DEVELOPMENT OF AN ANALOGUE COMPUTER FOR THE SIMULATION OF LINEAR CIRCUITS AND SYSTEMS

BY

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CERTIFICATION

I certify that this research work was carried out by **Fayose Rufus Sola** of the department of Physics Federal University of Technology Akure and had been approved as meeting the requirement for the award of M.Tech degree in Physics of the Federal University of Technology Akure, Ondo State, Nigeria.

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DEDICATION

This work is dedicated to God Almighty the giver of knowledge, wisdom and understanding.

The thesis is also dedicated to my dearest children – **Ayomide, Olamide** and **Olumide**

ACKNOWLEDGMENT

I give all thanks to God Almighty the giver of all knowledge and wisdom for seeing me through my studies and for making this research work a reality.

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To my darling wife **Banke** and loving children **Ayomide, Olamide** and **Olumide**, I say thank you a million times for your prayers and being there for me despite all odds. I appreciate your concern, love, care, support and understanding. I love you all.

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ABSTRACT

In order to understand better the physical world, scientists use mathematical models to predict the reaction of various physical systems, such as the response of a mass attached to a spring and to external stimuli. Such systems are reduced to appropriate differential equations. However, showing graphical representation of the solutions to these models becomes computationally complex. An analogue computer consisting of summers, integrators and coefficient potentiometer was constructed to solve this problem. Using different types of differential equations and assuming different initial conditions the system's response to sine, triangular and square waves at frequencies between 2Hz and 10kHz were determined. Applying the computer to a control transfer function, the response was obtained for different values of gain K from 0.3 to 0.9, it was shown that the overshoot, the delay time, the rise time, settling time and frequency response all varied significantly with K.

CHAPTER ONE

1.0 INTRODUCTION

The analogue computer is basically an electronic simulator in which continuously varying voltages are used to represent the system variable such as temperature, pressure, velocity etc the independent variable being the time. In solving such a problem physical quantities are translated into related mechanical or electrical quantities and uses mechanical or electrical equivalent circuits as an analogue for the physical phenomenon being investigated.. Analogue computer is a computer that operates on analogue data by performing physical process on the data. The analogue computer is also a device, which simulates the variables of a system in terms of similar physical quantities easy to generate and control continuously. An analogue computer comprises of a patch board that provides connections to large numbers of integrators, amplifiers and potentiometers. Adjusting the values of the potentiometers enables the gain of the amplifiers and the time constants of the integrators to be varied. Each amplifier or integrator has a summing junction, thus allowing several input signals to be connected to these devices. The variables of a control system are represented on an analogue computer by the output voltages of the amplifiers and integrators.

Analogue computers are very useful for solving mathematical equations, which describe the behaviour of a system. It is used extensively in the study of system dynamics; that is, the study of the transient and steady-state behavior of a wide variety of physical systems. Today all analogue computers are electronic and the quantity used is voltage. For example consider the differential equation:

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = cSin\omega t \qquad 1.1$$

This equation is merely a mathematical expression and is not necessarily related to electrical quantities. Suppose it is assumed that y is a voltage, then $\frac{dy}{dt}, \frac{d^2y}{dt^2}, cSin\ \omega t$ would be also be voltages.

Analogue computers are also very useful in simulation in that the results are usually presented in the form of a continuous graph of the required variables and the system performance is easily observable. It is useful in the real time solution of differential equation. An equation is solved on an analogue computer by using operations (such as addition, subtraction, integrations) on voltages that represent the variables of the equation, the amplifiers are connected together in such a way that the mathematical relations prescribed by the equation are performed and a time-varying voltage is measured to obtain the solution.

Though differential equations contain derivatives, mathematical differentiations are almost never performed on an analogue computer because differentiation produces voltages whose amplitudes are proportional to their frequencies. It in effect, amplifies high frequency noise to an unacceptable level.

The basic mathematical operations that are performed on an analogue computer may be classified under the following headings.

a) Summation

b) Integration

c) Multiplication

1.1 CLASSIFICATION OF ANALOGUE COMPUTER

The chart shown in figure1.1 illustrates the variety of devices embraced by the general category of analogue computers. Analogue computers may be divided into general purpose and fixed purpose machines, depending upon whether they provide generally valid mathematical solutions useful for multiple purposes or they solve the problem inherent in a specific situation only. General-purpose and fixed-purpose analogue computers are also categorized as 'indirect' and 'direct' types respectively. Indirect analogue computers solve algebraic and differential equations of the linear and non-linear types. These equations may represent physical systems of various kinds, but the main point is that a general-purpose indirect computer may be set up to meet any type of problem situation that can be expressed in mathematical terms. In contrast, the direct type of analogue computer sets up a direct analogy to the behaviour, form and parameters of the problem, such as is represented by a miniature aeroplane, for example.

1.2 GENERAL-PURPOSE (INDIRECT) ANALOGUE COMPUTERS

As shown in figure1.1, these primarily may be broken down into mechanical, fluid or electrical types. This range all the way from the simple slide rule, where lengths on a stick are analogous to the logarithms of numbers, to the sophisticated mechanical differential analyzer, which can take complex systems of differential equations in its stride. Various types of linkages and nomograms also belong to this category. The fluid type of general-purpose computer is rather rare. The electrolytic tank is the main representative of this group. In this device two or more electrodes are inserted into a tank filled with a conductive liquid (electrolyte) and electricity is applied to the electrodes. Depending on the configuration of the tank and electrodes, certain potential fields are created which 'simulate' known types of differential equations. By moving 'probes' through the tank the potential field can be explored and, hence, a solution of the equation represented by it can be obtained. The electrolytic tank also can be used as a fixed-purpose direct device, to simulate the potential field inside an electron tube, for example, where the exact equation representing the field is unknown or not easily soluble.

The electrical type of general-purpose analogue computer is perhaps betterly called 'electronic' since the electronic differential analyzer uses operational amplifier as its primary representative. This is the most flexible and useful kind of general-purpose analogue computer, since its electronic and electrical components can easily be interconnected to perform almost any type of mathematical operation.

1.3 FIXED-PURPOSE (DIRECT) ANALOGUE COMPUTERS

Like the general-purpose (indirect) computers, the fixed-purpose (direct) analogue computers may also be broken down into mechanical, fluid and electrical categories (Henry and Leslie 1963). The mechanical types are perhaps the easiest to comprehend since they are represented by a great variety of scale models. Not every model will serve as a scale model. A true scale model preserves the physical form and structure of its prototype while its physical dimensions are scaled down to convenient size. Fairly elaborate dimensional analysis is required in properly scaling down a machine or device.

Mechanical bombsights fall into the category of fixed-purpose analogue computers, though they are not generally of the direct type. The famous Norden used during the Second World War, for example automatically solves the equations involved in causing a bomb to hit its target and does not set up a direct analogy. It is therefore an indirect type of analogue computer.

The number of electrical and electronic fixed-purpose analogue computers is unlimited. A great number of small electronic companies manufacture specialized electrical analogue computers. Most fixed-purpose analogue computers are of the electronic type using operational amplifiers and they are tailored to do a specific job, which is in contrast to electronic differential analyzers. Present day special-purpose computers are miniaturized by the use of lightweight 'solid state' devices, such as transistors. For example, the Polaris missile has a solid-state analogue 'think' device, which monitors its flight performance.

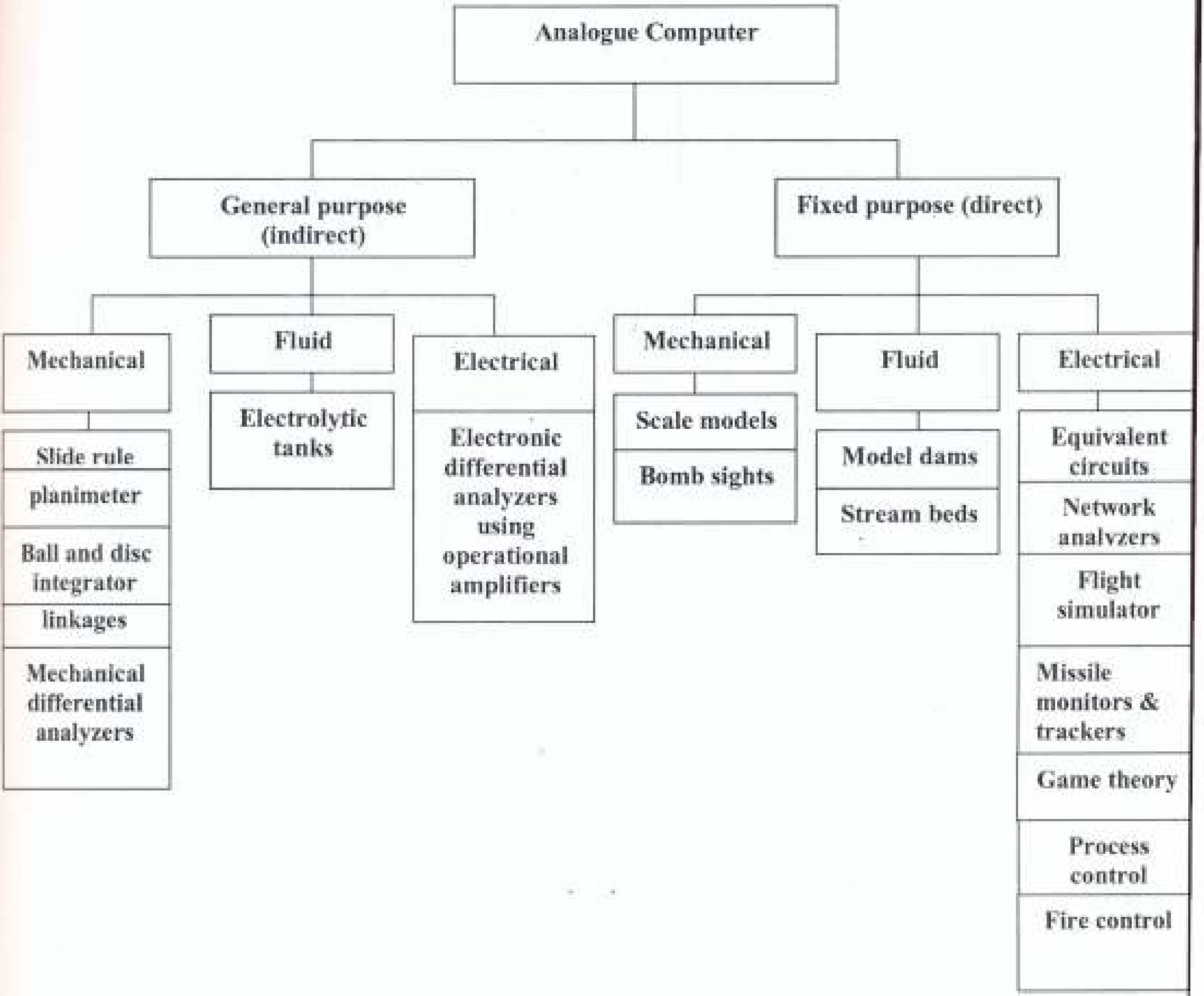

Figure 1.1: Classification of analogue computer

If the missile does not achieve sufficient velocity in the initial portion of its flight the analogue computer 'aborts' the flight. In addition to guiding flight, fixed-purpose analogue computers can simulate the flight of even the most complicated jet bomber. Other fixed-purpose analogue computer can utilize principles of game theory to solve problems in military tactics and strategy, do operations research, assist in the proper blending of petrol and control a variety of industrial and chemical processes.

1.4 ANALOGUE COMPUTER CHARACTERISTICS

An analogue computer sets up a mechanical analogy to the problem in question. The computing device may be mechanical (such as a planimeter or ball-and-disc integrator), electrical (a resistor network), electromechanical (a motor), or electronic (an amplifier). Regardless of its form, it must represent a quantity of the problem continuously and in a mathematically analogous manner. If the problem is simple, a single analogue device may represent the entire problem. Thus the varying rate of rotation of a shaft may continuously represent the speed of a car while the total amount (angle) the shaft has rotated can represent the total distance traveled. A more complex problem can be represented by a more sophisticated analogous device. Evidently, even a very difficult problem can be represented by relatively few highly sophisticated analogous devices, which solve various portions of the problem at the same time. Fewer devices, even if relatively complicated, means lower cost and less trouble in preparing a problem (programming) for the computer.

On the other hand, the fact that each device measures some physical or mathematical quantity creates a physical limitation to the accuracy of measurement, which is usually no more than one part in ten thousand in an analogue computer. A considerable advantage of the analogue computer is that it actually represents a physical problem or system and hence is capable of giving the designer genuine insight into the behaviour of that system under varying conditions. Analogue computers are best suited to serve as models and simulate some physical systems having varying stimuli.

1.5 ANALOGUE VERSUS DIGITAL COMPUTERS

The chart below shows the comparison between an Analogue Computer and a Digital Computer

ANALOGUE COMPUTER	DIGITAL COMPUTER
Sets up analogy of problem. Represents physical variable by continuous measurement of analogous quantity.	Breaks down problem into arithmetic Represents number by discrete, coded pattern (digital data).
Basic operation performed by relatively few single-purpose devices (integrators, multipliers, summers resolvers etc)	Operations performed by relatively many interchangeable arithmetic devices(adders,registers, accumulators)
Relatively few devices needed; hence comparatively low cost and ease of programming.	Many devices needed; hence high cost and difficult programming.
Distinct elements used for each operation	Identical elements used in sequence
Accuracy limited to about 1 part in 10^4	Unlimited accuracy to about 10^{12} or more
Data storage (memory) dispersed in various non-interchangeable devices	Data storage concentrated in space, interchangeable and unlimited in duration
Analogue computer serves as model and 'mirrors' relations of actual system; operations usually carried out in actual	Digital computer compounds arithmetic data, unrelated to system it represents. Time of operations usually

(real) time of physical system.	does not correspond to 'real' time.
Represents physical or mathematical	Can represent numbers, letters and other quantity symbols
Best suited to represent measurable quantity and simulate response of physical systems by mathematical analogies.	Best suited to handle discrete random processes, statistical data, and numerical problems of business and scientific nature.

1.6 SCOPE OF THE WORK

The scope of this research work is to design and construct an analogue computer to solve the following type of differential equations.

(*i*) $$\frac{d^2x}{dt^2} + a_1\frac{d^2y}{dt^2} + a_2\frac{dx}{dt} + a_3x = f(t)$$

(*ii*) $$\frac{d^2y}{dt^2} + a_4\frac{d^2x}{dt^2} + a_5\frac{dy}{dt} + a_6\frac{dx}{dt} + a_7y = 0$$

(*iii*) $$G = \frac{K}{s(s+1)(1+0.2s)}$$

The analogue computer system designed includes the following major units: summers, integrators and coefficient potentiometers. Also, sine wave, square wave, and triangular wave at low frequency from 0.01Hz to 200Hz were included in the design. A computer switch mode power supply was used as the dc voltage source. The output from the analogue computer is in the form of a voltage, which must be read and recorded by the use of oscilloscope or X-Y plotter.

CHAPTER TWO

LITERATURE REVIEW

2.0 DIFFERENTIAL EQUATION

A differential equation is an equation that describes the relationship between an unknown function and its derivatives. It describes the relationship between an independent varable,x, a dependent variable y, and one or more derivatives of y with respect to x. e.g.

$$x^2 \frac{dy}{dx} + y \sin x = 0 \tag{2.1}$$

$$xy \frac{d^2 y}{dx^2} + y \frac{dy}{dx} + e^x = 0 \tag{2.2}$$

Differential equations represent dynamic relationships, such as, quantities that change, and frequently occur in scientific and engineering problems. The order of a differential equation is the highest order derivative involved in the equation.

2.1 DEFINITION

Given that y is a function of x and that

$$y', y'', \ldots\ldots\ldots, y^n \tag{2.3}$$

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denote the derivatives

$$\frac{dy}{dt}, \frac{d^2 y}{dt^2}, \ldots \ldots, \frac{d^n y}{dt^n} \tag{2.4}$$

an ordinary differential equation (ODE) is an equation involving

$$x, y, y', y'', \ldots\ldots\ldots \tag{2.5}$$

When a differential equation of order n has the form

$$F(x, y, y', y'', \ldots\ldots.. y^{(n)}) = 0 \qquad 2.6$$

it is called an implicit differential equation whereas the form

$$F(x, y', y'', \ldots\ldots.. y^{(n-1)}) = y^{(n)} \qquad 2.7$$

is called an explicit differential equation. A differential equation not depending on x is called autonomous.

2.2 GENERAL APPLICATION

An important special case is when the equation does not involve x. This differential equation may be represented as a vector field. This type of differential equations has the property that space can be divided into equivalent classes based on whether two points lie on the same solution curve. Since the laws of physics are believed not to change with time, the physical world is governed by such differential equations. The problem of solving a differential equation is to find the function y whose derivatives satisfy the equation. For example, the differential equation

$$\frac{d^2y}{dt^2} + y = 0 \qquad 2.8$$

has the general solution

$$y = A\cos x + B\sin x, \qquad 2.9$$

where A, B are constants determined from boundary conditions. In the case where the equations are linear, this can be done by breaking the original equation down into smaller equations, solving those, and then adding the results back together. Unfortunately, many of the interesting differential equations are non-linear, which means that they cannot be broken down in this way. There are also a number of techniques for solving differential equations using a computer

Ordinary differential equations are to be distinguished from partial differential equations where y is a function of several variables, and the differential equation involves partial derivatives.

Differential equations are used to construct mathematical models of physical phenomena such as fluid dynamics or celestial mechanics. Therefore, the study of differential equations is a wide field in both pure and applied mathematics.

Differential equations have intrinsically interesting properties such as whether or not solutions exist, and should solutions exist, whether those solutions are unique. Applied mathematicians, physicists and engineers are usually more interested in how to compute solutions to differential equations. These solutions are then used to design bridges, automobiles, aircraft, sewers, etc.

2.3 LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

The first method of solving linear ordinary differential equations with constant coefficients is due to Euler, who obtained the solution to the differential equation of the form:

$$\frac{d^n y}{dx^n} + A_1 \frac{d^{n-1} y}{dx^{n-1}} + \ldots\ldots + A_n y = 0 \qquad 2.10$$

He showed that the solution is z given by

$$F(z)=z^n+A_1 z^{n-1}+.....+A_n=0 \qquad 2.11$$

This equation F(z) = 0, is the "characteristic" equation.

2.4 A SIMPLE ORDINARY DIFFERENTIAL EQUATION

The simplest differential equation is the ordinary linear differential equation of the first order with constant coefficients. For example:

$$\frac{dy}{dt}+f(t)y=0, \qquad 2.12$$

where f(t) is some known function. We may solve this simply by rearranging it (using the chain rule) as

$$\frac{d(e^{F(t)}y)}{dt}=0 \qquad 2.13$$

where

$$F(t)=\int f(t)dt \qquad 2.14$$

Integrating this, we have

$$y=Ae^{-F(t)} \qquad 2.15$$

where A is an arbitrary constant.

2.5 A SIMPLE MATHEMATICAL MODEL

Suppose a mass is attached to a spring. The spring exerts an attractive force on the mass proportional to the extension/compression of the spring x(t). Now, using Newton's second law we can write

$$M\frac{d^2x}{dt^2} = -kx \qquad 2.16$$

where M is the mass of the spring and k is the spring constant.

The solution is of the form $X = Ce^{kt}$, where C is a constant, and k is a complex number. Thus, using Euler's theorem the solution is of the form:

$$X(t) = A\cos t + B\sin t \qquad 2.17$$

To fix the unknown constants A and B, we need initial conditions, i.e. to specify the state of the system at a given time (usually taken to be t = 0).

Assuming for example that at t=0, X=1 and $\frac{dx}{dt}=0$ it can be shown that A=1 and B=0.

Therefore x(t) = cost. (This is an example of simple harmonic motion)

2.6 IMPROVING OUR MODEL

The above model of an oscillating mass on a spring is plausible but not really realistic. For a start, we have invented a perpetual motion machine which violates the second law of thermodynamics. To make the system more realistic, some friction is added, whose magnitude is proportional to its velocity (i.e. dx / dt). The new differential equation, expressing the balancing of the acceleration and the forces, is

$$M\frac{d^2x}{dt^2} = -c\frac{dx}{dt} - kx \qquad 2.18$$

where c is the coefficient of friction, and $c > 0$. Again looking for solutions of the form Ae^{kt}, the solution is of the form $x = e^{kt}$

where

$$K^2 + CK + 1 = 0 \qquad 2.19$$

If $c < 2$ the equation has two complex roots $a \pm ib$, and the solution (with the above boundary conditions) is:

$$x(t) = e^{at}\left(\cos bt - \frac{a}{b}\sin bt\right) \qquad 2.20$$

This is a damped oscillator, and the plot of displacement against time is shown in figure 2.1

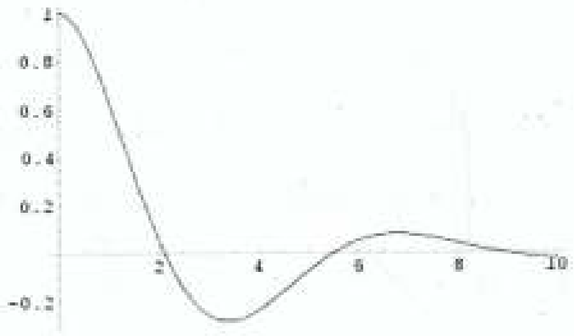

Figure 2.1: Graph of displacement against time

2.7 ANALOGUE COMPUTER BASICS

The modern analogue computer is based on an electronic circuit known as an operational amplifier. It is a direct-coupled high-gain amplifier to which feedback is added to control its overall response characteristics. It is used to perform a wide range of linear functions (and some nonlinear operations) and is often referred to as the basic linear (or more accurately, analogue) integrated circuit. Early operational amplifiers ("op amps" for short) used vacuum tubes, since that was the only available technology. Modern operational amplifiers are constructed as semiconductor integrated circuits. Either way, the general theory is the same.

The integrated circuit operational amplifier has gained wide acceptance as a versatile, predictable and economic system building block. It offers all the advantages of monolithic integrated circuits such as: small size, high reliability, reduced cost, temperature tracking and low offset voltage and current.

For the overall discussion of analogue computer circuits and operational amplifier behaviour in such applications, three assumptions are generally made about operational amplifiers:

- They have infinite voltage gain.
- They have infinite input resistance (or zero input current).
- They have zero output resistance (infinite output current capability).

Although these assumptions are not really correct, they are close enough that the circuit works well, so long as the electronic components connected to the amplifier to control its operation have reasonable values.

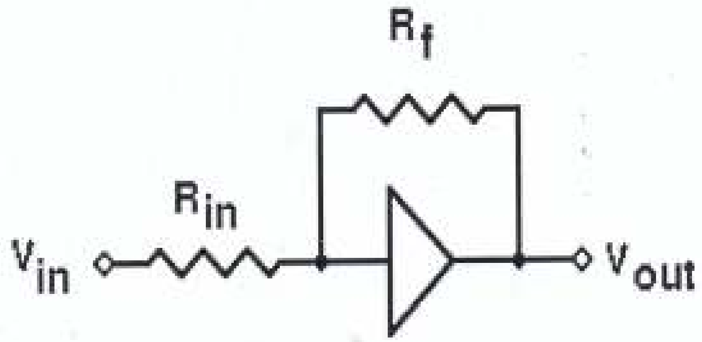

Figure 2.2: Analogue computer basic circuit

Figure 2.2 shows the basic circuit used in analog computers. The triangle represents operational amplifier. Associated with the amplifier are two resistors: an input resistor (R_{in}) and a feedback resistor (R_f). In addition, the amplifier inverts the signal. That is, a positive input signal will result in a negative output signal, and vice-versa. With this combination of characteristics, precision resistors and other components are used to accurately determine how the circuit will behave.

To determine the current through the two resistors, R_{in} and R_f we must determine the output voltage and the voltage at the amplifier input, where both resistors are connected. With an infinite voltage gain, any voltage at the amplifier's input will cause an excessive output voltage. Therefore, the voltage at the junction must always be zero. The amplifier output will provide whatever voltage is required to maintain that condition, and keep the currents through R_{in} and R_f the same.

$$\frac{V_{out}}{R_f} = -\frac{V_{in}}{R_{in}} \qquad 2.21$$

This in turn means that so long as the circuit is operating within its bounds (output voltage within the range of ±10 volts), the junction of these components will be a virtual ground.

From equation 2.21,

$$V_{out} = -\frac{R_f}{R_{in}} V_{in} \qquad 2.22$$

or,

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}} \qquad 2.23$$

Equations 2.22 and 2.23 imply that the voltage gain of the overall circuit is set entirely by the ratio of R_f/R_{in}. This is the secret of the operational amplifier: it uses extremely high gain combined with negative feedback in order to achieve accurate and predictable results. If precision resistors are used, precise and measurable results can be obtained.

2.8 BASIC OPERATIONAL AMPLIFIERS

The basic operational amplifier is represented by the symbol shown in figure 2.3. The amplifier has two inputs, which are represented, by V_+ and V_- and a single output, V_o. Positive and negative power supplies of equal magnitude are normally used though single-supply operation is possible. The power supply is shown in figure 2.3 as $\pm V_s$. The common zero of $+V_s$ and $-V_s$ is an important reference point to which V_+, V_- and V_o are referred. It does not appear explicitly on the amplifier symbol since a direct connection is not required. However, there are applications in which one or other of the amplifier inputs may be connected either directly or indirectly to the ground.

Ideal operation of the amplifier is shown in the transfer characteristics of figure 2.3. V_i represents the difference between the voltages applied to the two inputs V_+ and V_-. If V_i is positive, even by a small amount, the output V_o is positive and constant, having a magnitude slightly less than that of the supply voltage. Similarly, negative values of V_i produce a constant negative output.

In practice, a finite change in V_i is needed in order to change V_o from one level to another as shown by the dotted line in figure 2.4.

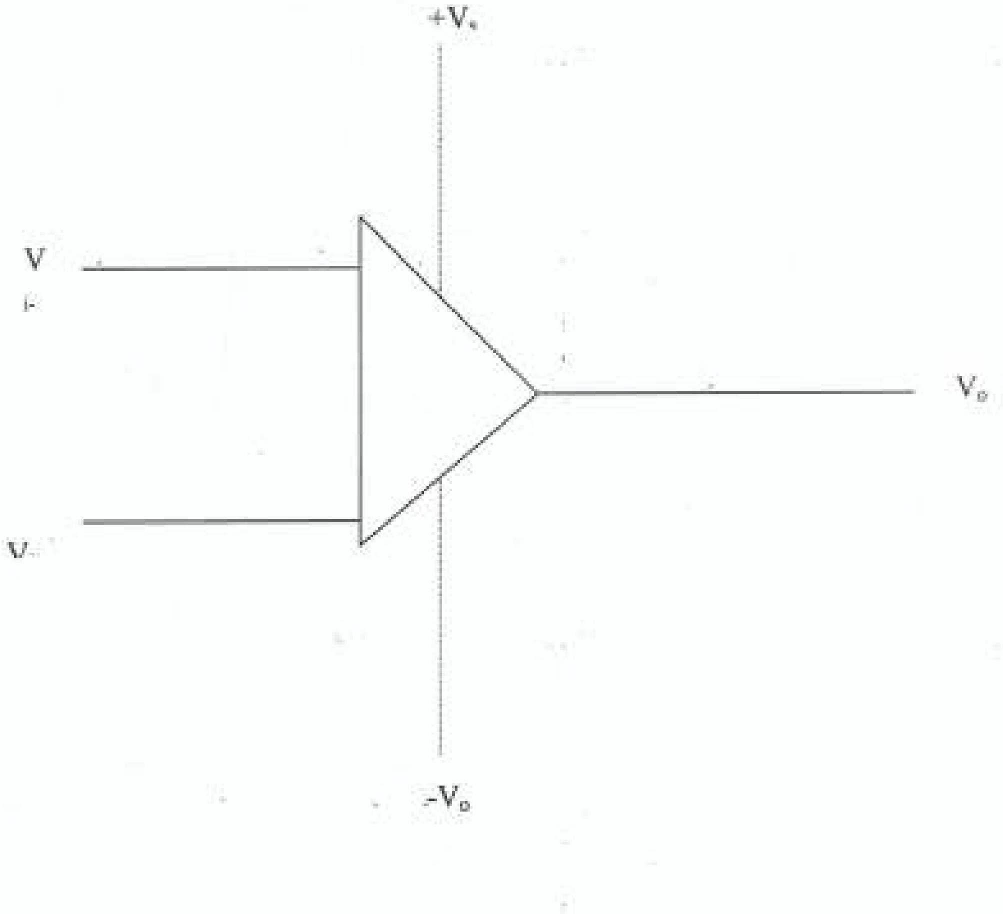

Figure 2.3: Basic Operational Amplifier Symbol

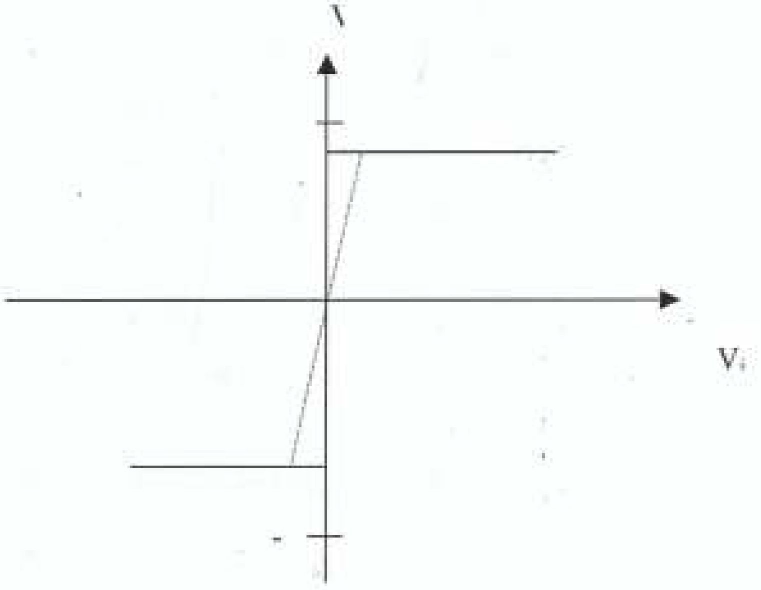

Figure 2.4: Ideal transfer characteristics (solid line) and practical approximation (broken line) $V_i = V_{i+} + V_{i-}$

For a characteristic having a finite slope, the input/output relationship is written as

$$V_o = A(V_+ - V_-) \qquad 2.24$$

where A is the gain of the amplifier in the region between the two output saturation voltages. The value of A is large for practical amplifiers (at least 50000) and theoretically infinite for ideal ones. A is known as the open loop gain, which is the gain of the amplifier without feedback. The inputs indicated by + and – in figure 2.4 are referred to as non-inverting and inverting respectively.

Operation without feedback is often referred to as open loop operation, which becomes closed loop operation when feedback is applied.

The operational amplifier was designed to perform mathematical operations. Although now superseded by the digital computer, operational amplifiers are a common feature of modern analogue electronics.

The operational amplifier is constructed from several transistor stages, which commonly include a differential input stage, an intermediate-gain stage and a push-pull output stage. The differential amplifier consists of a matched pair of bipolar transistors or FETs. The push-pull amplifier transmits a large current to the load and hence has a small output impedance.

The operational amplifier is a linear amplifier with V_{out} directly proportional to V_{in}. The DC open-loop voltage gain of a typical operational amplifier is 10^3 to 10^6. The gain is so large that most often feedback is used to obtain a specific transfer function and control the stability.

Cheap IC versions of operational amplifiers are readily available, making their use popular in any analogue circuit. The cheap models operate from DC to about 20 kHz, while the high-performance models operate up to 50 MHz. A popular device is the 741op-amp; its open loop gain is 100000 at dc. It drops off at a rate of o. 6 dB/octave above 5 Hz. Operational amplifiers are usually available as an IC in an 8-pin dual, in-line package (DIP). Some operational amplifiers ICs have more than one operational amplifier on the same chip.

The supply voltage to an operational amplifier is $\pm V_{CC}$.

V_{CC} is typically, but not necessarily, ±15 V. The positive and negative voltages are necessary to allow the amplification of both positive and negative signals without special biasing.

For a linear amplifier the open-loop gain is

$$\vec{V} = A(j\omega)(\vec{V}_+ - \vec{V}_-) \qquad 2.25$$

The open-loop gain can be approximated by the transfer function

$$A(j\omega) = A_o H_{low}(j\omega) \qquad 2.26$$

where A_o is the DC open-loop gain and H_{low} is the transfer function of a passive low-pass filter. We can write

$$A(j\omega) = \frac{A_o}{1 + {}^{j\omega}\!/\!_{\omega_c}} \qquad 2.27$$

where $A_o = 2\mathrm{x}10^5$ and $f_c = 5\mathrm{Hz}$

Two conditions must be satisfied for linear operation:

1. The input voltage must operate within the bias voltages:

$$-V_{cc}/\ A_o \leq (V_+ - V_-) \leq +V_{cc}/\ A_o\ .$$

2. For no clipping, the output voltage swing must be restricted to

$-V_{cc} \leq V_{out} \leq +V_{cc}$

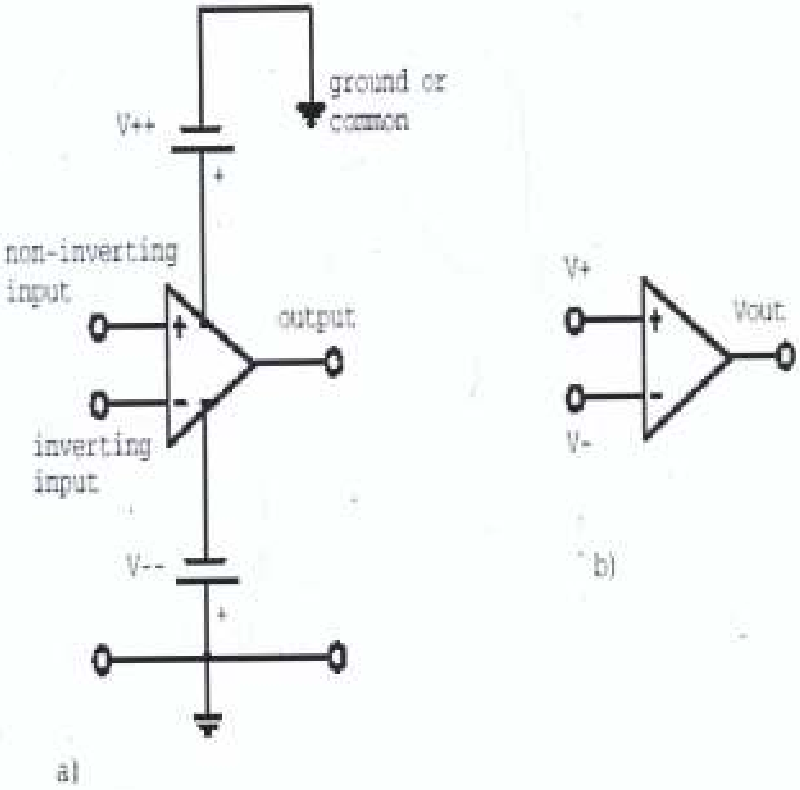

Figure 2.5 (a) complete diagram of an operational amplifier and (b) common diagram of an operational amplifier

2.9 IDEAL AMPLIFIER APPROXIMATION

The following are properties of an ideal amplifier.

1. Large forward transfer function, Z_{out}
2. Virtually nonexistent reverse transfer function,
3. Large input impedance, $Z_{in} \rightarrow \infty$ (any signal can be supplied to the op-amp without loading problems),
4. Small output impedance, $Z_{out} \rightarrow \infty$ (the output can drive a load impedance),
5. Wide bandwidth, and
6. Infinite gain, $A \rightarrow \infty$

If these approximations are followed two rules can be used to analyze operational amplifier circuits:

Rule 1: The input currents I_+ and $I-$ are zero,

The op amp does not draw any current

Rule 2: The voltages V_+ and $V-$ are equal,

$$V_+ = V- \; (A = \infty)$$

To apply these rules requires negative feedback.

Feedback is used to control and stabilize the amplifier gain. The open-loop gain is too large to be useful since noise will cause the circuit to clip. Stabilization is obtained by feeding part of the output back into the input. When used in this way, the closed-loop gain does not depend on the amplifier characteristics.

2.10 NON-INVERTING AMPLIFIERS

Figure 2.4 shows a non-inverting amplifier.

For this circuit

$$V_+ = V_- \Rightarrow V_{in} = \frac{R_I}{R_I + R_F} V_{out} \qquad 2.28$$

The gain is

$$\frac{V_{Out}}{V_{in}} = \frac{R_I + R_F}{R_I}$$

and

$$\frac{V_{0ut}}{V_{in}} = 1 + \frac{R_F}{R_I}$$

$$G(j\omega)=1+\frac{R_F}{R_I} \qquad 2.29$$

It is seen from equation 2.29 that the gain cannot be less than unity.

A special version of the non-inverting amplifier is the buffer amplifier (or voltage follower) in which R_I is omitted and $R_f = 0\Omega$. The voltage gain is unity. The advantage of the voltage follower is that the input impedance is very large ≈ 100MR and the output impedance is zero ohms.

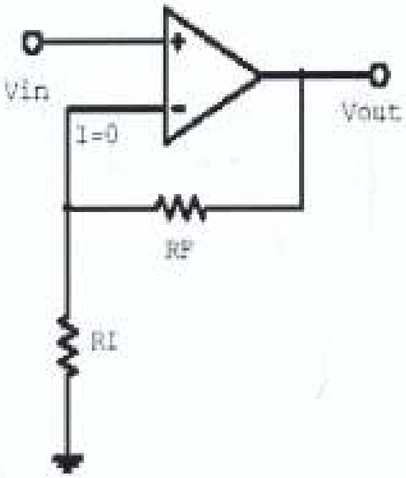

Figure 2.6 Non-inverting amplifier with feedback

2.11 INVERTING AMPLIFIERS

An inverting amplifier is shown in figure 2.6 where the resistors R_I and R_F are the input and feedback resistors respectively. The non-inverting input of the amplifier is connected to the common zero of the power supplies and the inverting input has a voltage V_- with respect to this. Let the currents in the input and feedback resistors be i_I and I_f and since the operational amplifier has a very high input resistance, it is considered that a negligible current flows into the inverting input. Hence, the steady-state error voltage V_- is approximately equal to zero. The currents will then sum to zero and applying ohm's law to each of the resistor:

$$i_I + i_f = 0$$

$$\frac{V_{in} - V_-}{R_I} = \frac{V_- - V_{out}}{R_F}$$

Since $V_+ = V_- = 0$

The gain is

$$\frac{V_{out}}{V_{in}} = \frac{-R_F}{R_I} \qquad 2.30a$$

and

$$G(j\omega) = \frac{-R_F}{R_I} \qquad 2.30b$$

This is an important and useful result since the relationship between V_{out} and V_{in} (a gain of $-R_F/R_I$) depends only on the values of the resistors and not on the characteristics of the amplifier itself. This is true only when the circuit is operating under such conditions that the assumption of very high input resistance and very high open gain is valid. Since V_- has become very small, the potential of the inverting input is very close to that of the common reference point that is referred to as a virtual earth.

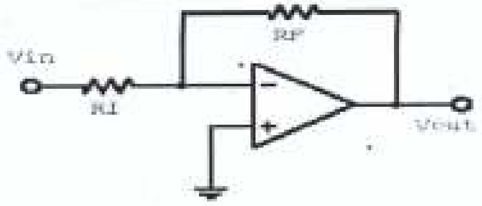

Figure 2.7: Inverting amplifier

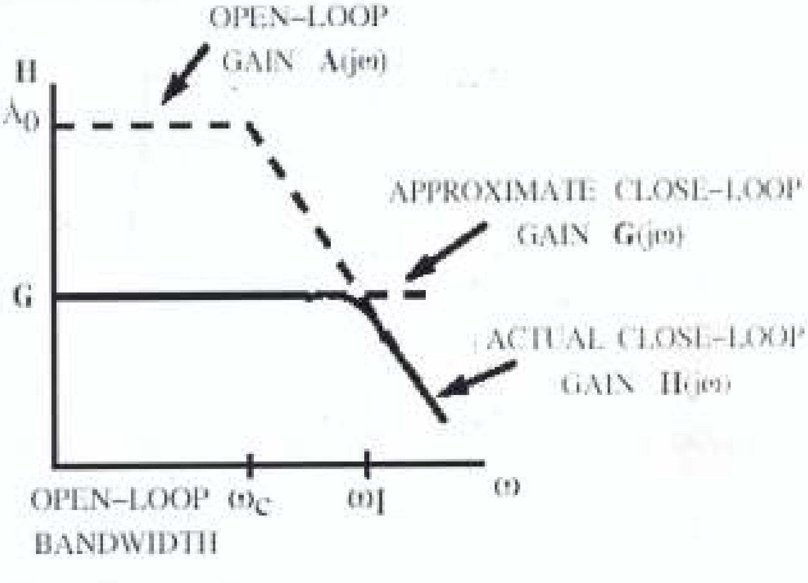

Figure 2.8: Inverting and non-inverting amplifier frequency response

The output is inverted with respect to the input signal.

A sketch of the frequency response of the inverting and non-inverting amplifiers are shown in figure 2.8

The input impedance of the inverting amplifier is R_1.

The extra resistor R is a current bias-compensation resistor. It reduces the current bias by eliminating non-zero current at the inputs.

2.12 DIFFERENTIAL AMPLIFIER

The function of a differential amplifier is in general to amplify the difference between two signals. The need for a differential amplifier arises in many physical measurements where response from dc to many megahertz is required. It is also the basic stage of an integrated operational amplifier with differential input.

Figure 2.9 represents a linear active device with two input signals V_1, V_2 and one output signal V_o, each measured with respect to ground. In an ideal differential amplifier the output signal V_o is given by

$$V_o = A_d(V_1 - V_2) = \frac{R_f}{R_1}(V_1 - V_2) \qquad 2.31$$

Where A_d is the gain of the differential amplifier. However, a practical differential amplifier cannot be described by the above equation because, in general, the output depends not only upon the difference signal V_d of the two signals, but also upon the average level, called the common-mode signal V_c, where

$$V_d \equiv V_1 - V_2 \qquad 2.32a$$

and

$$V_c \equiv \frac{1}{2}(V_1 + V_2) \qquad 2.32b$$

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2.12.1 Common-mode Rejection Ratio (CMMR)

The output of figure 2.9 can be expressed as a linear combination of the two input voltages.

$$V_0 = A_1 V_1 - A_2 V_2 \qquad 2.33$$

where A_1 is the voltage amplification from input 1 to the output when input 2 is grounded and A_2 is the voltage amplification from input 2 to the output when input 1 is grounded.

From equations 2.32a and 2.32b

$$V_1 = Vc + \frac{1}{2} V_d \qquad 2.34a$$

and

$$V_2 = V_c - \frac{1}{2} V_d \qquad 2.34b$$

If equations 2.34a and 2.34b are substituted into equation 2.33

$$Vo = A_d V_d + A_c Vc \qquad 2.35$$

where A_d is the voltage gain for the difference signal and it is given by

$$A_d \equiv \frac{1}{2} (A_1 - A_2) \qquad 2.36a$$

and A_c is the voltage gain for common-mode signal and given by

$$A_c \equiv A_1 + A_2 \qquad 2.36b$$

The relationship between A_d and A_c is referred to as the Common-mode rejection ratio (CMRR), which serves as a figure of merit and it is given by:

$$CMMR = \rho = \frac{A_d}{Ac} \qquad 2.37$$

Combining equations 2.35 and 2.37, the expression for the output is given by

$$Vo = A_d V_d (1 + \frac{1}{\rho} \frac{V_c}{V_d}) \qquad 2.38$$

From equation 2.38 it is seen that the amplifier should be designed so that ρ is large compared with the ratio of the common-mode signal to the difference signal.

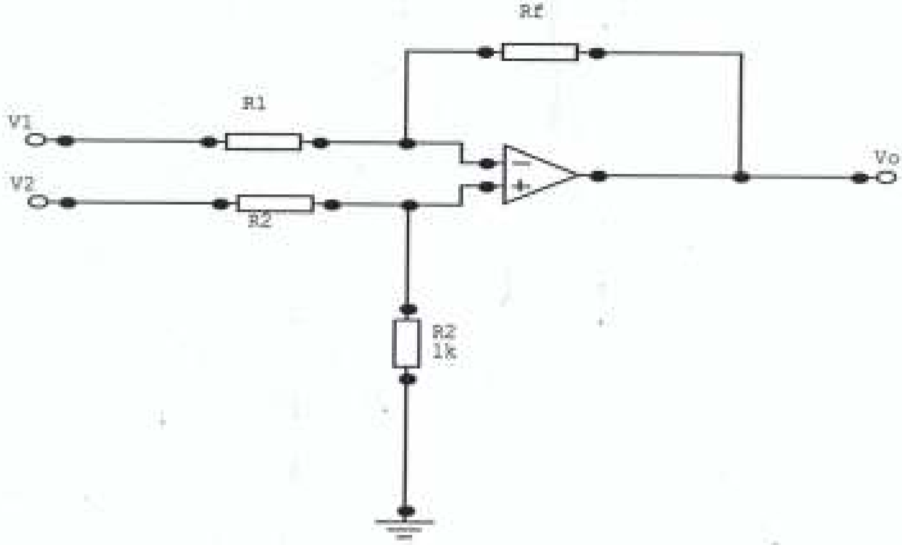


Figure 2.9: The differential Amplifier

2.13 INSTRUMENTATION AMPLIFIER

The instrumentation amplifier is shown in figure 2.10. It is highly useful in measurement systems. If the input current to the amplifiers is neglected, the same current will flow through R_1, R_2, and R_3, which can be represented by I, with voltages as, indicated in the figure:

$$I = \frac{V_{oa} - V_{ia}}{R_1} = \frac{V_{ia} - V_{ib}}{R_2} = \frac{V_{ib} - V_{ob}}{R_3}$$

For the first pair:

$$\frac{V_{oa}}{R_1} = V_{ia}(\frac{1}{R_2} + \frac{1}{R_1}) - \frac{V_{ib}}{R_2}$$

Similarly, for the second pair:

$$\frac{V_{ob}}{R_3} = V_{ib}(\frac{1}{R_3} + \frac{1}{R_2}) - \frac{V_{ia}}{R_2}$$

In view of the high gain of the buffer amplifier

$V_{ia} \approx V_a$ and $V_{ib} \approx V_b$

Hence:

$$V_{oa} - V_{ob} = V_a(1 + \frac{R_1}{R_2} + \frac{R_3}{R_2}) - V_b(1 + \frac{R_1}{R_2} + \frac{R_3}{R_2})$$

Since the coefficients are the same the expression reduces to

$$V_{oa} - V_{ob} = (V_a - V_b)(1 + \frac{R_1 + R_3}{R_2}) \qquad 2.39$$

IC_3 I is used as a differential amplifier of voltage gain R_f/R_i.

The overall gain will be given by

$$V_o = (V_a - V_b)(1 + \frac{R_1 + R_3}{R_2})\frac{R_f}{R_i} \qquad 2.40$$

One of these can be made adjustable in order to optimize common mode rejection.

Instrumentation amplifiers are available in the form of integrated circuit modules. One well-established device (Analogue Devices AD524) has built-in resistance for the equivalent of R_2. These can be selected by means of a link to give gains of 10,000 or 1000. Intermediate values can be obtained by using a resistor in place of the link.

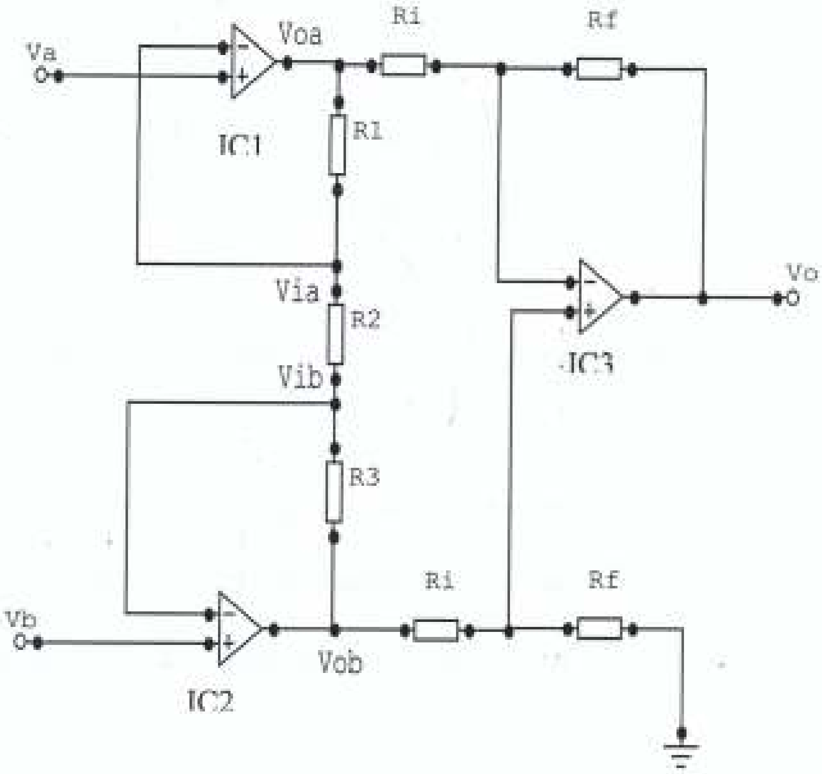


Figure 2.10: The Instrumentation Amplifier

2.14 Precision Design and Low Noise Techniques

An ideal operational amplifier would have an output of 0volt when its differential input voltage is zero. Real operational amplifiers are likely to have a d.c output voltage even when the input voltage is zero.

This is because it is impossible to make a differential amplifier with perfect symmetry. Because of this imperfection, there will always be a d.c voltage at the output of an Operational amplifier, when both inputs are grounded. This voltage is called an output-offset voltage. Such an output voltage is an error voltage, which is undesirable in many applications.

In an operational amplifier, output offset voltage is caused by three main sources, namely (Odo, 2000);

1. Operational amplifier output offset voltage due to input circuitry
2. Operational amplifier output offset voltage due to the output circuitry
3. Output offset error due to the external components used to bias the operational amplifier.

2.14.1 Output offset voltage due to input circuitry

The input stage of most analogue amplifiers is usually a symmetrical arrangement of transistors. These transistors need a small input current to "bias" them for correct operation. If the transistor at each input is a bipolar transistor, this bias current must flow into each of the two input terminals.

However, since the two transistors at the input terminals of an operational amplifier cannot be perfectly matched, the currents that flow into the input terminals of the transistors are therefore not exactly equal. These unequal bias currents give rise to an offset in the output, and input offset current I_{IO} at the input.

A straight forward way of assessing the effect of nonzero input current upon circuit performance is to determine the output with zero input signals. We shall do this for the inverting and non-inverting amplifiers. As far as d.c conditions are concerned, figure 2.11 represents both non-inverting and inverting circuit configurations for an operational amplifier with feed back (Crecraft et.al,1993). R_3 represents the equivalent resistance of the bias current path from the non-inverting input to the 0V rail. In the non-inverting case, the signal is fed in here, so this bias path will include the source resistance if the signal is directly coupled.

Both R_F and R_1 provide paths for the bias current to the inverting input. In the inverting case, the signal is fed in between R_1 and 0V, and R_1 includes the source resistance if the signal is directly coupled. If the path from the non-inverting input to the 0V rail includes a series capacitor (in either inverting or non-inverting cases) then R_1 is infinite as a bias current path, and all the current must flow through R_F

Because of the unavoidable mismatch at the input circuitry of an op-amp, I_p and I_n differ slightly. The input bias current I_B is defined as the average of the two:

$$I_B = (I_p + In)/2$$

While their difference is called input offset current (I_{IO}):

$$I_{IO} = I_n - I_p$$

The polarity of I_{IO} depends on the direction of mismatch of the input transistors. Usually I_B is about 5 to 10 times greater than input offset current I_{IO}.

2.14.1.1 Effect of input currents i_b and i_{io}

To evaluate the offset in output voltage caused by the input bias currents (I_B and I_{IO}), we make use of figure 2.11

At node X,

$$I_2 = I_1 + (I_B + I_{IO}/2)$$

Substituting for I_1 and I_2

$$(V_0 - V_n)/R_F = V_n/R_1 + (I_B + I_{IO}/2)$$

Therefore,

$$V_0 = V_n(1 + R_F/R_1) + (I_B + I_{IO}/2)R_F \tag{2.41}$$

Because of the very high d.c open-loop gain of the op-amp, $V_n \approx V_p$. Thus

$$V_n \approx V_p = -(I_B - I_{IO}/2)R_3$$

So substituting V_n in equation (2.41)

$$V_0 = -(I_B - I_{IO}/2)R_3 (1 + R_F/R_1) + (I_B + I_{IO}/2)R_F$$

Collecting terms gives:

$$V_0 = I_B[R_F - R_3(1 + R_F/R_1)] + I_{IO}[R_F + R_3(1 + R_F/R_1)]/2 \tag{2.42}$$

To make equation (2.42) independent of I_B, we make

$R_3(1 + R_F/R_1) = R_F$. That is,

$$R_3 = \frac{R_F}{(1 + {}^{R_F}\!/_{R_1})} = \frac{R_F R_1}{R_F + R_1} = R_F \,//\, R_1$$

When this is done, then $V_0 = I_{IO}R_F$ due to input currents alone.

Many technologies are used in fabricating the differential input stage of an operational amplifier. Examples are bipolar input stage, JFET input stage, and MOSFET input stage. Operational amplifiers are also available entirely in CMOS technology. In all these technologies, manufacturers have devised several ways to keep I_B and I_{IO} as small as possible.

Table 2.1: List of common operational amplifier and their I_B and I_{IO} characteristics

Op-amp	Technology	I_B	I_{IO}	Manufacturer
LM308 & 312	BJT	1.5nA	0.2nA	National Semiconductor
LM312, OP-08	BJT	1nA	0.08nA	Precision Monolithic
LM 312 OP-08	BJT	50pA	10pA	National Semiconductor
LF385/6/7	JFET	30pA	3pA	National semiconductor
TL070/080	BiFET	30pA	5pA	Texas Instrument
AD549	JFET	≤100fA		Analogue Devices.
CA3130	BiMOS	2pA	0.1pA	Raytheon
CA080/1/2/3/4	BiMOS	15pA	5pA	-
ICL761x/2x/3x	CMOS	1pA	0.5pA	Intersil
TLC271/2/4/7	CMOS	1pA	1pA	Texas instrument

From the above data, we see that operational amplifiers made of FET and CMOS technologies have lower I_B and I_{IO} than BJT input op-amps.

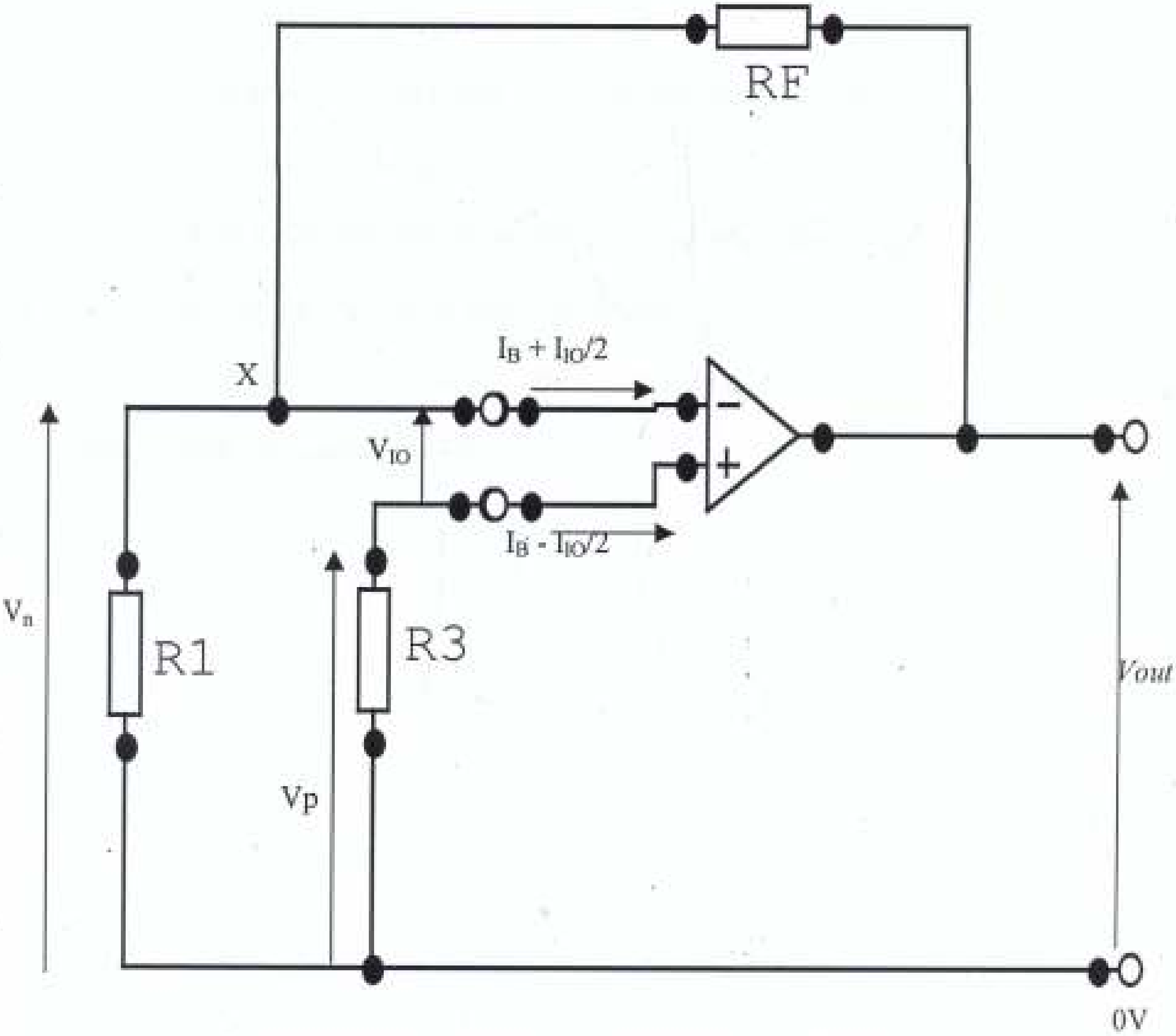


Fig 2.11: D.C bias paths for a real Operational Amplifier.

1.14.1.2 Input bias current variation with temperature

Fig 2.12 compares typical bias current and bias current variation with temperature for various technologies (Franco 1988). It is noted that the FET input operational amplifiers as compared with BJT-input types, namely, an approximately exponential increase of gate current with temperature.

A well-known rule of thumb states that gate current doubles with about every 10°C of temperature increase. Thus the advantage that FET-input op-amps hold over their bipolar counterparts at room temperature (the condition for which the ratings in figure 2.12 are given) disappears at sufficiently high temperatures.

Knowledge of the intended temperature range of operation is an important factor when selecting the optimal device.

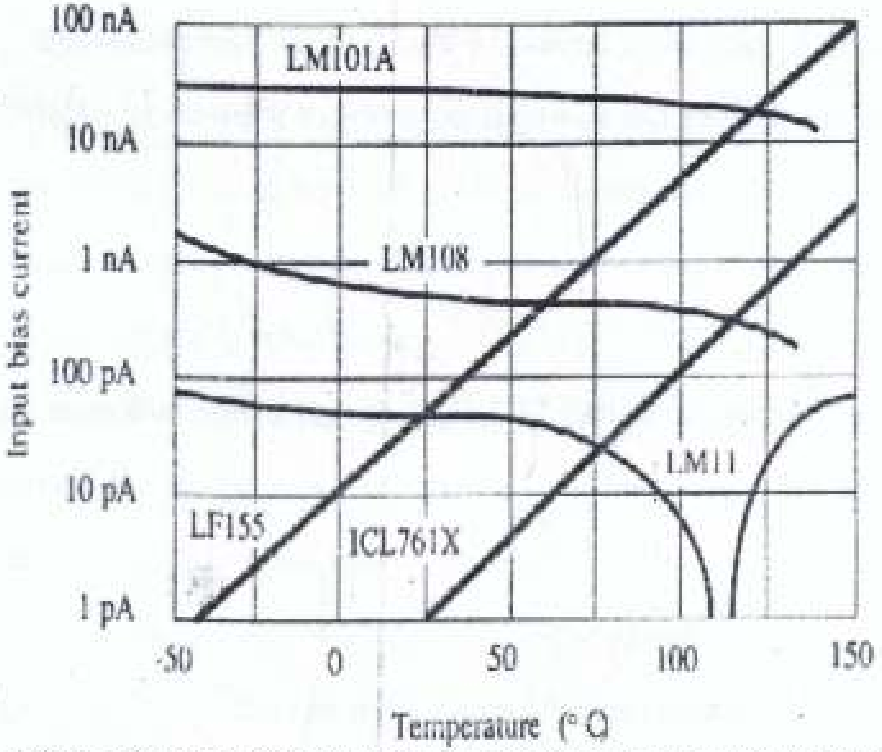


Fig 2.12: Comparison of typical bias currents for various op-amp types (Franco, 1988): LM101A (conventional bipolar), LM108 (bipolar superbeta), LM11 (bipolar darlington superbeta), LF155 (biFET), and ICL761X (CMOS).

2.14.2 Effect Of Input Offset Voltage, V_{io}

Input offset voltage V_{IO} is the voltage, which must be applied across the input terminals of an operational amplifier in order to bring its output voltage to zero. It is caused by manufacturing tolerances. There exists an output-offset voltage V_{OO} as a result of V_{IO}. To nullify this error voltage, we need to have a circuit at the input of the op-amp that will give the flexibility of obtaining V_{IO} of proper amplitude and polarity. Any circuit that enables us to achieve this is known as Input-offset voltage compensation network.

As is often quoted in many electronics text, the error caused by V_{IO} is given as:

$$V_{OO} = (1 + R_F/R_1)\, V_{IO}$$

Manufacturer data sheets provide typical as well as maximum values of V_{IO} at room temperature. For the 741C, V_{IO} = 2mV (typ) and 6mV (max) ; for the 741E, V_{IO} = 0.8mV (typ) and 3mV(max). These figures are typical of many low cost operational amplifiers, although types are available with much lower offset ratings.

2.14.2.1 Changes in V_{io} due to temperature drift

The V_{IO} ratings are usually given at room temperature, and, like other parameters of the op-amp, V_{IO} is temperature dependent. The temperature coefficient of V_{IO} is the change in V_{IO} brought about by a 1°C temperature change and is represented by $\frac{\Delta V_{IO}}{\Delta T}$.

LM 301A is rated at $\Delta V_{IO}/\Delta T = 6\mu V/°C$ (typ) and $30\mu V/°C$ (max). These ratings are typical of the BJT family. There are some operational amplifier types, which are specifically designed for low input offset voltages and low temperature coefficient of V_{IO}. As an example, the OP-07 family of ultra low-offset–voltage operational amplifier comes in versions with $V_{IO} = 10\mu V$ and $\Delta V_{IO}/\Delta T = 0.2\mu V/°C$.

On the basis of the average temperature coefficient one can estimate the value of V_{IO} at a temperature other than 25°C as:

$$V_{IO}(T) \approx V_{IO}(25^{\circ}C) + \Delta V_{IO}/\Delta T(T-25^{\circ}C)$$

FET input operational amplifiers traditionally have been plagued by poorer input offset voltage and offset drift characteristics than their bipolar counterparts. However with advances in design and fabrication, this difference has become less pronounced.

2.14.3 Input Guarding

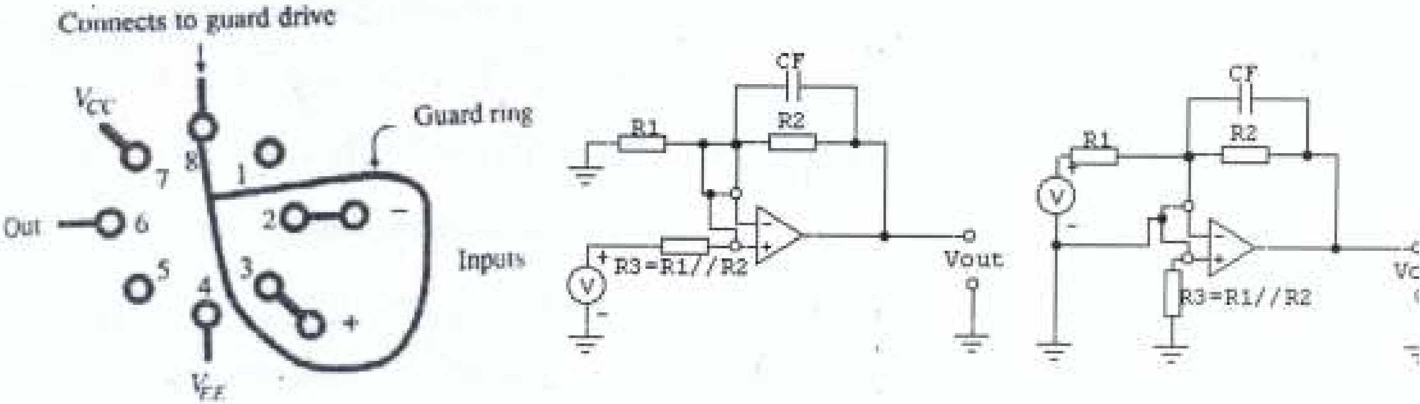
When an operational amplifier is wired on a printed circuit board, there is every possibility for current to leak across the circuit board. Therefore, an operational amplifier with very low input bias currents must be properly guarded to protect it from leakage currents. If this is not properly done, the leakage current can easily exceed the input bias current itself, thereby degrading circuit performance.

Leakage currents can be reduced in two ways, namely;

- Mounting the operational amplifier on a Teflon IC socket
- Putting the input terminals in a guard ring if the operational amplifier must be soldered directly on the board (see fig 2.13).

A guard consists of a conductive ring surrounding the input terminals and driven at the same potential. The guard ring will absorb leakages from other points on the board and thus prevent leakage to the operational amplifier inputs. Secondly, the guard ring acts as a shield against noise pickup and reduces the common-mode input capacitance seen by the inputs (Franco, 1988).

To combat the stray capacitance of the ring itself, a small compensating capacitance (of a few pF) is connected between the output and the inverting input. When a sensor and its amplifier are far apart, a coaxially shielded cable with good insulation must be used in addition to the guard ring. The cable shield must be kept at the same potential as the guard ring.



(a) (b) (c)

Fig 2.13: Input guard layout and guard connections for the non-inverting (b), and inverting (c) configurations

2.15 OFFSET ERROR COMPENSATION

Manufacturers of precision operational amplifier employ various techniques to minimize sources of error in their products. One of such means is a provision made to compensate for all sources of error from I_B and V_{IO}.

Offset error compensation, also referred to as offset nulling, offset trimming and offset balancing, can be achieved in more than one way depending on the operational amplifier type as well as the configuration used.

Offset nulling techniques are classified as internal and external.

2.15.1 Internal offset nulling

Fig 2.14 is the circuit diagram for internal offset nulling. The nulling procedure is straightforward. The potentiometer R_V is adjusted to drive V_O to zero. However, it should be noted that for internal offset nulling to be carried out, the operational amplifiers must have provision for it. For example the popular 741 operational amplifier has this facility accessible on its pins 1&5.

When a precision circuit involves more than one operational amplifier, experience has shown that introducing offset nulling into just one of the devices can null the overall error. However, for this technique to succeed, the offset voltage adjustment range must be wide enough to span the combined effect of all the other offsets. This is because the overall error is the result of the algebraic sum of all the individual offset errors. It is worth noting that not all operational amplifiers have provision for internal offset nulling. Even for those with this provision, the nulling scheme varies among types. For example the LF356 requires a 25KΩ pot with the wiper connected to Vcc. It is always advisable to consult the data sheets to find the recommended scheme for any device in use.

Though internal offset nulling is quite easy to implement, it affects other characteristics of the operational amplifier. For example in LM321 & OP-10, it reduces thermal drift. While in some other types, it degrades the thermal drift, the CMRR and the

PSRR. Unfortunately, data sheets rarely provide adequate information in this regard. When in doubt, external offset nulling (which is being discussed next) offers a more predictable alternative.

2.15.2 External offset nulling

External nulling (shown in Figure 2.15) is based on the injection of an adjustable d.c voltage into the circuit to compensate for the existing offset error. External nulling does introduce additional imbalance in the input stage; hence there is no degradation in temperature drift, CMRR and PSRR.

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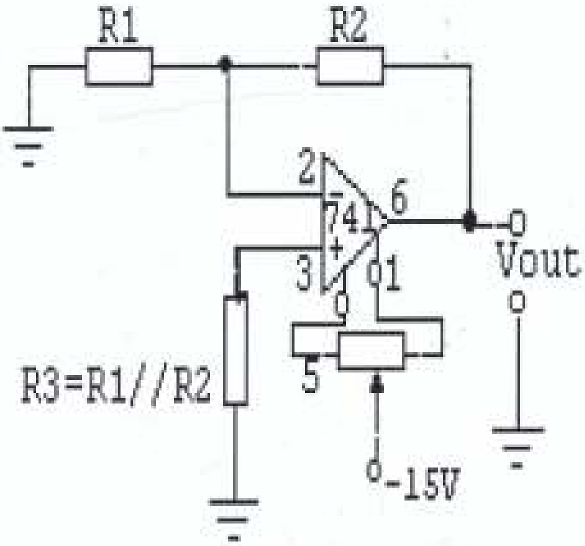

Fig 2.14: Resistor feedback configuration with internal offset nulling

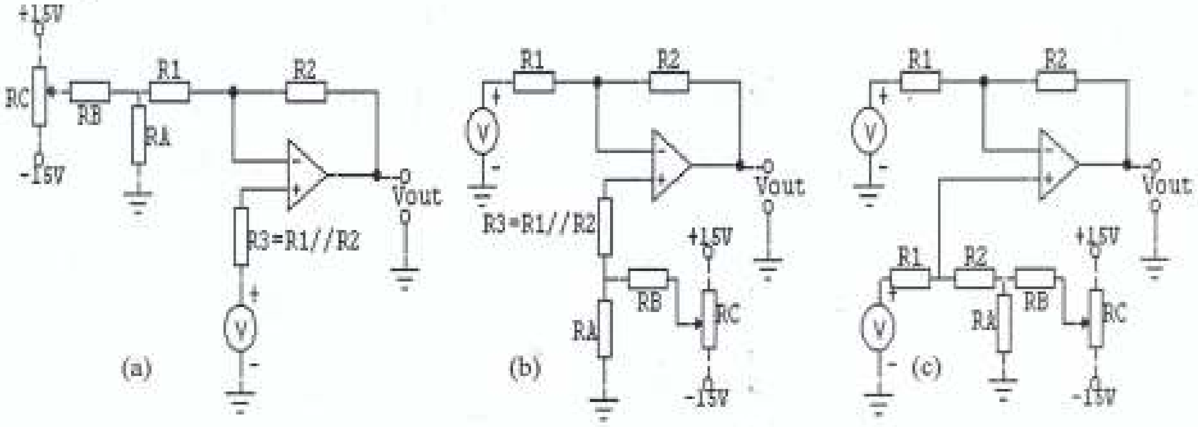

Fig 2.15: External offset nulling for (a) non-inverting amplifier, (b)inverting amplifier and (c)difference amplifier

2.15.3 Auto zero techniques

Offset nulling, whether internal or external, is done at one specific set of temperature, common-mode, and supply voltage conditions. As these conditions change so does the offset error. Manufacturers of precision operational amplifiers have developed special types of operational amplifier that are capable of continuous offset correction, irrespective of the temperature and supply voltage condition.

These special types of operational amplifiers are known as chopper-stabilized operational amplifier.

The main properties of the autozero chopper-stabilized operational amplifier are

1. They are built on CMOS technology.
2. Their V_{IO} and temperature coefficients are five times better than the best precision bipolar op-amp.
3. They deliver their full speed and bandwidth.
4. Being a CMOS device, most of them have a severely limited supply voltage (typically 15V. total supply). Because of this, it cannot run from conventional ±15Vsupply. The only exceptions to this are the Maxim MAX430/2 and Teledyne TSC915 and TSC76HV52, which operate from ±15V supplies.
5. Many autozero operational amplifiers have restricted common-mode input voltage. For example, the popular ICL7650 has a guaranteed common-mode input range of -5V to 1.5V when running from its usual ±5V supply. For the improved ICL7652, the range is from -4.3V to 3.5V; that is a wider range, but it does not include the negative rail, so it cannot be used as a single supply operational amplifier.

The high voltage types are much better. For example, the MAX432 has a guaranteed common-mode range of -15V to 2V when running from ±15V supplies.

6. They have poor output current sourcing capability – sometimes as little as 1 - 2mA in the sourcing (positive output) direction.

7. They have disastrous saturation characteristics. Recovery is very low – up to a second. The "cure" is to sense when the output is approaching saturation, and clamp the input to prevent it. Most auto-zeroing operational amplifiers provide a "clamp" output for this purpose, which can be tied back to the inverting input to prevent saturation.

Saturation can be prevented in chopper operational amplifier without a "clamp" pin (and in ordinary op-amps as well) by bridging the feedback network with a bi-directional zener (two zeners in series) (Horrowitz and Hill 1995). This will clamp the output at the zener voltage, rather than letting it limit at the supply rail; this works best in the inverting configuration.

8. Chopper stabilized operational amplifiers are plagued with the problem of clock induced noise. It is caused by charge coupling from the MOS switches and can cause spikes at the output. In low frequency applications, we can (and should) RC – filter the output to a bandwidth of a few hundred hertz, which will make these spikes disappear (Horrowitz and Hill 1995). This spiky noise is of no importance in integrating applications (e.g. integrating A – D converters) or in applications where the output is low (e.g., a thermocouple circuit with a meter at the output). Infact, if we only want very low frequency (below 1Hz) output response, a chopper operational amplifier will actually have less noise than a conventional low noise operational amplifier (Horrowitz and Hill 1995).

2.16 OUTPUT OFFSET ERROR DUE TO OPERATIONAL AMPLIFIER OUTPUT CIRCUITRY

There are limits to the amount of voltage and current that an operational amplifier can deliver to a load, and there are limits as to how quickly this voltage can change. These limitations become sources of error when using operational amplifier in ac circuits.

Though most operational amplifiers have over-current/short-circuit protection, it is a good design practise to always ensure that operational amplifier circuits are terminated with high impedance device so as to prevent excess current been drawn from the device (Odo, 2000). Otherwise, if the operational amplifier must drive a low impedance load, then, a unity buffer must come between the amplifier and the load.

2.17 ERROR DUE TO THE EXTERNAL NETWORK COMPONENT

Operational amplifiers like some other monolithic IC's needs to be biased alongside with some resistors, inductors, diodes and capacitors for proper operating condition. The degree of precision of circuits realized from such arrangements depends largely on the accuracy and thermal stability of the operational amplifier and indeed the entire discrete components used.

For example, the common-mode-rejection ratio of a differential amplifier is known to be greatly affected when the ratio of the two pairs of resistor used at the differential input do not match. The accuracy and linearity of the ramping action of integrators is also largely dependent on the properties and stability of the capacitors used in the circuit.

Following the above facts, it can be seen that the choice of component is vital in any circuit implementation.

Components are generally specified with an initial accuracy, as well as the change in value with time and temperature. Complete specifications also include the effects of temperature, cycling and soldering, shock and vibration, short-term overload and effect of moisture.

Table 2.2 gives the specification for the commonly used two resistor types. From this table, it is obvious that metal film resistors are better than carbon composition resistors for precision work.

Table 2.2: Variation of Resistor properties (Horowitz and Hill 1995 pg. 1055).

Material	Temperature coefficient (tempco)	Soldering temperature/load cycle	Shock and vibration	Moisture
Metal film Resistors	50ppm/°C	-55°C to 175°C	0.1%	0.5%
5% Carbon composition Resistor	80ppm/°C	25°C to 85°C	2%	6%

2.18 MATHEMATICAL OPERATIONS

Because operational amplifier can among other things add, subtract, multiply, divide, integrate or differentiate electrical input signals they can be used to perform any mathematical operations. They perform these operations by the appropriate connection of resistors and capacitors to the external pins of the integrated circuit (IC) package.

2.18.1 SUMMING AMPLIFIER

Consider the current-to-voltage converter shown in figure 2.16. Application of our ideal amplifier rules gives

$V_+ = V_- = 0 \Rightarrow 0 - V_{out} = IR_F$

Therefore $V_{out} = -IR_F$ and the circuit acts as a current-to-voltage converter.

Figure 2.17a shows several current sources driving the negative input of an inverting amplifier. Summing the current into the node gives

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_{out}}{R_F} \qquad 2.43$$

Therefore

$$V_{out} = -(\frac{R_F}{R_1})V_1 - (\frac{R_F}{R_2})V_2 - (\frac{R_F}{R_3})V_3 \qquad 2.43.1$$

If $R_1 = R_2 = R_3$ (=R),

we have

$$V_{out} = -\frac{R_F}{R}(V_1 + V_2 + V_3) \qquad 2.43.2$$

and the output voltage is proportional to the sum of the input voltages.

For only one input and a constant reference voltage as shown in figure 2.17b

$$V_{out} = -\frac{R_F}{R}V_{in} - \frac{R_F}{R}V_{ref} \qquad 2.44$$

where the second term represents an offset voltage. This provides a convenient method for obtaining an output signal with any required voltage offset.

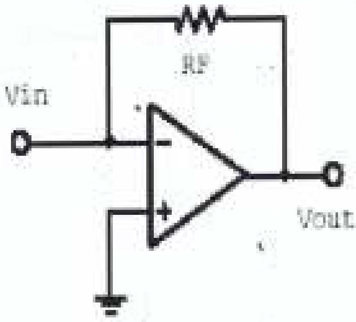

Figure 2.16: Current-to-voltage converter

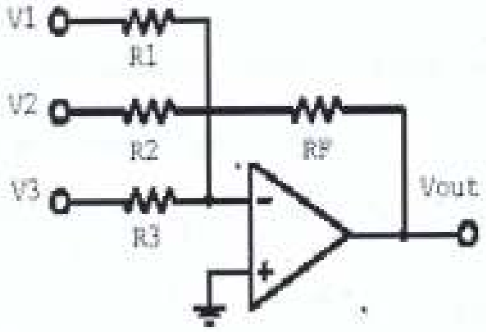

Figure 2.17a: Current summing amplifier

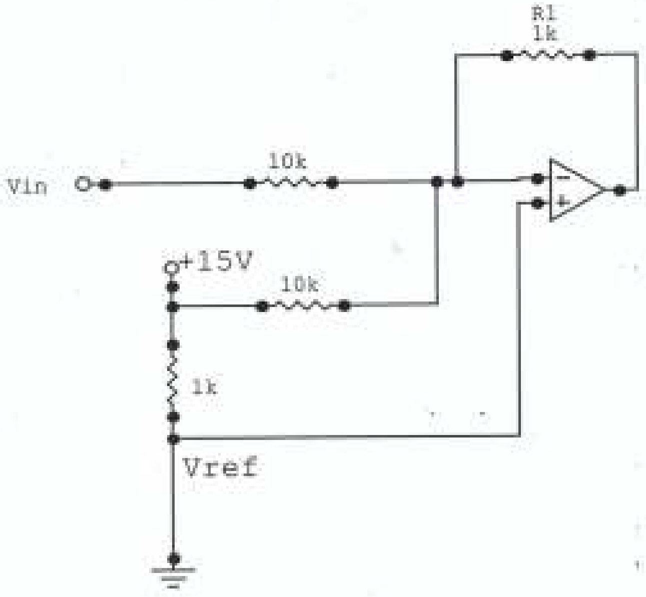

Figure 2.17b: One input summing amplifier

2.18.2 DIFFERENTIATION CIRCUIT

Although this circuit is not used in analogue computer simulation because of its inherent instability and susceptibility to noise, it can be used as a high pass filter.

To obtain a differentiation circuit we replace the input resistor of the inverting amplifier with a capacitor as shown in figure 2.18a

Replacing R_1 with $Z_C = 1/(j\omega C)$ in the voltage gain gives

$$G(j\omega)=\frac{V_{out}}{V_{in}}=-\frac{R}{Z_C}=-j\omega RC \tag{2.45}$$

or

$$V_{out}=-j\omega RCV_{in}=-RC\frac{dV_{in}}{dt} \tag{2.46}$$

The product RC is called the time constant of the circuit.

The frequency response is shown in figure 2.18b

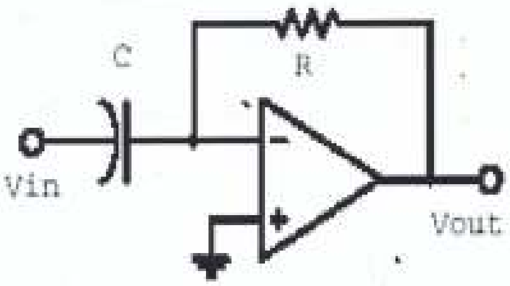

Figure 2.18a: Differentiation circuit

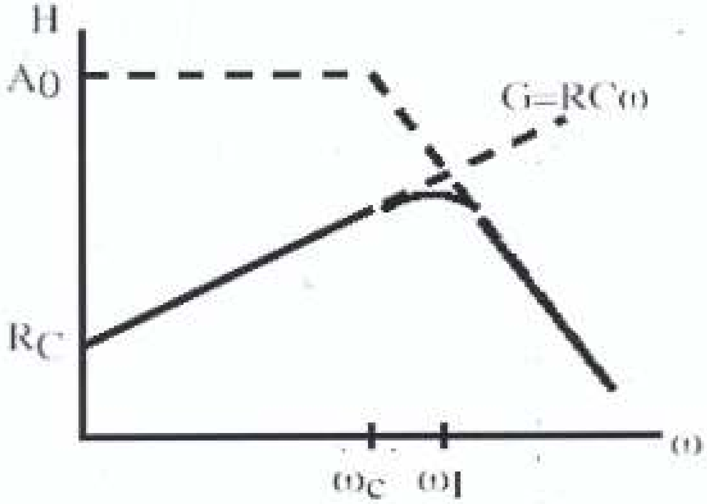

Figure 2.18b: Frequency response differentiation circuit

2.18.3 INTEGRATION CIRCUIT

Integration is obtained by interchanging the resistor and the capacitor of the inverting amplifier as shown in figure 2.19a. The capacitor is now in the feedback loop. Using the circuit in figure 2.19a, the output voltage is the time integral of the input voltage

Analysis of the circuit gives

$$G(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_C}{R} = \frac{-1}{j\omega RC}$$

or

$$V_{out}(t) = \frac{-V_{in}}{j\omega RC} = \frac{-1}{RC}\int V_{in}(t)dt + V_o(t_o) \qquad 2.47$$

where V_o is the initial voltage on the capacitor C and the product RC is called the time constant of the circuit.

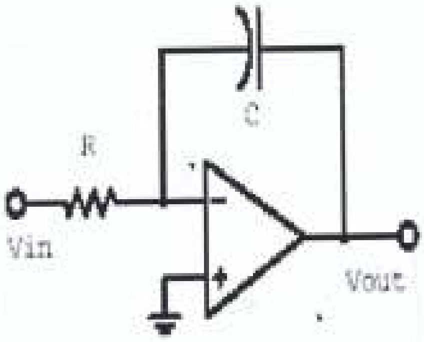

Figure 2.19a: Integration circuit

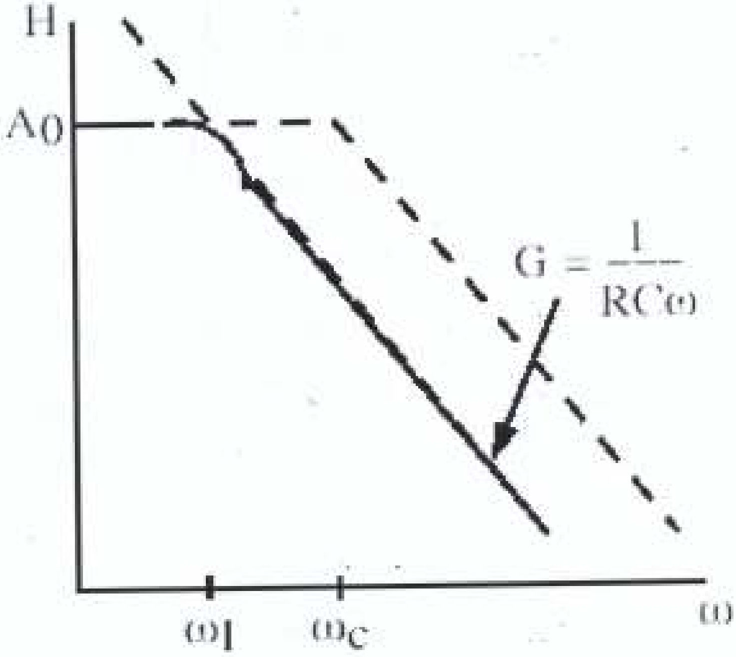

Figure 2.19b: Frequency response of Integration Circuit

2.18.4 THE COEFFICIENT POTENTIOMETER

Multiplication of a variable by a constant coefficient less than unity can be achieved by using a linear potentiometer. In an analogue computer these are usually ten-turn helical potentiometers or switch attenuators with a total resistance of the order of 50kΩ.

From figure 2.20,

$$e_i = iR \text{ and } e_o = ikR$$

$$\frac{e_o}{e_i} = k \qquad 2.48$$

where $k \leq 1$ and is called the potentio

Potentiometers in analogue computer may be ungrounded or have one side of the potentiometers grounded. A potentiometer can only provide a multiplication factor, which lies between zero and unity i.e.

$$0 < k < 1$$

When a coefficient that is greater than unity is required, is it necessary to factorise the coefficients into parts:

(a) One, which is less than unity, which will be used as a potentiometer setting

(b) One, which is greater than unity, which will be incorporated into the gain of the following amplification stage.

For example, if the coefficient happened to be 25, then k=25 is too large for a potentiometer setting, so 25 is factorised as

$$25 = 0.5 \times 50$$

that is,

Coefficient value = potentiometer setting x gain of next stage

It is then possible to adjust the value of k from zero to 50, which provides plenty of working room.

Potentiometer can also be used as a voltage divider but it is not associated with the amplifier input resistor.

When using potentiometer, it is necessary to make a correction for the loading effect of the input impedance at the operational amplifier to which it is connected.

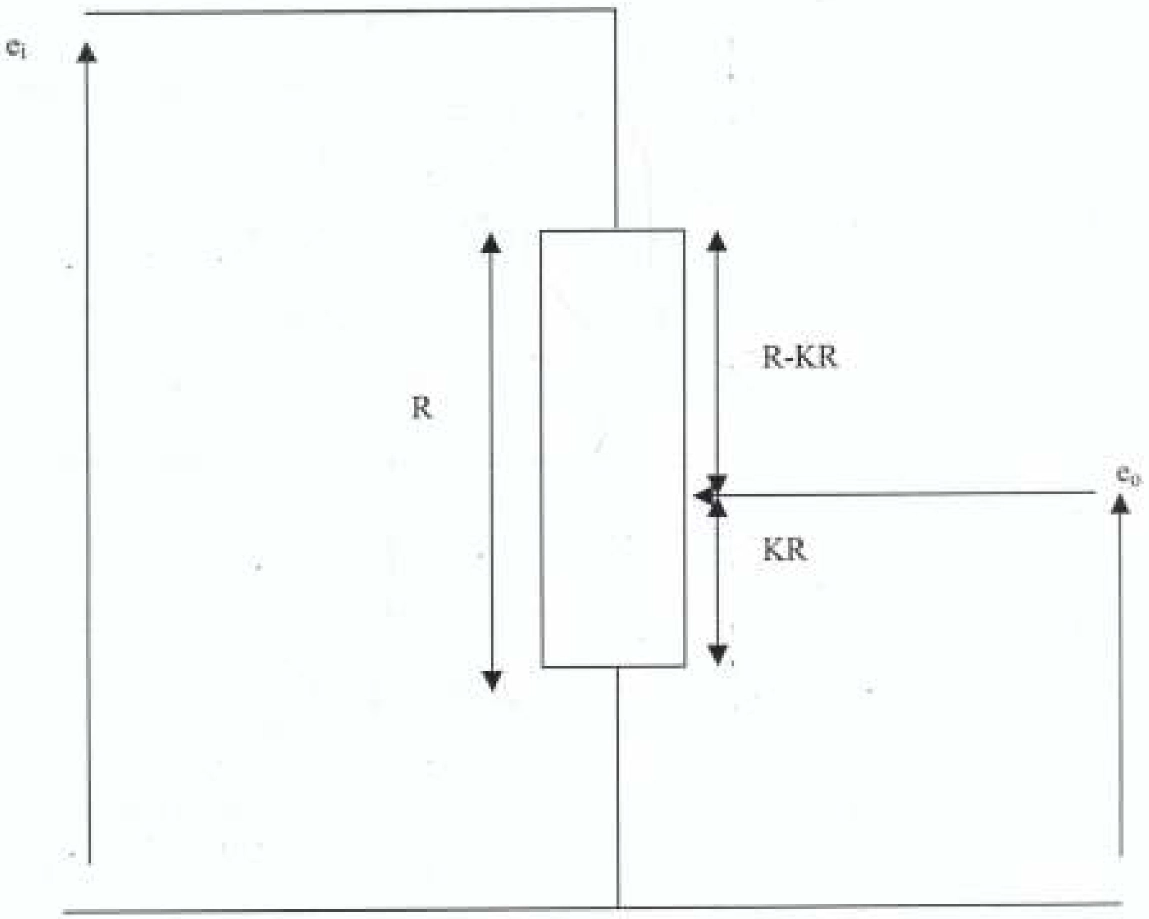

Figure 2.20: The Coefficient Potentiometer

2.19 AMPLITUDE SCALING

The output from an analogue computer may be a voltage representing the response of a controlled system. However, the amplitude of the output voltage, regardless of the magnitude of the physical quantity it represents, has to be within the voltage range of the final operational amplifier. There are cases (or problems) where the numerical values of variables exceed the permitted operating range of amplifiers thereby driving them into saturation. Under such conditions, the amplifier is overloaded and its output is no longer a linear function of its input. At this point it is necessary to scale the magnitude of the voltage to fall within the operating range of the amplifiers and this process is called Amplitude scaling.

Amplitude scaling is a process, which ensures the maximum output voltages are kept within the permissible range. The constant, which relates one unit of the variable to the computer reference voltage, is called the Amplitude Scale Factor.

$$\text{Scale Factor} = \frac{1M.U.}{Maximum\ value\ of\ the\ \text{var}iable}$$

ILLUSTRATION:

Amplitude-scale the following differential equation and obtain the corresponding scaled diagram providing outputs for x, dx/dt and d^2x/dt^2.

$2d^2x/dt^2 + 9dx/dt + 20x = 35$

Given that $x_{max} = 5$, $dx/dt = 10$, $d^2x/dt^2{}_{max} = 20$ and $x_o = x_o$.

SOLUTION:

The scale variables are

$$\left(\frac{x}{5}\right),\left(\frac{1}{10}\frac{dx}{dt}\right)and\left(\frac{1}{20}\frac{d^2x}{dt^2}\right)$$

If equation is rearranged

$$-\frac{d^2x}{dt^2} = -17.5 + 4.5\frac{dx}{dt} + 10x \qquad 2.49$$

The scaling for $\frac{d^2x}{dt^2}$ is

$$-\left(\frac{1}{20}\frac{d^2x}{dt^2}\right) = \frac{-17.5}{20}(1) + 4.5x\frac{1}{2}\left(\frac{1}{10}\frac{dx}{dt}\right) + 10x\ X\ \frac{1}{4}\left(\frac{x}{5}\right) \qquad 2.50$$

Also the scaling for $\frac{dx}{dt}$ and x are respectively

$$\frac{d}{dt}\left(\frac{1}{10}\frac{dx}{dt}\right) = 2\left(\frac{1}{20}\frac{d^2x}{dt^2}\right) \qquad 2.51$$

and

$$\frac{d}{dt}\left(\frac{x}{5}\right) = 2\left(\frac{1}{10}\frac{dx}{dt}\right) \qquad 2.52$$

writing these equations in terms of potentiometer coefficients and amplifier gain gives

$$-\left(\frac{1}{20}\frac{d^2x}{dt^2}\right) = -[0.875]1(1) + [0.225]10\left(\frac{1}{10}\frac{dx}{dt}\right) + [0.25]10\left(\frac{x}{5}\right) \qquad 2.53$$

$$\frac{d}{dt}\left(\frac{1}{10}\frac{dx}{dt}\right) = [0.2]10\left(\frac{1}{20}\frac{d^2x}{dt^2}\right) \qquad 2.54$$

$$\frac{d}{dt}\left(\frac{x}{5}\right) = [0.2]10\left(\frac{1}{10}\frac{dx}{dt}\right) \qquad 2.55$$

The scale diagram is shown in figure 2.21

Table 2.3: System characteristics of Amplitude and Time Scaling

	Amplitude Scaling	Time Scaling
Overshoot	3.13%	6.25%
Delay Time	0.25s	0.55s
Rise Time	0.35s	0.58s
Settling Time	3.55s	4.25s
Freq. response	0.37Hz	0.37Hz

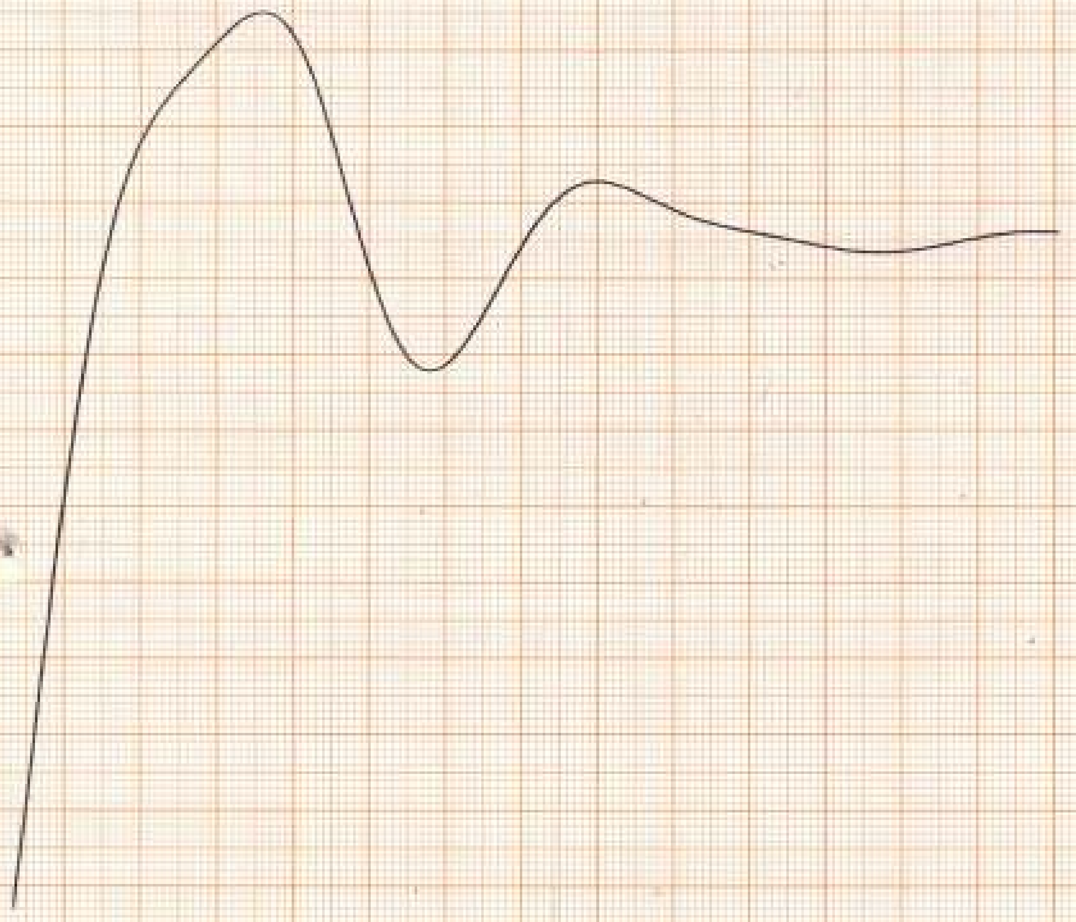

Figure 2.22: Response of the Amplitude Scale diagram of figure 2.21

2.20 TIME SCALING

Output signals from an analogue computer are usually displayed graphically on a chart recorder, an X-Y plotter or a storage oscilloscope. There are times when the actual solution time of a differential equations may be so fast that the recorder is unable to follow the response time accurately, alternatively the solution may also take an excessively long time. In either case, observing the response can be difficult or inconvenient. These problems are overcome by changing from real time to a machine time used by the computer.

The process of computing a problem either faster or slower than the real time is called the Time Scaling. The factor, which relates the computer time to the real time, is called the Time Scale Factor (α), which is given by,

$$\alpha = \frac{\text{time taken by the computer}}{\text{time taken by the actual process}}$$

$$\alpha = \frac{\tau}{t} \qquad 2.56$$

where,

τ is the Computer time, t is the real time and α is the time scale factor

Then,

$d\tau = \alpha dt$

If the solution is slowed down, then

$$\alpha_1 = \frac{\tau_1}{t}$$

where $\tau_1 > t$

hence $\alpha_1 > 1$

If the computer solution is speeded up, then

$$\alpha_2 = \frac{\tau_2}{t}$$

where $\tau_2 < t$

hence $\alpha_2 < 1$

Real time t can, therefore, be expressed in terms of α, the time scaling factor and τ, computer time, as,

$$t = \frac{\tau}{\alpha}$$

The reasons for Time Scaling are:

1. Physical systems may take anything from micro seconds to years to undergo an operation while computer solutions normally take a few seconds
2. The frequency of output signals should be within the accurate range of the variable recorders and output equipments.
3. The gains associated with the inputs of integrators should always be kept below 40.

An example is given to illustrate the time scaling:

A factor of 10 is used to time scale the following equation, which has already been amplitude scaled.

$$-\frac{d}{dt}\left(\frac{x}{5}\right)=[0.75]10\left(\frac{x}{5}\right)+[0.4]10(1)$$

SOLUTION

Dividing both sides by β

$$-\frac{d}{\beta dt}\left(\frac{x}{5}\right)=\frac{[0.75]10}{\beta}\left(\frac{x}{5}\right)+\frac{[0.4]10}{\beta}(1)$$

Substituting $d\tau$ for βdt and 10 for β

$$-\frac{d}{d\tau}\left(\frac{x}{5}\right)=\frac{[0.75]10}{10}\left(\frac{x}{5}\right)+\frac{[0.4]10}{10}(1)$$

This yields the time scaled equation

$$-\frac{d}{d\tau}=[0.75]1\left(\frac{x}{5}\right)+[0.4]1(1) \qquad 2.57$$

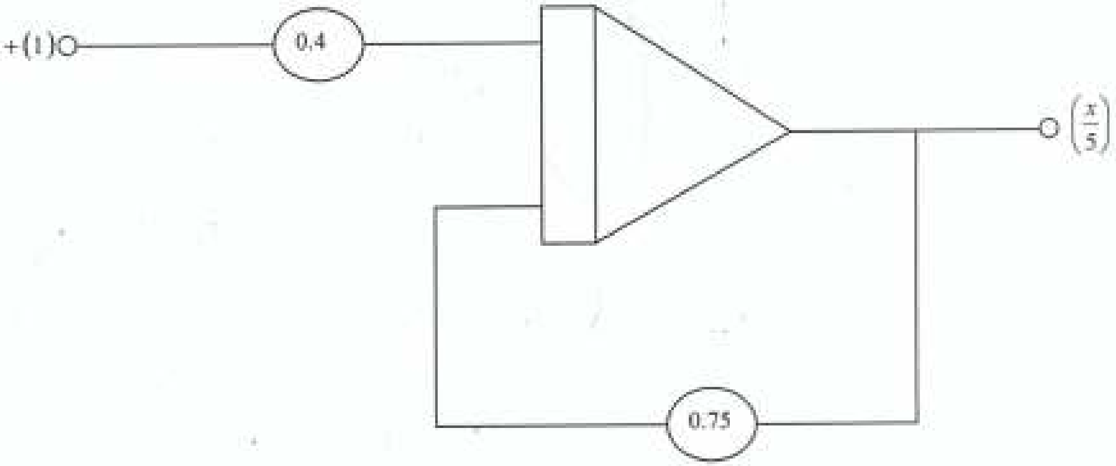

Figure 2.23: Time scaled diagram of equation $-\frac{d}{d\tau} = [0.75]1\left(\frac{x}{5}\right) + [0.4]1(1)$

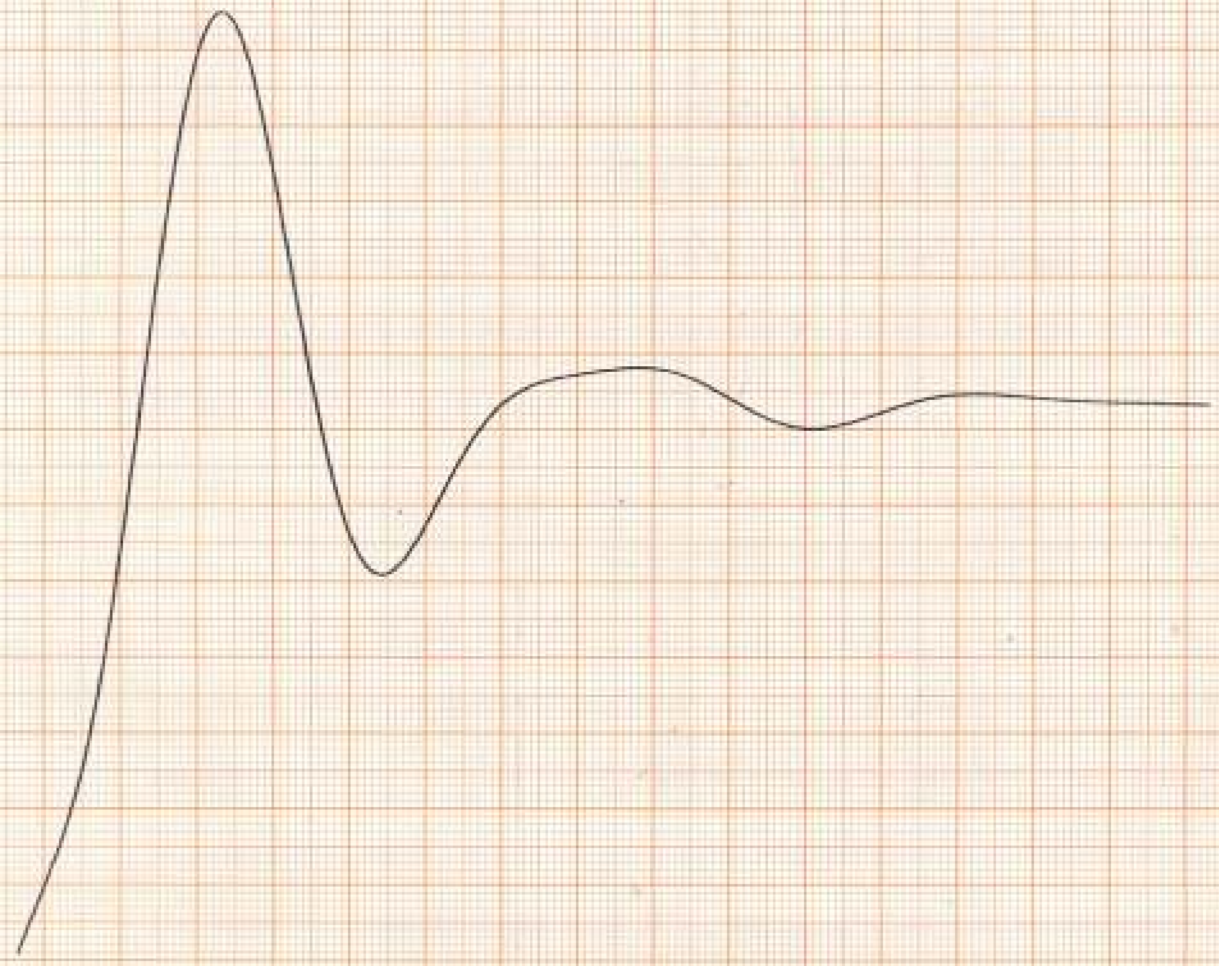

Figure 2.24: Response of the Time Scale diagram of figure 2.23

2.21 WAVE GENERATOR

A comparator with hysteresis may be configured as shown in figure 2.25. This looks very much like a convectional operational amplifier configuration, but it should be noted that feedback is applied to the non-inverting input. In view of the resulting positive feedback, V_o will always be at either the upper or lower saturation voltage (figure 2.25). Assume initially that V_1 and V_o are both positive; clearly, the non-inverting input of the amplifier will be positive and V_o is held at its positive saturation value. Only when V_o is taken sufficiently negative in order to absorb all the current flowing through R_f will the non-inverting input be taken negative thereby causing the amplifier to 'flip over' into its negative saturation state. A subsequent similar reversal will take place when V_i is taken sufficiently positive. This change of state will clearly occur when

$$\frac{V_i}{R_i} = -\frac{(V_o)_{sat}}{R_f} \qquad 2.58$$

where sat denotes the saturation value. The comparator with hysteresis can readily be combined with an operational integrator in order to provide a triangular and square wave generator as shown in figure 2.26. V_T will be a triangular wave with nearly equal positive and negative slopes and V_s will be a square wave, which is positive during the falling ramp of V_T and negative during the rising ramp of V_T. Matching of the positive and negative saturation voltages, and hence positive and negative slopes of V_T can be improved by using Zener diode and a current limiting resistor at the output of the comparator. The limiting voltages become the operating voltages of the Zener diodes. The overall D.C. level of the waveform may be adjusted by returning the inverting input of the comparator amplifier to an appropriate variable voltage.

Different gradients for the rising and falling ramps may be obtained by means of diodes which select different integrator input resistances in the two cases.

The falling ramp may be made very fast, giving an approximation to a sawtooth waveform, by using a high-speed switching device in order to discharge the inverting capacitor rapidly.

2.22 SINE WAVE OSCILLATOR

Sine wave oscillator circuits are characteristed by positive feedback giving a closed loop gain very close to unity and a frequency sensitive network, which ensures that the required conditions for oscillation are satisfied at the required frequency. Many configurations have been used; only the Wien bridge based oscillator, which is widely used, is discussed.

The basic circuit of the Wien bridge is shown in figure 2.27, where $R_1C_1R_2R_2$ constitute the Wien bridge and the amplifier provides an adjustable positive gain. The output from the filter V_f will be related to the output of the amplifier V_o by

$$V_F = V_O \frac{Z_2}{Z_1 + Z_2} \qquad 2.59$$

where Z_1 is the impedance of R_1 and C_1 in series and Z_2 is the impedance of R_2 and C_2in parallel; that is:

$$Z_1 = R_1 + \frac{1}{j\omega C_1} \qquad 2.60$$

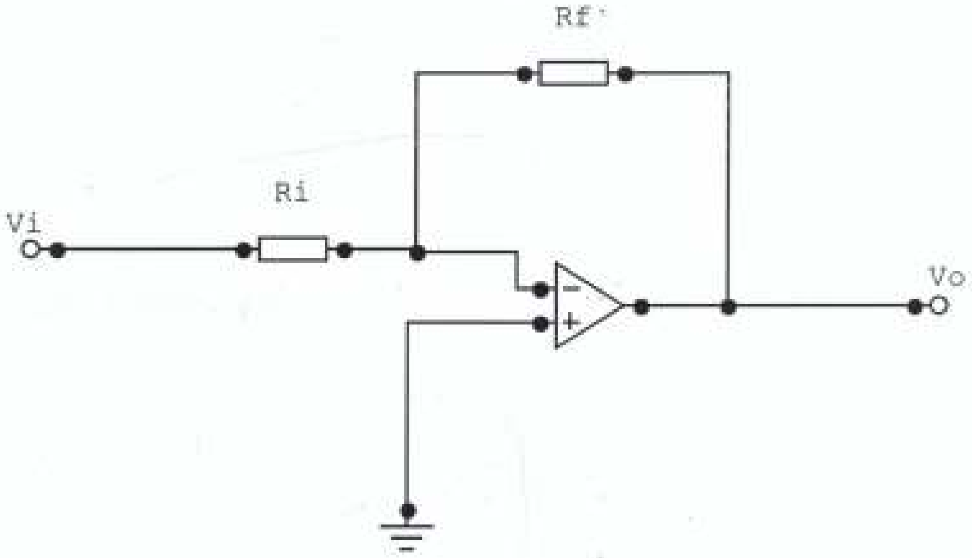

Figure 2.25:Comparator with hysteresis

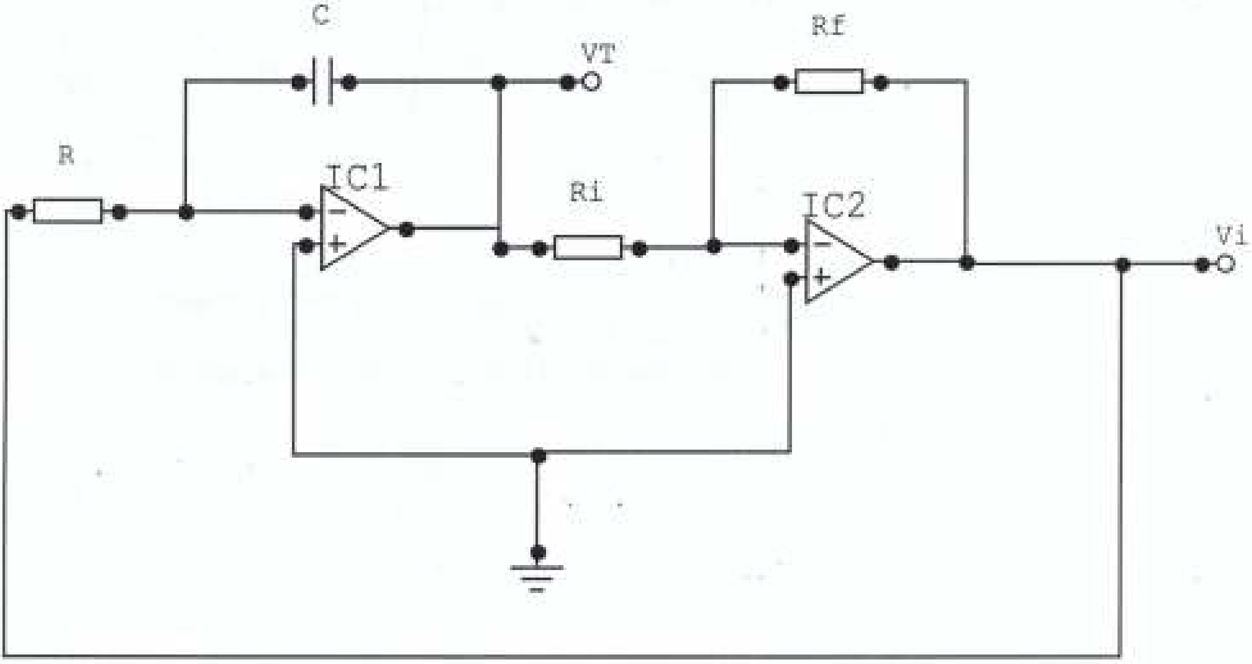

Figure 2.26: Basic Triangular Waveform Generator

$$Z_2 = \frac{R_2\left(\frac{1}{j\omega C_2}\right)}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega C_2 R_2} \qquad 2.61$$

Substituting and simplifying gives

$$V_f = V_o \frac{R_2}{(R_1 + R_2 + \frac{R_2 C_2}{C_1}) + j(\omega R_1 R_2 C_2 - \frac{1}{\omega} C_1)} \qquad 2.62$$

If $R_1 = R_2 = R$ and $C_1 = C_2 = C$, which is convenient but not essential:

$$V_f = V_o \frac{R}{3R + j(\omega R^2 C - \frac{1}{\omega C})} \qquad 2.63$$

For oscillation to be maintained there must be no net phase shift so the imaginary part of equation 2.61 will be zero at the frequency of oscillation; hence

$$\omega = \frac{1}{RC} \quad \text{or} \quad f = \frac{1}{2\pi RC} \qquad 2.64$$

At this frequency the gain of the network will be 1/3 so the amplifier must provide a gain of 3 for oscillations to be maintained. From equation 2.28,

$$\frac{R_f + R_i}{R_i} = 3$$

and hence,

$$R_f = 2R_i$$

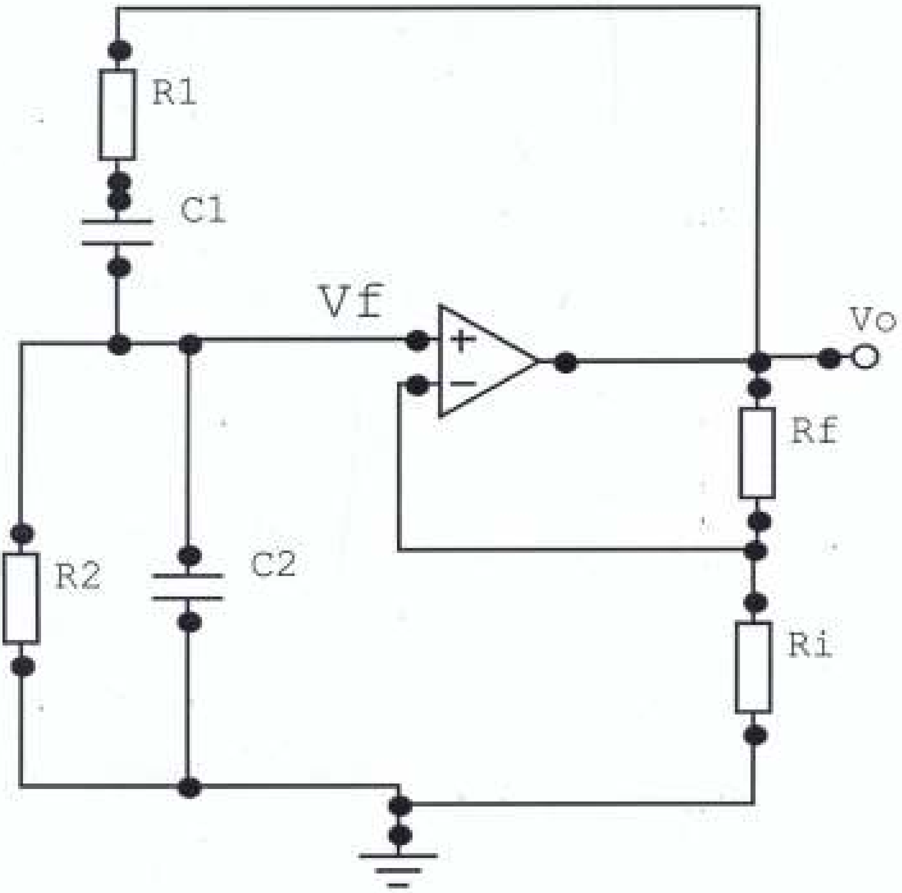

Figure 2.27: Basic Wien bridge oscillator

CHAPTER THREE

DEVELOPMENT AND CONSTRUCTION OF THE ANALOGUE COMPUTER SYSTEM

3.1 SUMMER DESIGN

From the summer circuit in figure 3.1 and equation 2.43.1 given by

$$V_{out} = -\left(\frac{V_1}{R_1}R_f + \frac{V_2}{R_2}R_f + \frac{V_3}{R_3}R_f\right) \qquad 3.1$$

$$= -\left(K_1V_1 + K_2V_2 + K_3V_3\right) \qquad 3.2$$

Where $K_1 = \frac{R_f}{R_1}$; $K_2 = \frac{R_f}{R_2}$ *and* $K_3 = \frac{R_f}{R_3}$

For unity gain inverting amplifier

$R_1 = R_2 = R_3 = R_f = 100k\Omega$

$V_{out} = -(V_1 + V_2 + V_3)$

All the resistances are of carbon film types with ±1% tolerance. Each summer has four inputs. There are a total of eight summers on the board.

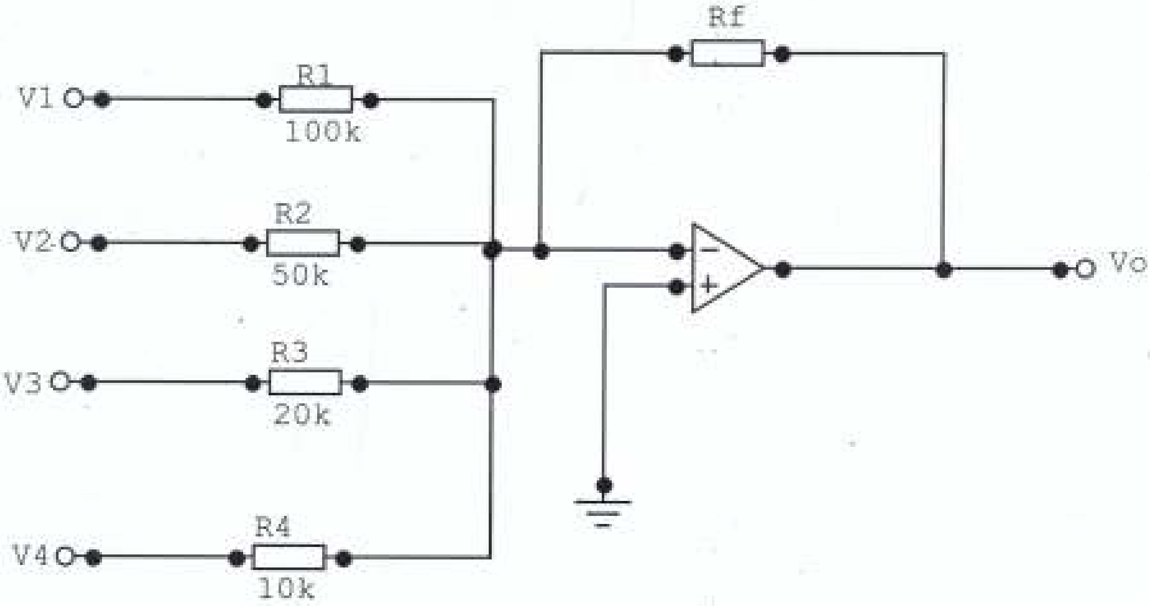

Figure 3.1: Summer design circuit

3.2 INTEGRATOR DESIGN

The principle of integrators has been described in section 2.18.3. The final expression derived was equation 2.47

$$V_{out}(t) = \frac{-1}{RC}\int V_{in}(t)dt + V_o(t_o) \qquad 3.3$$

Where V_o (o) is the initial voltage on the capacitor at t = 0. In our design each integrator is a four input summing-integrator circuit. Polycarbonate capacitor (PCC) was used as feedback component because it has a tolerance value of ± 1% which is very good for signal processing. Selecting 1μf polycarbonate capacitor, the resistor value can be calculated thus.

Using,

where $K = \frac{1}{RC} = 1s^{-1}$ K is the gain of the integrator,

when K = 1

$R = \frac{1}{KC} = \frac{1}{1x1x10^{-6}} = 1M\Omega$ Also when K = 2,

$R = \frac{1}{KC} = \frac{1}{2x1x10^{-6}} = 500k\Omega$ similarly, when K = 5, R = 200kΩ and when K = 10, R = 100kΩ.

3.3 TRIANGLE/SQUARE WAVE GENERATOR DESIGN

Two operational amplifiers are used as shown in figure 3.2 with slight modification from figure 2.24 to give simultaneous triangle and square wave output signals with a perfect symmetric or 50% duty circle. IC1 is wired as an integrator with time constant set by R_4 and C, which will give a ramp-type output for a fixed input voltage. IC2 is used as a comparator to give an output that switches between $+V_{o(sat)}\ and - V_{o(sat)}$ where $V_{o(sat)}$ is the saturated output level of the operational amplifier. The frequency of operation depends on the time constant of the integrator and also the values of R_1 and R_2. These two resistors determine the amplitude of the triangle wave. The frequency of oscillation is given by

$$f = \frac{R_1 + R_2}{4(R_3 + R_4)R_5C} \qquad 3.4$$

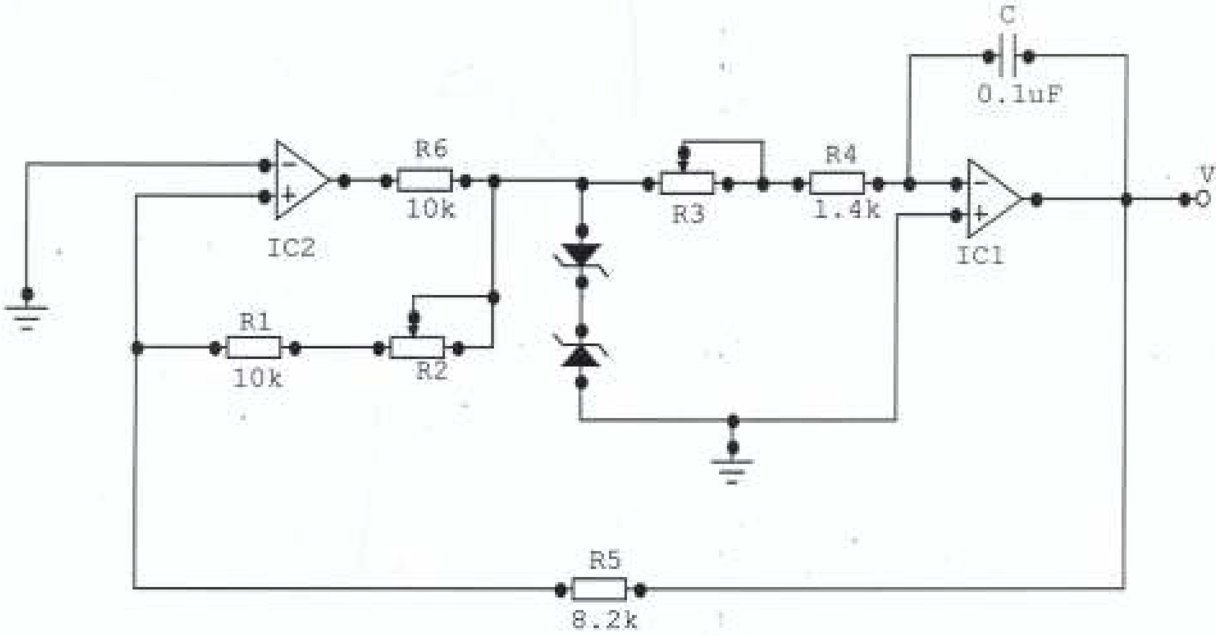

Figure 3.2: Triangle and Square wave generator

3.4 SINE WAVE DESIGN

Equation 2.64 indicates that f_0 can be tuned by varying either R or C. However, since this and the other equations were derived under the assumption of equal value reactive components, to vary f_o both resistors or both capacitors are varied simultaneously. However, this is an awkward constraint.

Let R_p and C_p denote the components in the parallel portion while R_s and C_s denote those in the series portion of the reactive network.

$$\beta = \frac{V_p}{V_o} = \frac{1}{1 + \frac{R_s}{R_p} + \frac{C_p}{C_s}} \quad for \quad f = f_0 \qquad 3.5a$$

Where,

$$f_0 = \frac{1}{2\pi \sqrt{R_s R_p C_s C_p}} \qquad 3.5b$$

The condition for sustained oscillation becomes

$$\frac{R_2}{R_1} = \frac{R_s}{R_p} + \frac{C_p}{C_s} \qquad 3.6$$

According to equations 3.5b and 3.6 and varying the value of one of the components, say R_p, will indeed vary f_o, However, the condition for neutral stability will no longer hold. This indicates that oscillation will either decay or become excessively distorted.

Since R_p is terminated on a virtual ground, f_o is still as in equation 3.5b so we can vary f_o by varying R_p. To find the condition for neutral stability, we note that by the superposition principle

$$V_o = -(R_2/R_1)V_{o2} + (1 + R_2/R_1)V_{p1}$$

where R_2 is the effective resistance in the negative feedback path, V_{p1} is signal at the non-inverting input of A_1, and V_{o2} is A_2's output.

Moreover,

$$V_{o2} = -(R_3/R_p)V_{p1}$$

substituting yields $V_o = AV_{p1}$, where

$$A = 1 + \frac{R_2}{R_1} + \frac{R_2R_3}{R_1R_p} \qquad 3.7$$

Imposing the neutral stability condition for oscillation i.e $A\beta = 1$ where β is feedback element, we obtain

$$\frac{R_2}{R_1} + \frac{R_2R_3}{R_1R_p} = \frac{R_s}{R_p} + \frac{C_p}{C_s} \qquad 3.8$$

To satisfy this condition we set

$$\frac{R_2}{R_1} = \frac{C_p}{C_s} \qquad 3.9a$$

after which equation yields

$$R_3 = \frac{R_sR_1}{R_2} \qquad 3.9b$$

Thus R_p is varied to program f_o without disturbing the neutral stability of the circuit.

In the practical realization of figure 3.3, C_s and C_p have been made equal. This implies that R_1, R_2, R_3 and R_s must also be equal. The operational amplifiers (LF351) are of the bipolar-JFET input type to cope with the large value of R_p. With the component values shown, f_o is variable from 214.6Hz to 21.46kHz. For easier control, the potentiometer is of logarithmic type. Adjusting the ratio R2/R1 can compensate for mismatches between Cs and Cp. Once this is done, R_3 is separately adjusted for a flat amplitude response over the entire frequency range. A 1kΩ thermistor was used in series with 2M pots for Rp.

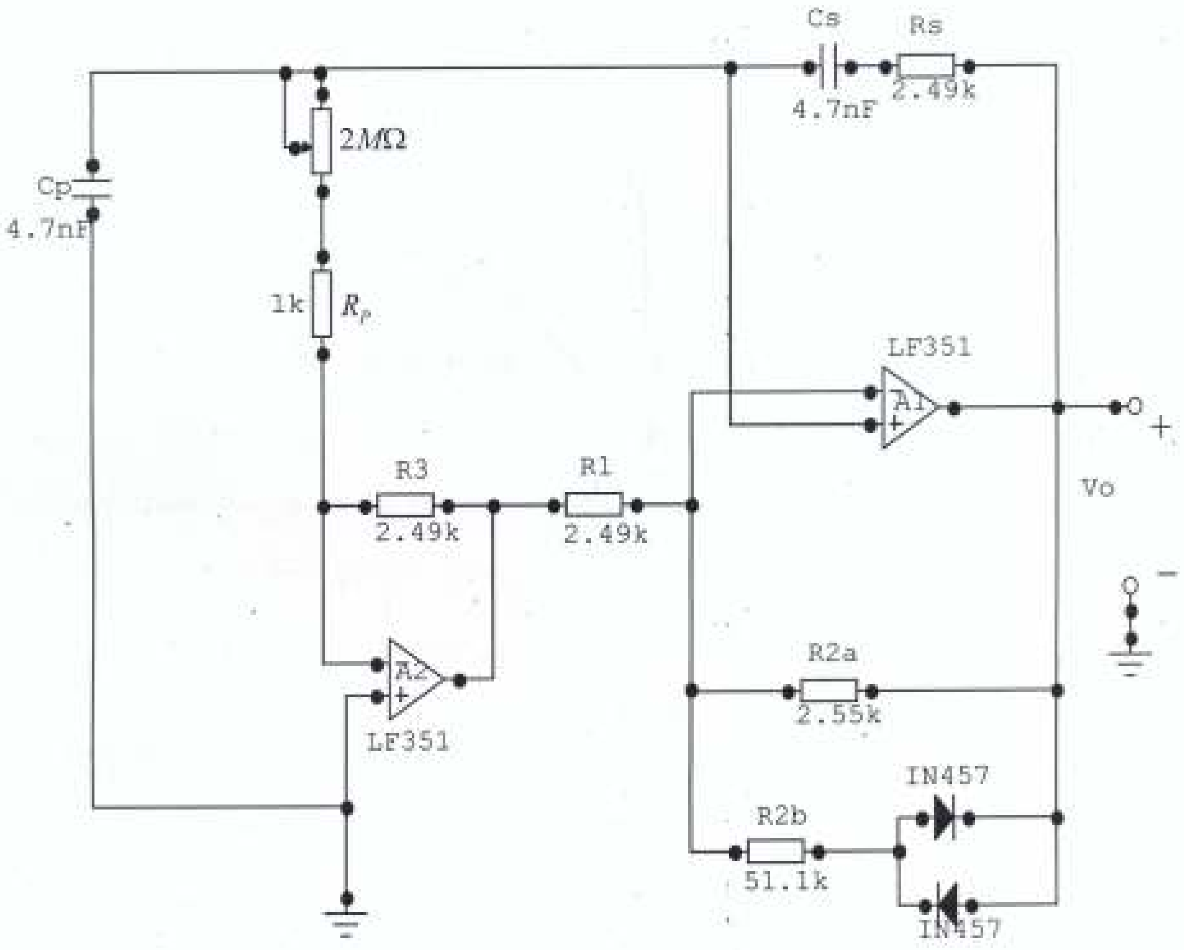

Figure 3.3: Sine Wave Oscillator

3.5 SOLVING PROBLEMS ON AN ANALOGUE COMPUTER

An analogue computer can be used to solve a wide variety of problems including nonlinear functions and time-dependent coefficients. However the equations to be solved here is linear, constant-coefficient, differential equations. They are equations, which describe the behaviour of linear, time-invariant, control system.

Usually the problem to be solved is to measure or record the response of a control system to the application of a particular input signal. The problem is solved by modeling the control system on the analogue computer, applying relevant input signals and recording the voltage output representing the control system response.

Amplifiers, integrators and potentiometers are used to construct the model and the system differential equation dictates the way in which these components are connected together to simulate the control system.

3.6 THE X-Y PLOTTER

The X-Y plotter is an electromechanical device, which gives good recordings of two variables against each other. It consists of two position mechanisms controlling the motion of a pen over a chart. The two axes of motion are perpendicular to each other and correspond to the X and Y coordinates.

A simple X-Y plotter consists of a pen, which is driven, in the Y direction by one servo-motor mounted on the pen carriage. A second servo-motor controls the displacement of the pen carriage in the X direction. The X and Y inputs have different frequency responses because of large difference in the mass of the two servo systems. Time-base units are normally incorporated in X-Y plotters to give Y-t or X-t plots where desired. The major advantage is that it produces X-Y plots of one variable against another variable. The major disadvantage is the relatively poor speed of response, due to the inertia of the moving parts.

X-Y plotters are precision devices and for good and accurate performance they must be well maintained.

3.7 RESPONSE OF THE INTEGRATOR TO VARIOUS INPUT WAVES

Sine, triangular and square waves were in turn applied to the input of the integrator at low frequency from 0.25Hz to 200Hz and the output signal was monitored with an oscilloscope. The results are shown in figures 3.4 to 3.6.

In figure 3.4 the input is a sine wave at a frequency of 100Hz. The figure shows that the output is a sine wave.

Also in figure 3.5 square wave was applied to input of the integrator at the frequency of 250Hz. This produces triangular wave signal at the output.

Similarly as shown in figure 3.6 triangular wave was applied at the frequency of 100Hz to the input of the integrator and this also produces a sine wave signal.

In each case, the result obtained from the integrator agrees with the expected waveform.

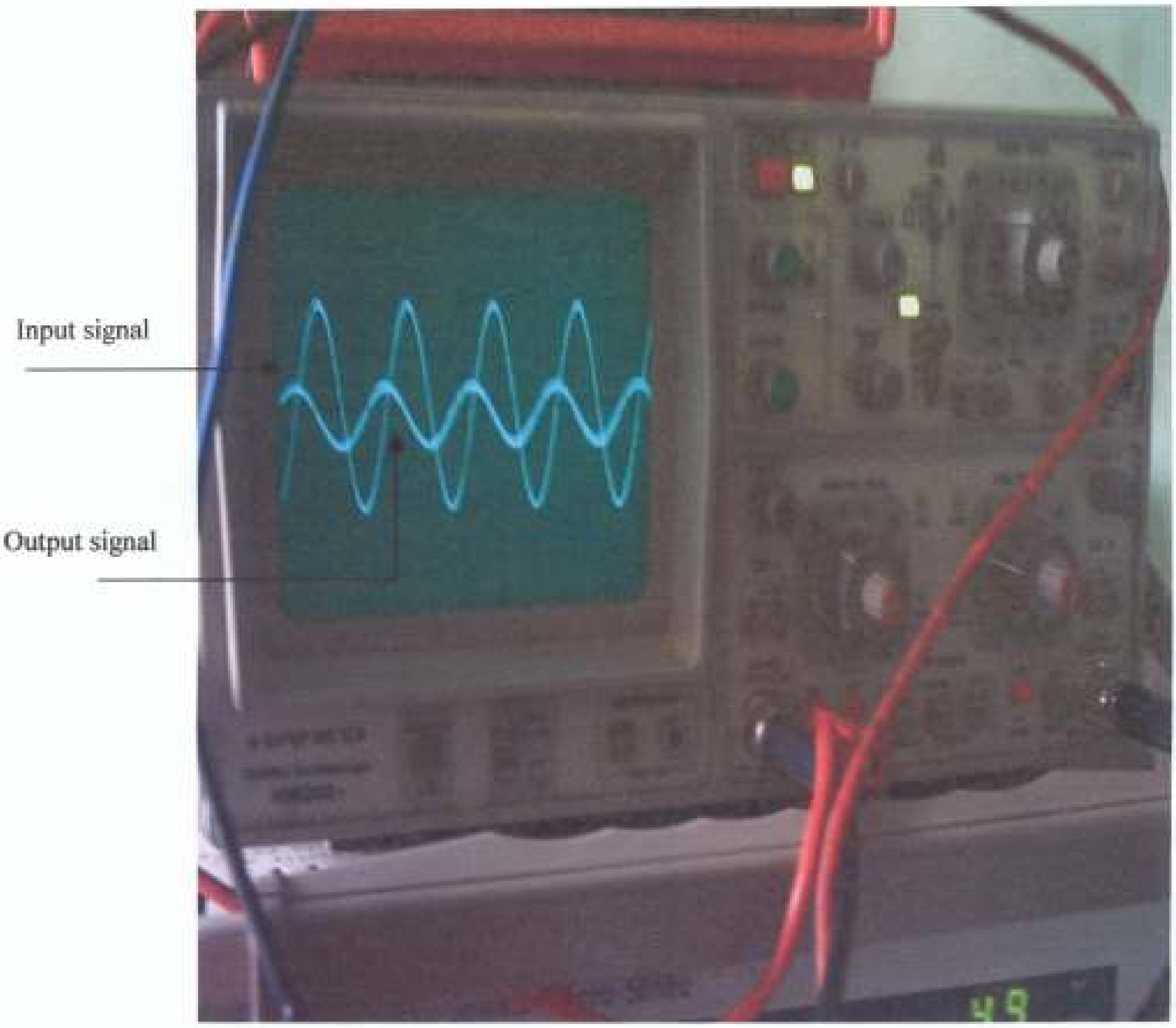

Figure 3.4: Sine wave output

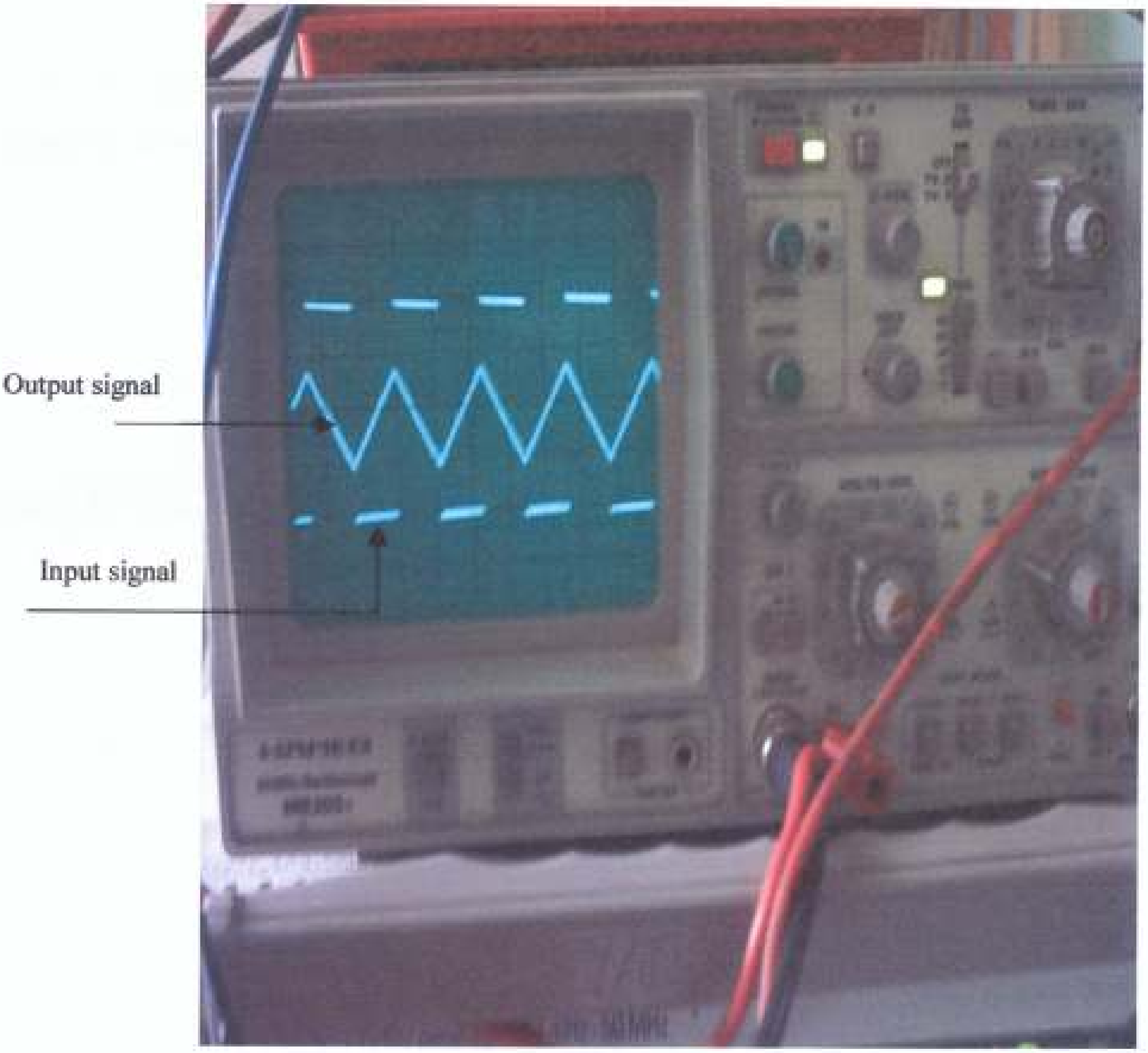

Figure 3.5: Triangular wave output

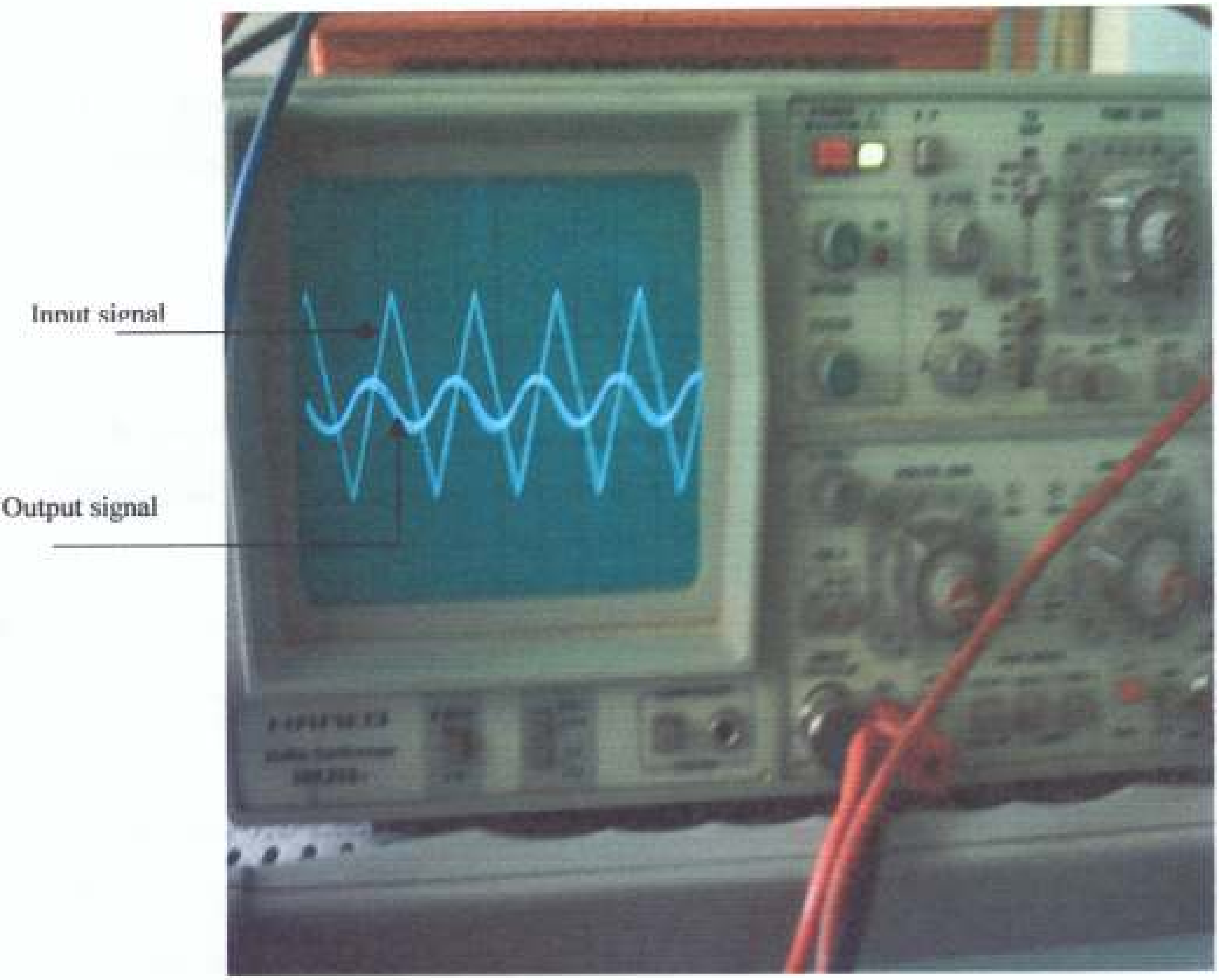

Figure 3.6: Sine wave output

3.8 SIMULATION

Before wiring the analogue computer to solve a differential equations, certain procedures leading to a wiring diagram were followed. The process is outlined is as follows:

1. The differential equation is manipulated to solve for the highest order derivative present in the equation.

2. It is assumed that a voltage representing the highest order derivative is available.

3. With the assumption of step 2, the equation is integrated to produce a voltage representing the next lower order derivative. The integrator is drawn and labeled showing its output with the next lower order derivative (including a minus sign for the phase inversion caused by the integrator)

4. Additional integrators and amplifiers are drawn which are necessary to generate all variables in the differential equation. These are combined in accordance with the equation obtained in step 1.

5. When all the voltages representing all the terms on the right side of the equation in step 1 have been summed together, the result is equal to the highest order derivative that we assumed we had in step 2. Therefore, this sum of terms is connected to the input of the first integrator (the one in step 3). Following the above procedure, the analogue computer constructed was used to solve the following problems:

1 Differential equations 2. Simultaneous Differential equations 3.Control Transfer function

These are demonstrated in experiments 1 to 5

Differential equations

The following differential equations were investigated

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Experiment 1

$$\frac{d^2x}{dt^2}+\frac{dx}{dt}+x=5 \qquad 3.10$$

Following step 1 of the procedure, and solving first for: $\frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2}=5-\frac{dx}{dt}-x \qquad 3.11$$

In accordance with step 2, it is assumed that $\frac{d^2x}{dt^2}$ is available, the term is integrated thus producing $-\frac{dx}{dt}$. This term was made the input of another integrator. The output is x. The scaling capability of the integrator is used to generate 5x. Since –x is needed, the next step will be to use an amplifier to provide a phase inversion. Also a coefficient potentiometer is used to scale the voltage by 0.1. The final step is to combine the voltages representing the variables of equation 3.11 in the same way they are combined on the right side of the equation. The final circuit diagram is shown in figure 3.7.

There is always more than one way to connect an analogue computer diagram for solving a particular equation. Generally speaking, the best way is the one that uses the fewest number of amplifiers and integrators. Therefore a separate summing amplifier was not used to combine the three terms on the right hand side of equation 3.11. It must be noted that nowhere in the circuit is there a voltage proportional to $\frac{d^2x}{dt^2}$. If the interest was to observe this term as a function of time, then the diagram must be revised to include a separate summing amplifier for generating $\frac{d^2x}{dt^2}$.

Figure 3.8 shows the solution 5x of the differential equation computed by the Analogue computer setup of figure 3.7 and as displayed on an X-Y plotter.

The analytic solution of equation 3.11 has the general solution

General solution = complimentary function + particular integral

To find the complimentary function the RHS = 0

$$m^2 + m + 1 = 0$$

$$m = \frac{-1 \pm \sqrt{1 - 4x1x1}}{2x1}$$
$$= \frac{-1 \pm \sqrt{-3}}{2}$$
$$= -0.5 \pm 0.8i$$

The complementary function is

$$x = e^{-0.5t}\left(A\cos 0.8t + B\sin 0.8t\right)$$

For the particular integral, Let x = C

$$then \frac{dx}{dt} = 0 \, and \, \frac{d^2x}{dt^2} = 0$$

Substituting into equation 3.10

C = 5

The general solution is

$$x = e^{0.5t}\left(A\cos 0.8t + B\sin 0.8t\right) + 5$$

The result from the analogue computer agrees with the expected waveform.

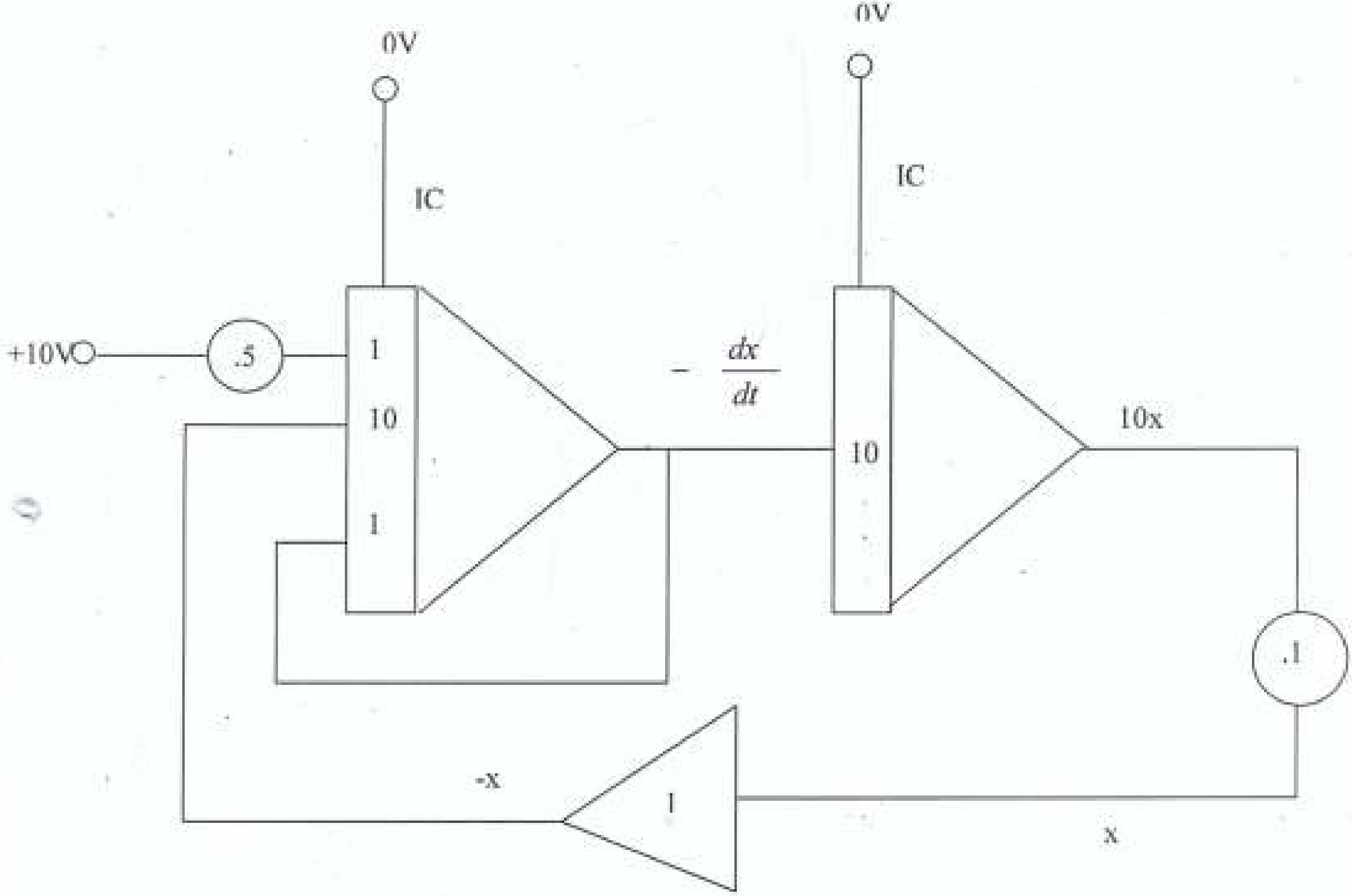

Figure 3.7: Analogue Computer flow diagram for solving the equation 3.11

①

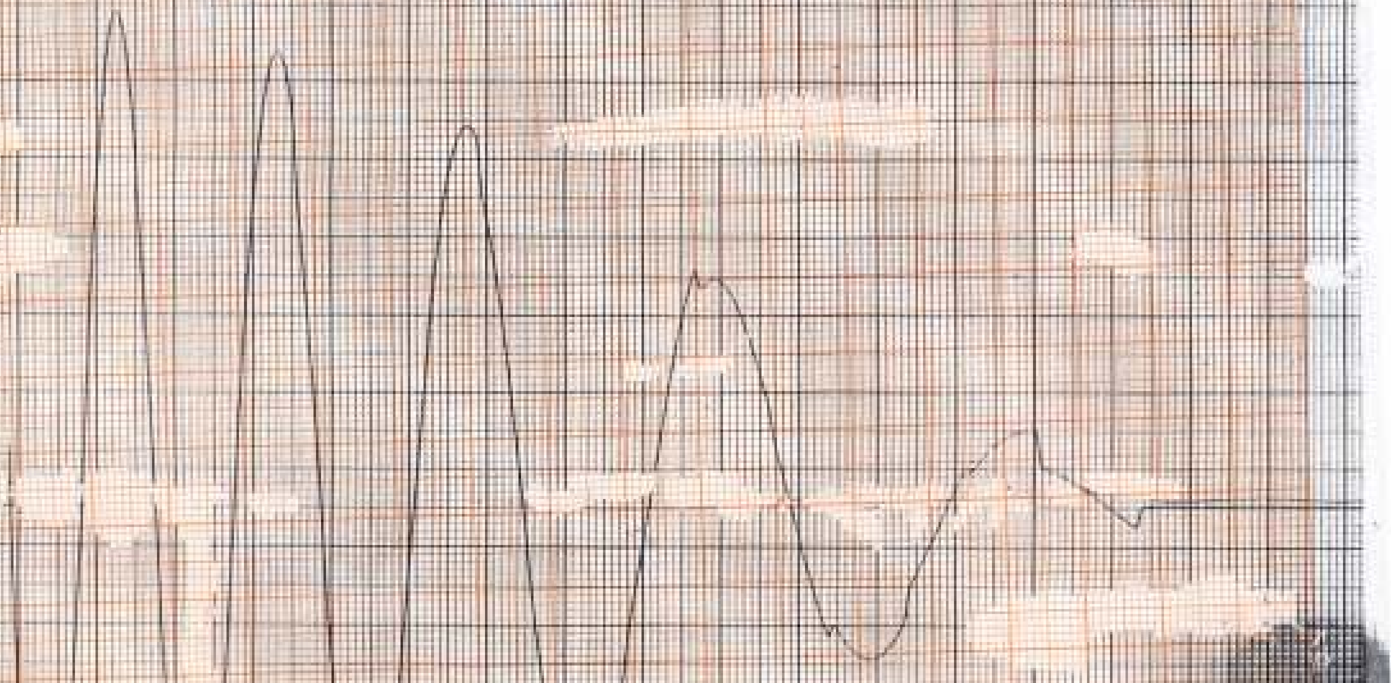

Figure 3.8: Response of differential equation 3.11 display by X-Y plotter

Experiment 2.

$$\frac{d^2x}{dt^2}-\frac{dx}{dt}=f(t)\mathit{triangular\ wave} \qquad 3.12$$

Where f(t) is a triangular waveform

$$\frac{d^2x}{dt^2}=f(t)+10[0.1]1\frac{dx}{dt} \qquad 3.13$$

The flow diagram for equation 3.13 is shown in figure 3.9. The signal output obtained from an X-Y plotter is shown in figure 3.10.

The waveform from the X-Y plotter agrees with the expected result.

Experiment 3

$$\frac{d^2x}{dt^2}-\frac{dx}{dt}=f(t) \qquad 3.14$$

where f(t) is a sine wave

$$\frac{d^2x}{dt^2}=f(t)+10[0.1]1\frac{dx}{dt} \qquad 3.15$$

The flow diagram for equation 3.15 is shown in figure 3.9.

The signal output obtained from an X-Y plotter is shown in figure 3.11. The waveform obtained from9 the plotter agrees with the expected result.

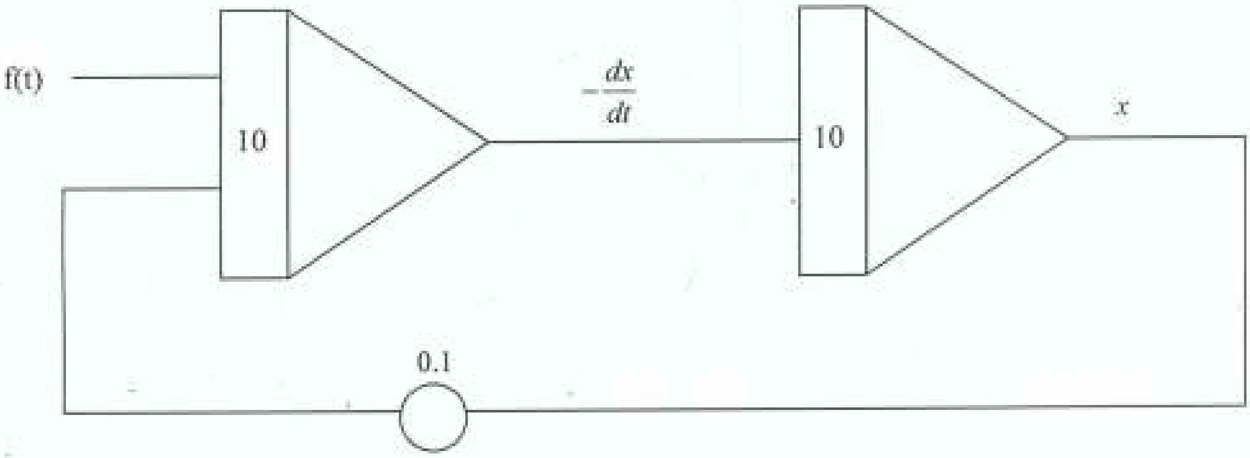

Figure 3.9: Analogue Computer flow diagram for solving the equation 3.13

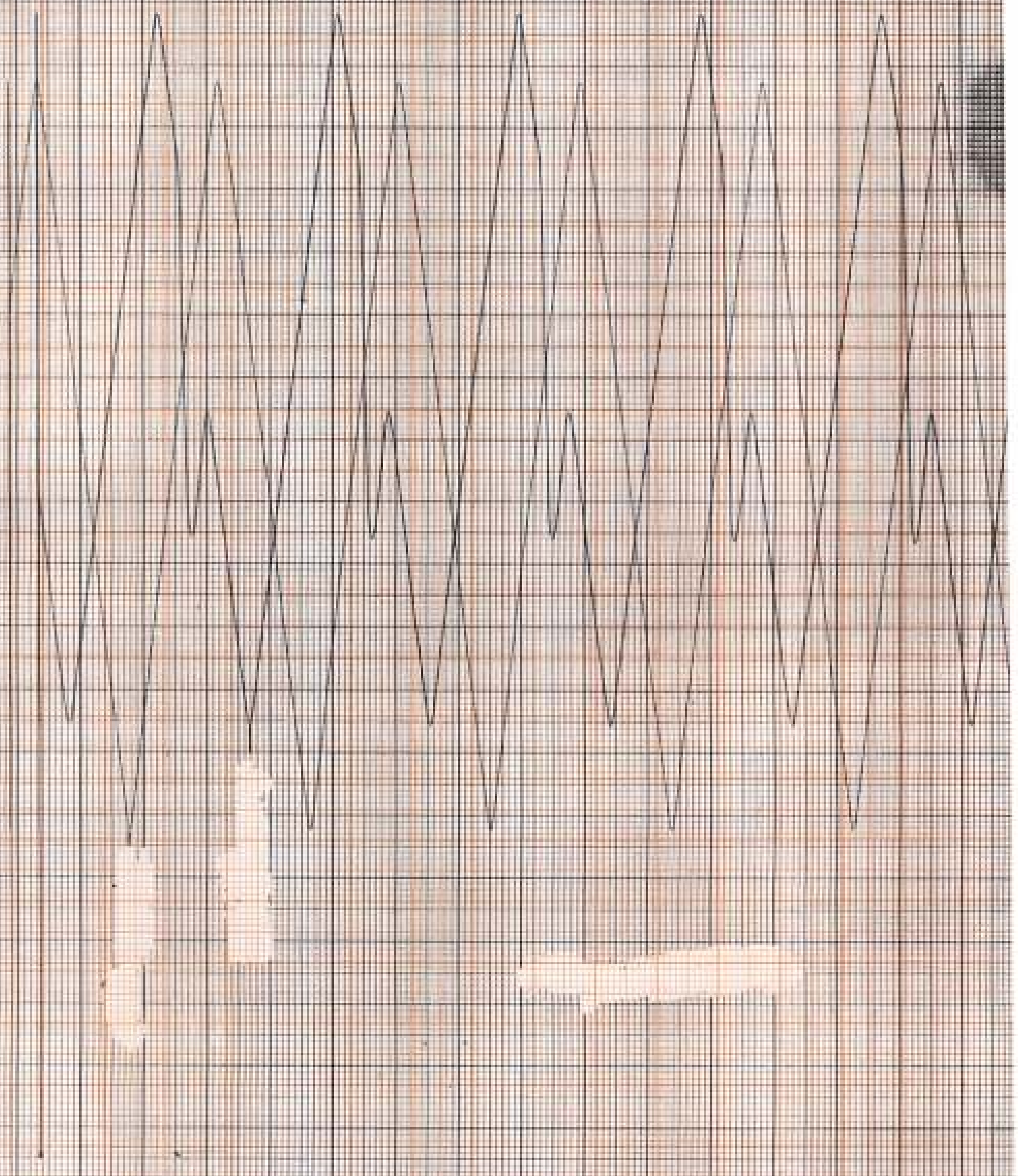

Figure 3.10: Response of differential equation 3.11 display by X-Y plotter

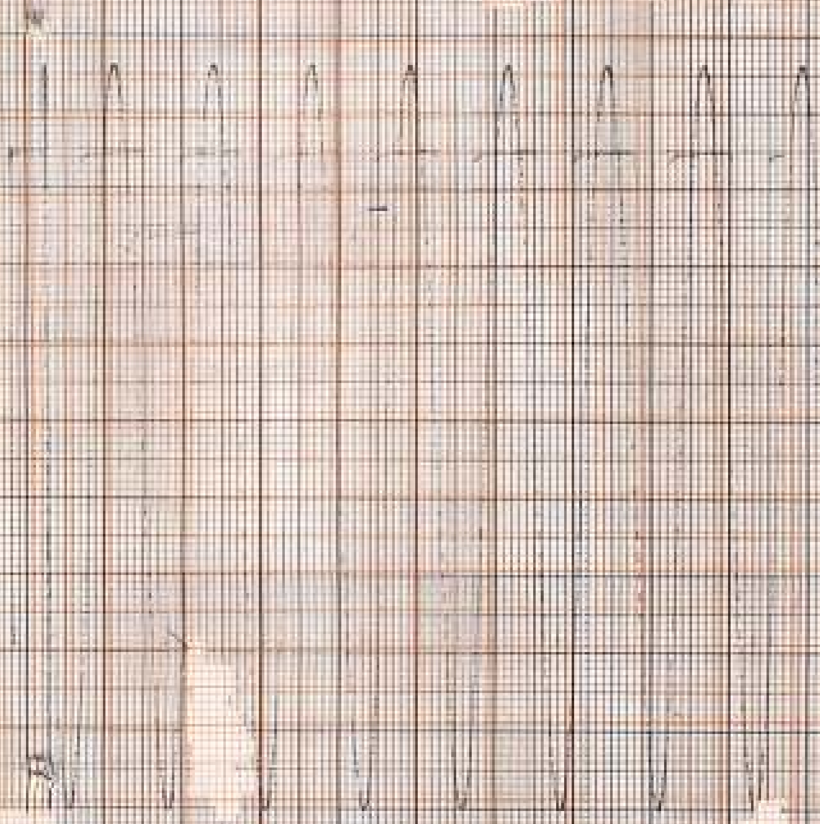

Figure 3.11: Response of differential equation 3.15 display by X-Y plotter

Simultaneous Differential equations

Experiment 4

$$\frac{d^2x}{dt^2}+2x-y=0 \qquad 3.16$$

$$\frac{d^2y}{dt^2}+2y-x=0 \qquad 3.17$$

This simultaneous differential equation was solved using the constructed analogue computer.

The flow diagram for the simultaneous differential equation is shown in figure 3.12

The signal output obtained from an X-Y plotter of the flow diagram of figure 3.12 is shown in figure 3.13. The result obtained as shown on the X-Y plotter was in agreement with the expected waveform.

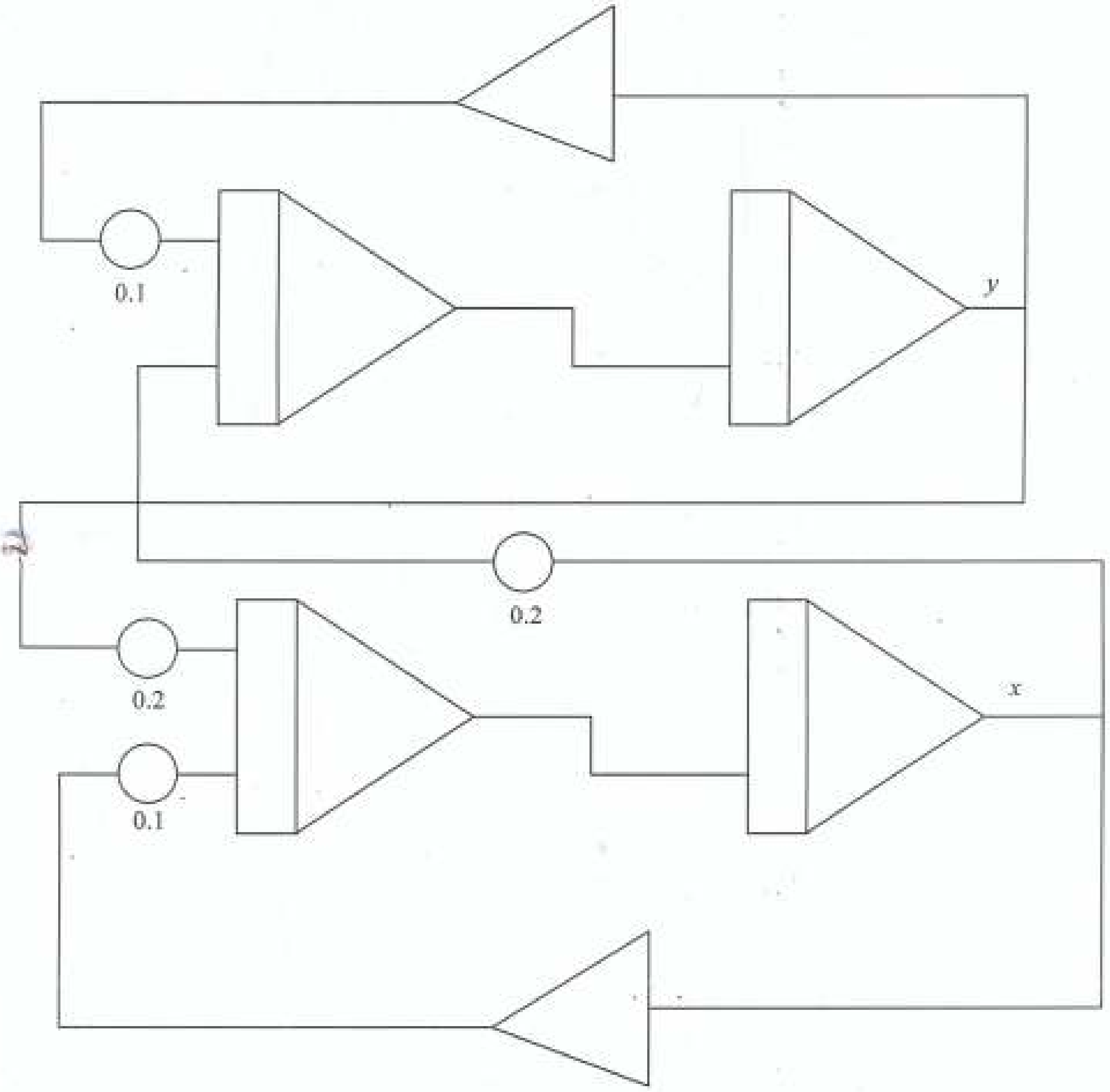

Figure 3.12: Analogue Computer flow diagram for solving the equation 3.20

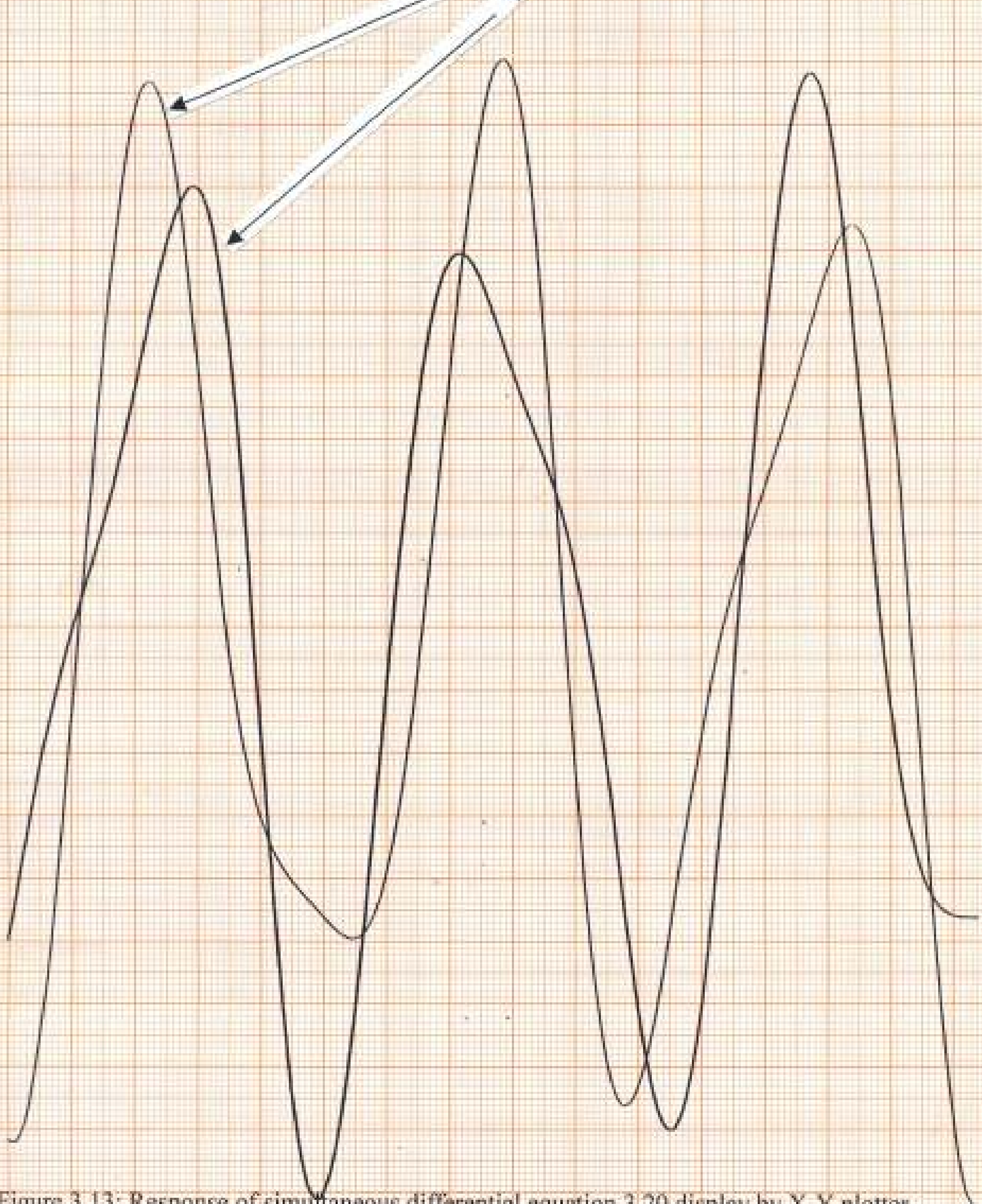

Figure 3.13: Response of simultaneous differential equation 3.20 display by X-Y plotter

Control Transfer function

Experiment 5

This control transfer function was investigated

$$G = \frac{K}{s(s+1)(1+0.2s)}$$

The transfer function was turned into differential equation yielding

$$0.2D^3c + 1.2D^2c + Dc = Ke \qquad 3.18$$

$$r - c = e \qquad 3.19$$

Solving this equation for the highest-order derivatives gives

$$0.2D^3c = Ke - 1.2D^2c - Dc \qquad 3.20$$

The flow diagram for equations 3.20 is shown in figure 3.14.

Figures 3.15 to 3.19 shows the signal outputs obtained from an X-Y plotter at different values of the gain (K) ranging from 0.3 to 0.9. The overshoot, delay time, rise time, settling time and frequency response for the various values of the gain K are given in Table 3.1

Table 3.1: Response of control Transfer Function at different values of gain K

	K=0.3	K= 0.5	K = 0.6	K = 0.7	K = 0.9
Overshoot	0.25%	5.00%	6.25%	6.435%	9.75%
Delay Time	0.10s	0.15s	0.20s	0.10s	0.00s
Rise Time	0.18s	0.10s	0.15s	0.12s	0.00s
Settling Time	0.30s	2.30s	5.95s	0.00s	0.00s
Freq. Response	1.18Hz	1.11Hz	0.59Hz	1.60Hz	1.54Hz

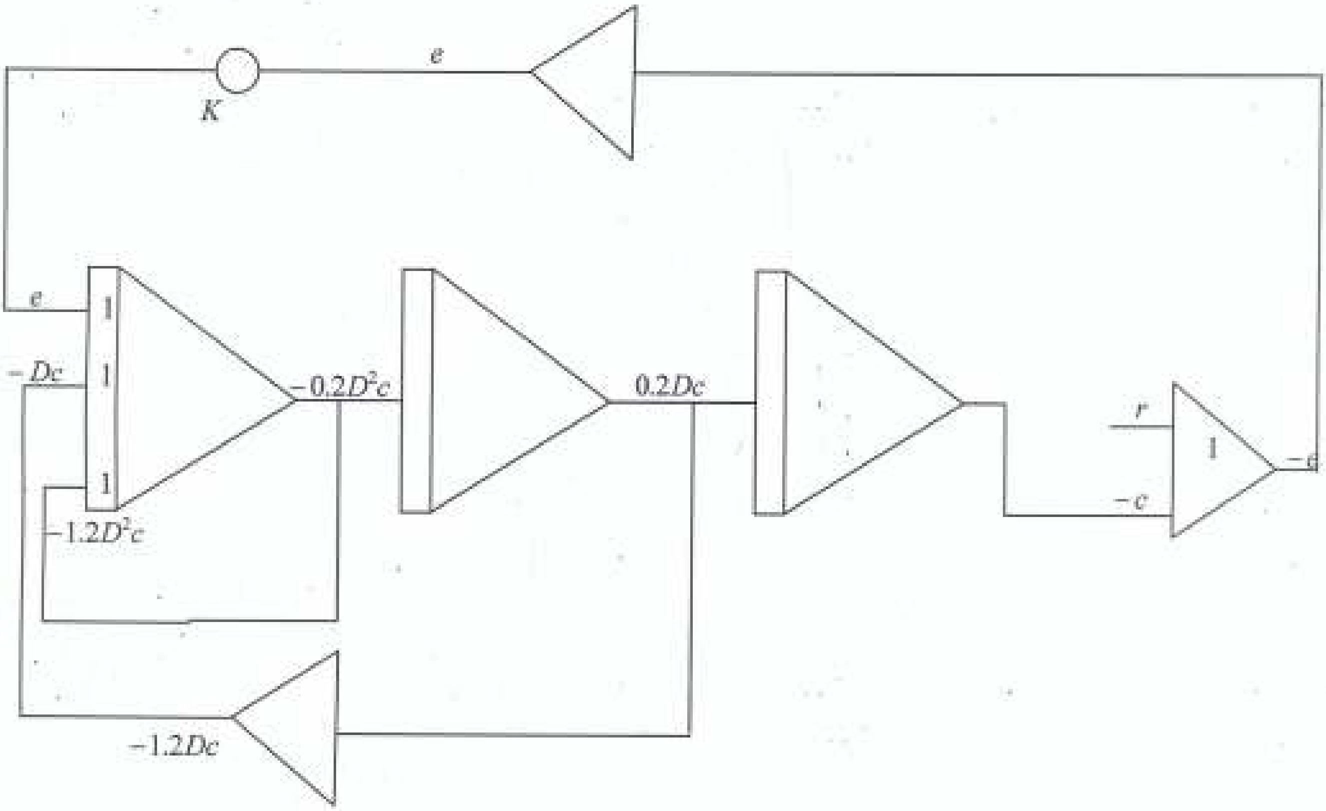

Figure3.14: Analogue Computer setup of equations 3.18 and 3.19

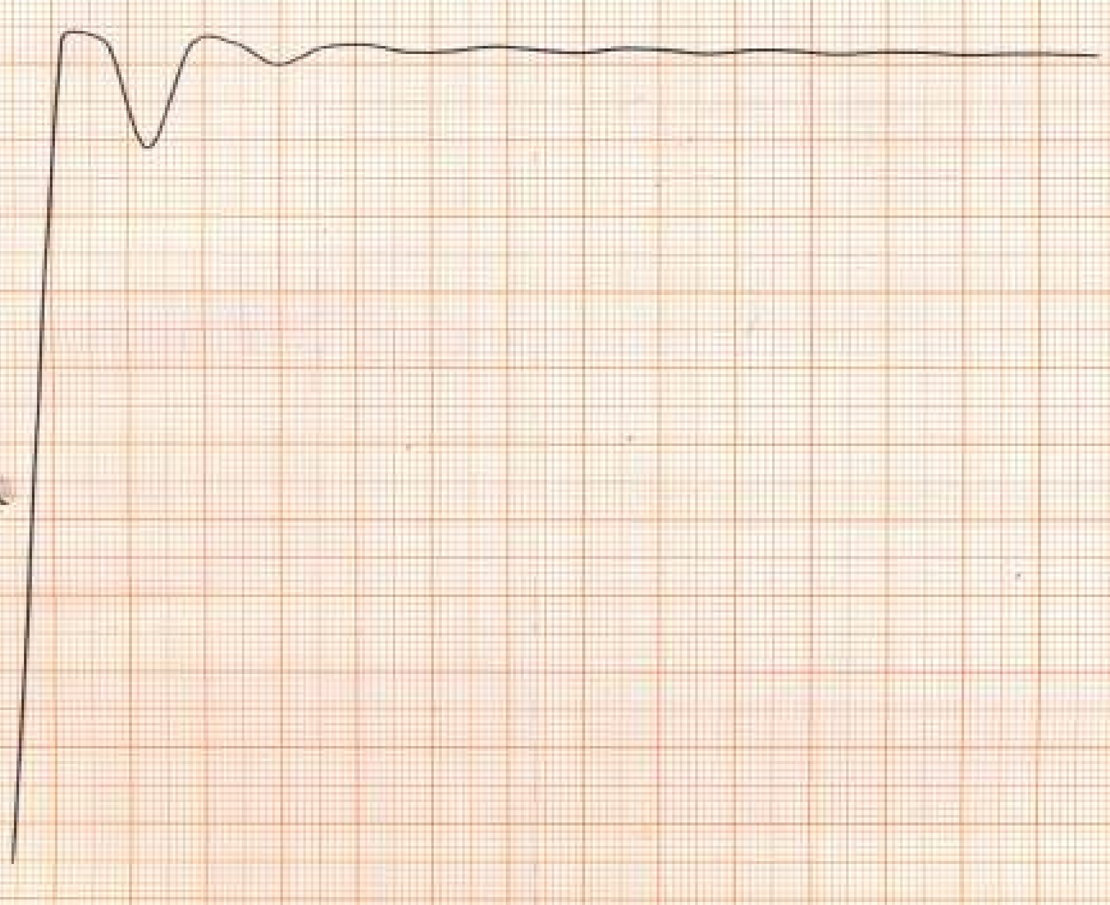

Figure 3.15: Response of Control Transfer Function of equation 3.20 with gain 0.3 display by X-Y plotter

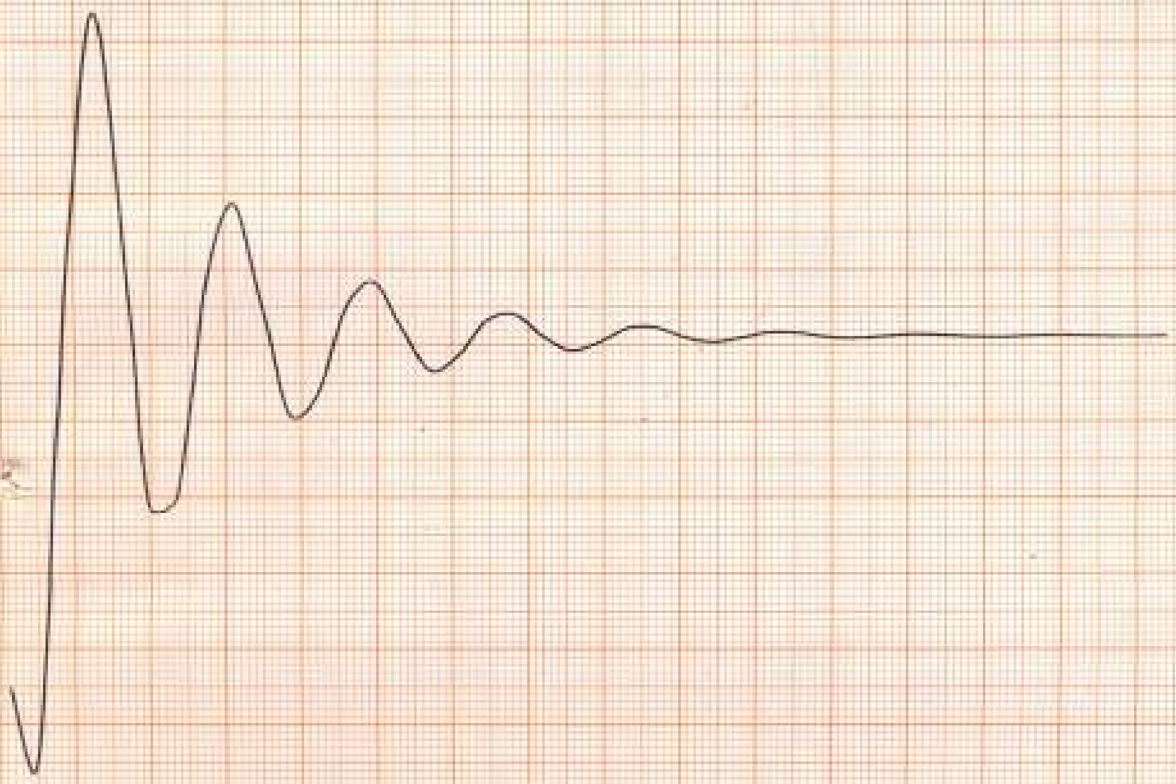

Figure 3.16: Response of Control Transfer Function of equation 3.20 with gain 0.5 display by X-Y plotter

Figure 3.17 Response of Control Transfer Function of equation 3.20 with gain of 0.6 display by X-Y plotter

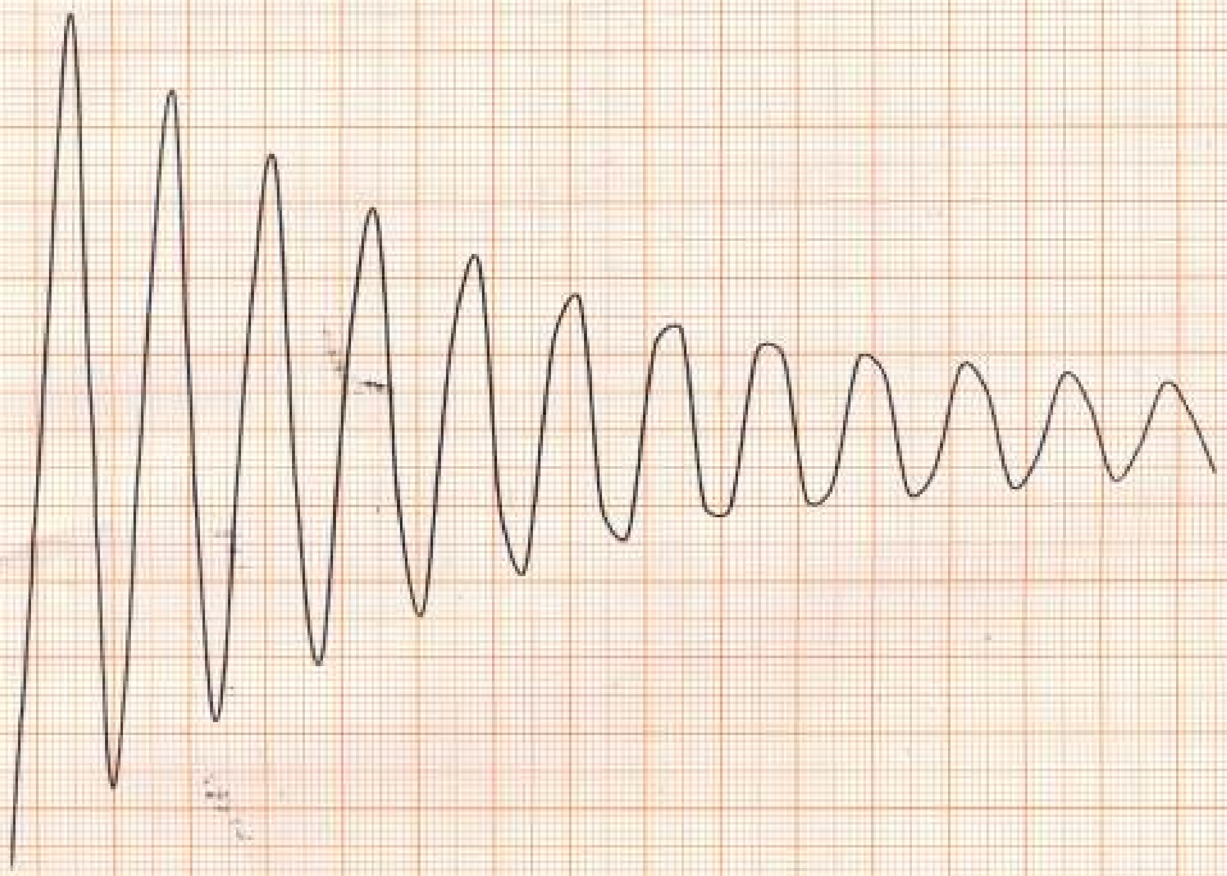

Figure 3.13: Response of Control Transfer Function of equation 3.20 with gain 0.7 display by X-Y plotter

Figure 3.13: Response of Control Transfer Function of equation 3.20 with gain 0.9 display by X-Y plotter

The accuracy of an analogue computer simulation depends on the following:

1) The quality of the computing amplifiers
2) The tolerance of components
3) The potentiometers settings
4) The accuracy of the output devices
5) The problem itself

3.9 SYSTEM PERFORMANCE

The performance of a system has the following characteristics

Overshoot: This is the maximum difference between the transient and steady-state response for a unit-step input. It is a measure of relative stability and it is often expressed as a percentage of the final value of the output.

Delay Time: This is a measure of the speed of response and is the time required for the response to reach 50% of its final value.

Rise Time: This is the time taken for the response to rise from 10 to 90 per cent of its final value.

Settling Time: This is the time required for the response to reach and remain within a specific percentage of its final value usually 2 to 5 per cent.

CHAPTER FOUR

4.1 DISCUSSION OF RESULTS

From the response of the different signals (Square wave, Sine wave and Triangular wave) applied as the forcing functions shown in figures 3.4 to 3.6, the output tried to match the input waveform for every wave that we put into the system. This is most noticeable at low frequency around 20Hz. It was noticed that the output can never really match the input. When triangular waves and sine waves are put into the circuit, the inability to match the input results in a phase shift. In every case, as the frequency increased, the phase change also increased. The X-Y plotter used to produce the graphs could not sample fast enough, but the reaction at low frequency for all waves was really interesting. This phase shift, and inability to match the input can be described in a couple of different ways. In physics terms, this shift is the momentum of the mass that is attached to the spring. As the input wave changes values, the mass cannot instantaneously follow. Because of its momentum, it must first be brought to rest and then start moving in the other direction. In systems terms, the phase shift is simply a result of the circuit. The circuit does not have a completely linear phase, and this non-linear phase will cause distortions in any waveform as it travels through the circuit.

From the response of x of the differential equation 3.11 as displayed on the X-Y plotter shown in figure 3.8 of the uncompensated system to the set input, it is seen that the system is underdamped and exhibits considerable oscillation before settling to its steady-state condition.

The response obtained from X-Y plotter for the simultaneous differential equation 3.16 and3.17 shown in figure 3.13 shows that the nature of the waveform looks like sine wave. From the figure 3.13 wave x is leading y by 103^0, which was in complete agreement to the solution of equations 3.16 and 3.17.

The response of the Control Transfer Function of equation 3.20 as displayed on an X-Y plotter is shown in figure 3.15 to 3.19.The desired value of the gain is selected by adjusting the setting of the potentiometer. Varying the values of the gain K the overshoot, delay time, rise time, settling time and the frequency response of each of the waveform is obtained which is shown in table 3.1. When the gain K = 0.3 the overshoot, delay time, rise time, settling time and frequency response was 0.25%, 0.10s, 0.18s, 0.30s and 1.18Hz. Also when the gain K = 0.5 the overshoot, delay time, rise time, settling time and frequency response was 5.00%, 0.15s, 0.10s, 2.30s and 1.11Hz respectively. Similarly when gain K = 0.6 the overshoot, delay time, rise time, settling time and frequency response were respectively 6.25%, 0.20s, 0.15s, 5.95s and 0.59Hz. The result obtained shows that the peak overshoot varies directly as the gain, that is, the peak overshoot increases as the gain increases. Analogue simulation of a control system is a useful way to investigate the stability, peak overshoot and the settling time of the system. The output of a stable control system ultimately reaches and maintains some steady-state position while the output of an unstable system oscillates indefinitely.

4.2 CONCLUSION

After adjusting our design, it was found that the analogue computer efficiently gave solutions to the differential equation that we were trying to solve. Analogue computers are definitely not as accurate as digital computers, but they are incredibly fast at solving differential equations. Because analogue computers solve differential equations speedily, the frequency or shape of the forcing function could be changed, the output waveform is produced with an unnoticeable delay.

Another advantage of analogue computer simulation is that solution to problems are produced in real time which implies that solutions can be time-scaled so that they occur in either "fast time" or "slow time"

Also analogue computer simulation is advantageous in that the user is not required to analyze the system model mathematically and produce an analytical expression for the solution.

It is only necessary to patch together components that affect voltage levels in the same way that the variables are affected in the mathematical model of the system, apply power and observe the output wave forms at different points in the circuit.

However, there are disadvantages to analogue computers. The principal disadvantage is that the user must be reasonably well acquainted with the computer hardware for a maximally effective simulation. He must be aware of amplifier limitations, noise sources, frequency response and the capabilities and operations of the output display devices.

In the end, analogue computers are an effective way to model real world physical systems.

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