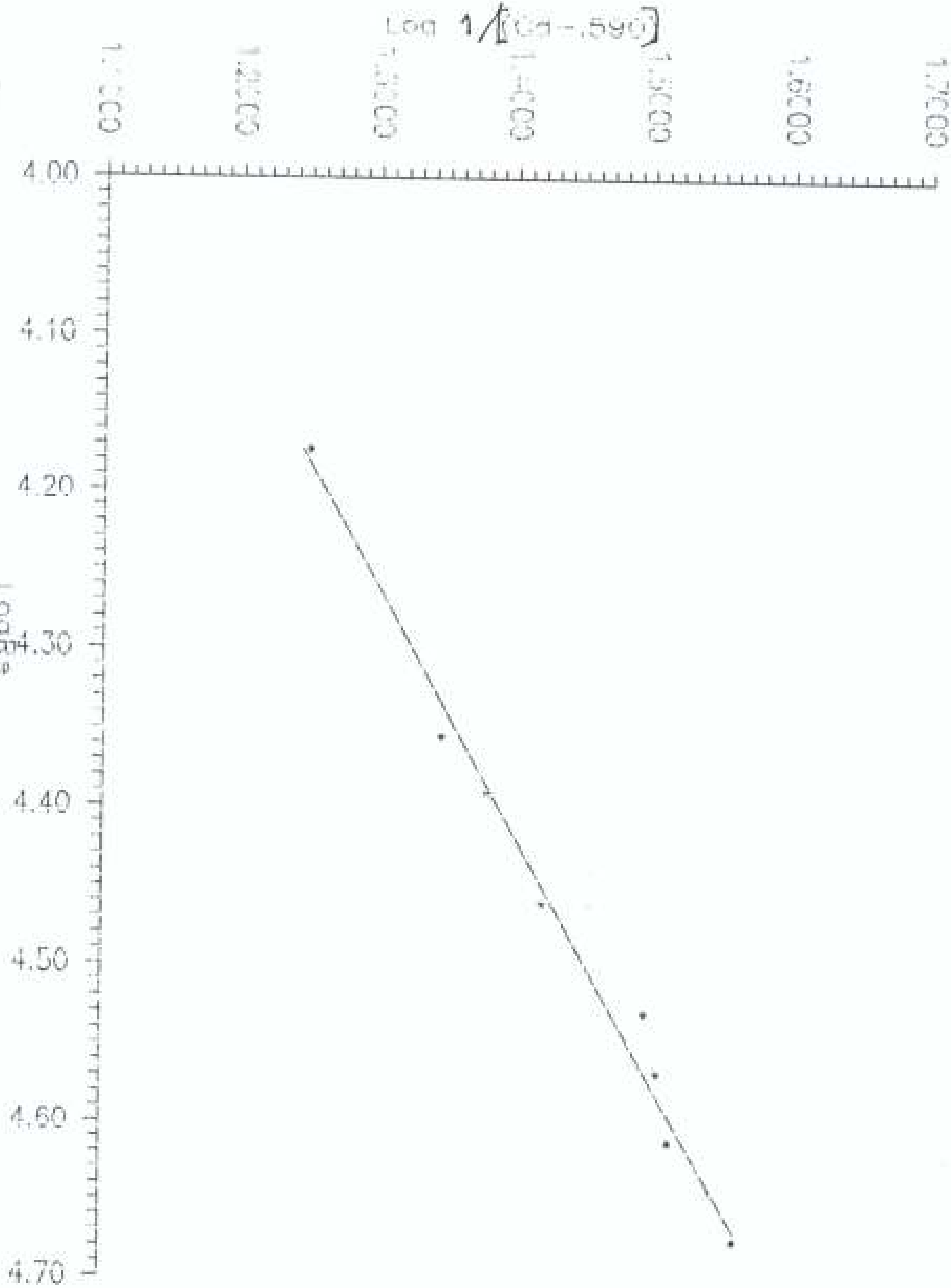


FIG. 3:5: REPRESENTATION OF RESULTS FOR 1cm ORIFICE IN NON-DIMENSIONAL TERMS



Log 1/[Ca−.590]
4.00
4.10
4.20
4.30
4.40
4.50
4.60
4.70
Log Rs

CHAPTER FOUR

4.0 ERROR ESTIMATION, DISCUSSION AND CONCLUSION

4.1 DEALING WITH ERROR IN THE SYSTEM

Error Estimation on q and h

It will be recalled that the flow-rate q for a range of values of h, q will be found from

$$q = \frac{M}{\rho T} \quad \text{..............} \quad (4.1)$$

where M is the mass flow in time T and ρ is the density of water. By application of equation 2.26 and equation 2.27 we see that neither these terms exerts more influence than the others on the accuracy of determination of q, so that none needs any special attention. Equation 4.1 leads in this case simply to

$$\frac{(\Delta q)^2}{q} = \frac{(\Delta M)^2}{M} + \frac{(\Delta \rho)^2}{\rho} + \frac{(\Delta T)^2}{T} \quad \text{..........} \quad (4.2)$$

The head h is measured directly, but we should note from equation 2.17 that expected relationship of form

$$q = \text{Constant} \times h^{1/2}$$

So that for a given proportional error in q we could accept twice that error in h.

Part of the errors in the determination of M, T and h can be accounted for by straight forward calibrations. For the determination of the errors that may arise from roughly assuming the measured values of head h, the standard deviation, σ of the sample was estimated to be 3.56, which gave us an idea that the uncertainty of the estimate was fairly low. These was used to make a rough estimate of how many readings we ought to take at each test to give us a reasonable precision in the mean value h. From statistical table we would expect 99% of such means to be within three times these values of the true mean. Therefore, for better results three reading at each test level was taken, which ensured that the mean values h were nearly all within about 1% of the true values.

From equation [$q = A(2gh)^{½}$] it is calculated that, with a 1cm diameter orifice, the time T taken to collect a quantity M of water equal to the capacity of the weighing tank (4335g) vary from about 22 to 70s over range of head from 100cm to 10cm (C_d = 0.6, these times will be longer by about 40%). There was available a good-quality stop-clock with graduations at 0.20 interval. The stop-watch had an admissible error of ± 1.8s for 10 minutes of operation under normal conditions (at temperature not above 30°C and pressure is not above 800mmHg). Therefore, a systematic slowness of about 0.003% variation, which is seen, without further analysis, to be too small to have any significant effect on the accuracy of determination of flow-rate. The only remaining unaccountable error of any substance arises from uncertainty of assigning the exact instant of attaining balance of the weighing tank. The yard-arm, used here as a null device is provided with a pointer Fig 3.1, so that the balance point is indicated by the passage of the pointer past a fixed mark. This is in fact a dynamic system, but having regard to the relatively long times taken for the tank to fill, any error in locating the balance-point seems likely to be small. The average combined in q is probably less than the error in h, so that accuracy of determining C_d from equation 3.17 depended primarily on the latter.

Error Estimation on C_v

From equation 2.21 the COEFFICIENT OF VELOCITY for the orifice, C_v, is given by

$$C_v^2 = \frac{x^2}{4yh} \quad or \quad C_v = \frac{x}{2\sqrt{yh}}$$

From equation 2.26 and equation 2.27

$$\left(\frac{\Delta C_v}{C_v}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \frac{1}{4}\left(\frac{\Delta y}{y}\right)^2 + \frac{1}{4}\left(\frac{\Delta h}{h}\right)^2 \quad \text{.................} (4.3)$$

For the best estimate of the combined error in C_v, x can be measured up to 50cm from the orifice and y up to 15cm. Since the point at which the velocity reaches the value V_1 is just outside the orifice, this point must be the origin of the co-ordinates of x and y. The correct location of the point with regard to x is uncertain, though no significant error in y will result if measurement is taken from the horizontal through the centre of the orifice. Using the procedure adopted by (Massey, 1970) we move the origin of x-scale to a point equivalent to half of the orifice diameter (5mm) from the plane of the orifice edge, with the assumption that the error in this correction is unlikely to make the accuracy of determination of C_v appreciably worse. Bearing in mind the difficulty of locating exactly the centre of the jet with the probe in Fig.2.12, we estimate that the maximum error in an x or y measurement would be about 2%, so that the highest combined error in C_v from a single measurement would be expected to be about 2.5%.

Since C_v is probably about $0.95C_c$ we cannot therefore expect to find the loss in velocity with great accuracy from a single measurement.

DISCUSSION

Having plotted results, as in Fig 3.4 and Fig 3.5, and reduced them to an empirical equation 3.4 we consider their implications.

We have already seen that the values for the orifice lie on a single curve when plotted in non-dimensional co-ordinates, that is C_d against R_e. Accordingly, it can be inferred that the effect of the parameter h/d is indeed negligible for the range in the experiment (h/d > 5), as suspected earlier. The general trend of the results in clear enough and the precision seems to be have been approximately as expected in the preliminary error analysis. It was there shown that the standard deviation of individual values of C_d would not be expected to be greater than about 0.25%, using equation 3.4 as the ideal relationship, the standard deviation of the results is found to be about 0.33%.

According to equation obtained by Lewitt [1958], gives a curve C_d versus R_e very similar to ours, as shown in Fig 4.1. However, Addison [1960] suggests the equation

$$C_d = 0.592 + \frac{4.5}{R^{0.5}} \quad \ldots\ldots\ldots\ldots (4.5)$$

which is traceable to Lea [1883], whose collection of large body of experimental data yields the equation.

$$C_d = 0.592 + \frac{4.8}{R_e^{0.5}} \quad \ldots\ldots\ldots\ldots (4.6)$$

and the one obtained in the late 19th century is given by equation

$$C_d = 0.593 + \frac{126}{R^{0.5}} \quad \text{.......................}(4.7)$$

rather than that given by Addison and which is also shown in Fig 4.1. The scatter in Lea's data is quite wide and would encompass our own results. It was noted that the asymptotic values C_d at large R_e suggested by our work (0.590) is close to 0.592 obtained by Lea, but the results R_e differ markedly. Nevertheless, the influence of the second term is small and difference between the equation negligible except at low R_e. At the lowest R_e in our tests, however, it has become almost 3.5%.

It is probable that some of the scatter in the results collected by Lea is due to the influence of the parameter $^{h}/_{d}$, for which the values are not given. Moreover, a multitude of minor factors (of which the inevitable radius at the lip of the "sharp-edged" orifice would perhaps be the most effective) could influence the results in what seems to be an unrewarding region where C_d is rather insensitive to R_e. It is seen that the experiment covers only a small part of the characteristic curve, which has the shape shown in Fig 1.3, according to Lea. It appears that the region between Re of 10^2 and 10^5 may be a transition zone between two regions, one in which C_d is proportional to R_e and one in which it is independent of R_e.

The values of the velocity coefficient C_v do not show a trend, and the average value is similar to those reported in the literature. It was expected that the highest combined error of a single determination would be about 4.5%. Most of the values of C_v are based on the average of three readings, increasing the precision to roughly 1.5% [$4\frac{1}{2}/(3)^{1/2}$]but the average deviation from the mean of the values in table 3.4 is actually about 6%. The mean value is 0.956 showing that only 2.5% of reduction in discharge below the theoretical value can be attributed to a loss in

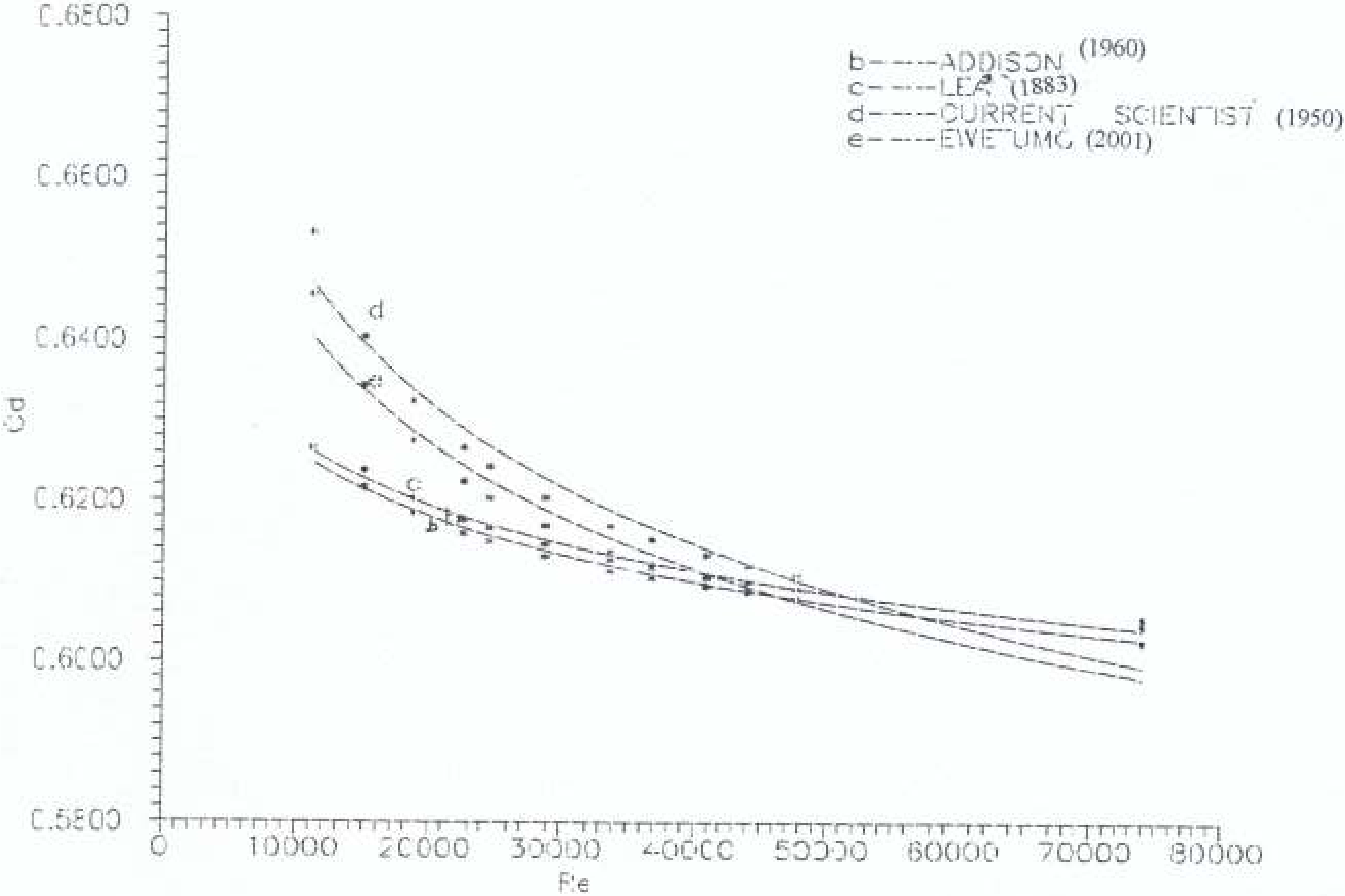


Fig. 4.1 COMPARISON OF RESULTS OF EXPERIMENT WITH OTHERS

velocity. It follows that the reduction is principally due to the contraction of the jet during acceleration.

CONCLUSION

The main results from the present work are follows.

* The coefficient of discharge C_d of a circular sharp-edged orifice is a function only of Reynolds number R_e for an incompressible fluid, when the ratio of the head h to the orifice diameter d is greater than about 5.

* The values obtained for C_d satisfy the empirical equation

$$C_d = 0.590 + \frac{69.980}{R^{0.740}}$$

over the range 14,000 < Re < 48,000, with a standard deviation about 0.33% (10 values).

* These values are not inconsistent with others reported in the literature, but they correspond to a range of R_e where there is a good deal of scatter in the values published.

* The coefficient of velocity C_v appears to be independent of Re, the mean value obtained being 0.956, with standard deviation of about 0.03.

* The discrepancy between the actual discharge and the theoretical value is almost entirely due to the contraction of the jet.

Therefore, the Reynolds Number R_e between 14,000 and 48,000, the coefficient of discharge C_d for a circular sharp-edged orifice is found to satisfy the relation

$$C_d = 0.590 + \frac{69.98}{R_e^{0.746}}$$

and to be independent of the ratio of the head of the fluid to the diameter of the orifice. The coefficient of velocity C_v is largely independent of R_e, with a mean value of 0.956.

The results are in reasonable agreement with published values but the range of R_e is rather narrow and further tests are recommended.

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