OPTIMUM DESIGN OF A FOUR-CYLINDER INTERNAL COMBUSTION ENGINE CRANKSHAFT

A THESIS

BY

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CERTIFICATION

We, the undersigned hereby certify that this research thesis has been carried out by **DADA, Ayodeji Sunday** for the partial fulfilment of the requirements for the award of Master of Engineering (M.Eng) in Mechanical/Production Engineering, Federal University of Technology, Akure and the work has not been submitted elsewhere for award of a degree.

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DEDICATION

I dedicate this work to the Almighty God, His Son Jesus Christ - the wisdom and power of God and the Holy Spirit my teacher, counsellor and sustainer.

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I give all glory and praise to the Almighty God the planner, Designer and Executor of the plan, and purpose of my life, for sustaining me in this phase of my life, that is, during the span of my M.Eng programme at FUTA.

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We are all moving forward, backward never! Best wishes and God Bless.

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ABSTRACT

The scarcity and need for efficiency in today's competitive world have forced engineers to show greater interest in economical and better designs.

This project is based on the analysis for optimum design of a four-cylinder internal combustion engine crankshaft. It considers the stressing, bending and torsional fatigue analyses using a high-duty cast iron BY 40-10, which can be worked into optimal shapes.

For the optimization procedure, the design parameters are identified as the main journal diameter and length, crankpin diameter and length, web width, and thickness. The objective function is the volume of the material for the shaft.

Using an appropriate optimization technique and computer aided method, the optimal shape is developed for the set operating conditions.

The values of the optimum design parameters are: Crankpin diameter= 0.042m, main journal diameter= 0.036m, Crankpin length =0.029m, main journal length = 0.027m, web width= 0.060m, and web thickness = 0.012m.

The project is useful in solving practical engineering problems in crankshaft design and it has the potential of being extended to other automotive components. It furnishes local fabricators and crankshaft refurbishing firms with useful information for improved crankshaft performance.

CHAPTER ONE

1 Introduction

1.1 Engineering Design

Engineering consists of a number of well-established activities, including analysis, design, fabrication, sales, research and development of systems. The process of designing and fabricating systems has been developed and used for centuries, and the existence of fine buildings, bridges, highways, automobiles, airplanes, space vehicles and other complex systems is an excellent testimonial. However, evolution of these systems has been slow. The entire process is both time-consuming and costly, requiring substantial human and material resources. Therefore, the procedure has been to design, fabricate and use a system regardless of whether it was the best one. Improved systems were designed only after a substantial investment had been recovered. These new systems performed the same or even more tasks, cost less, and were more efficient. Jasbir (1989).

1.2 Shafts Design

Shafts are used in a variety of ways in all kinds of mechanical equipment. Typical uses are power shafts, cam shafts, line shafts, and so on. As a result of industrial application, particular definitions are associated with shafts for a definite purpose. These definitions are as follows:

(i) **Shaft**: A rotating member used for the purpose of transmitting power.

(ii) **Axle**: A stationery member used as a support for rotating elements such as wheels idler gears, and so on.

iii) **Spindle**: A short shaft or axle (for example, head-stock spindle of a lathe)

(iv) **Stub Shaft**: (Also called head shaft). A shaft that is integral with an engine, motor, or prime mover, and is of such size, shape, and projection as to permit its easy connection to other shafts.

(v) **Jack Shaft** : (Also called countershaft). A short shaft that connects a prime mover with a line shaft or a machine.

(vi) **Line Shaft**: (also called power transmission shaft) A shaft that is directly connected to a prime mover and is essentially used to transmit power to a machine(s).

vii) **Flexible Shaft**: Permits transmission of motion between two points (for example, motor and machine) where the rotational axes are at an angle with each other. The amount of power transmitted is of relatively low level.

Depending on the loading, shafts are subjected to constant bending and/or torsional stress or a combination of these stresses caused by fluctuating loads.

Shafts are subjected to (a) fixed loading and (b) fluctuating loads. The associated considerations: stress consideration and endurance, therefore, will play a significant role in its design. A shaft designed from the view point of these consideration will possess adequate strength. However, of equal importance (sometimes of greater importance) is the consideration of shaft rigidity or stiffness. A shaft having too large a lateral deflection can cause excessive bearing wear or failure. A large lateral deflection is also responsible for lowering the critical speed, which may cause the shaft to vibrate violently if its revolution per minute are at or near this speed. (Aaron et al, 1975)

1.3 Reasons for Optimizing

1.3.1 Engineering Design versus Analysis

It is important to realize the difference between engineering analysis and design activities. The analysis problem is concerned with determining behaviour of an existing system, or a trial system being designed for a given task - Crankshaft, a case study. Determination of the behaviour of the system implies calculation of its response under specified inputs. Therefore, sizes of various parts and their configuration are given for analysis problem.

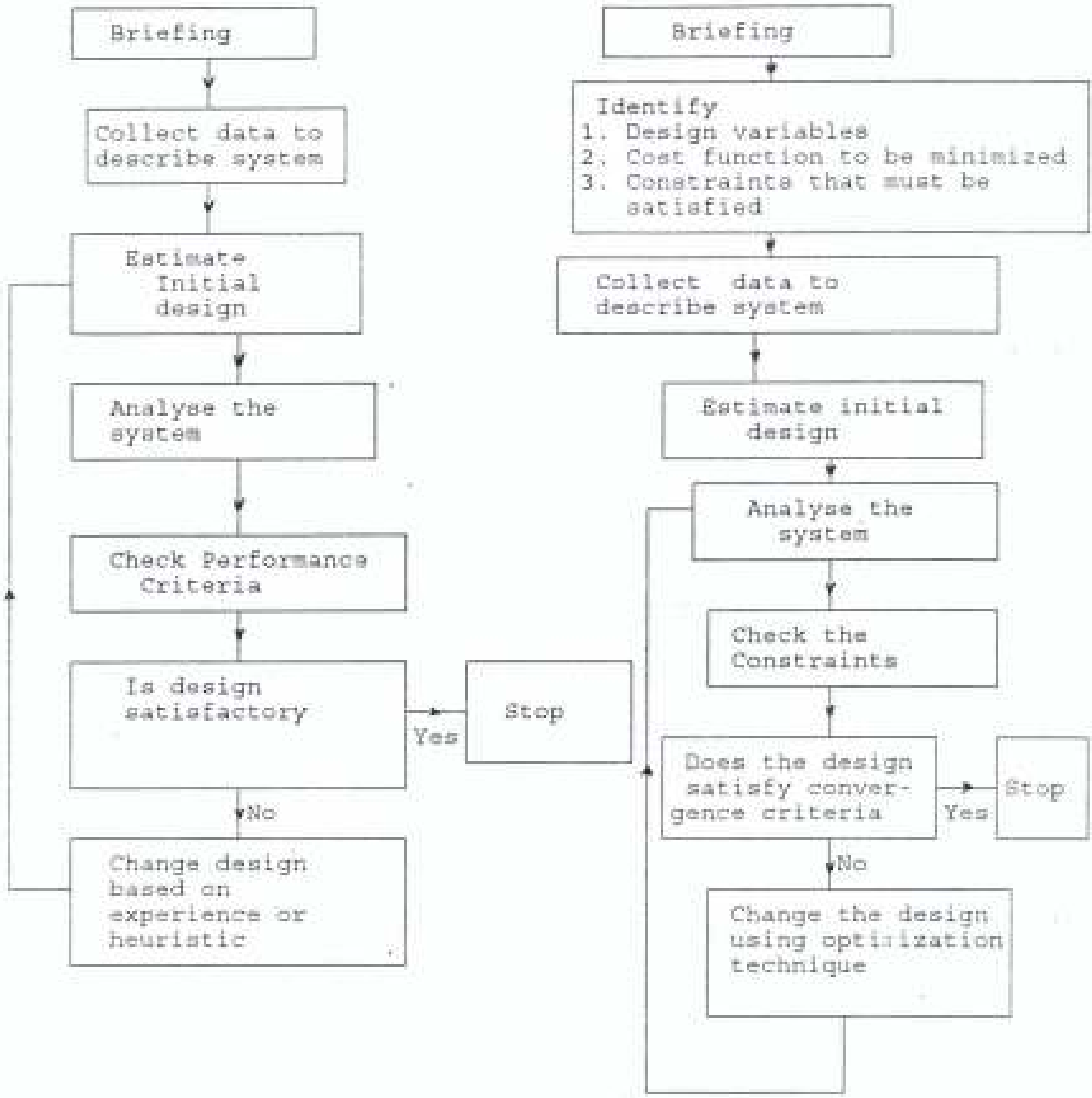
On the other hand, the design problem is to calculate sizes and shapes of various parts of the system to meet performance requirements.

The design of system is a trial and error procedure. We estimate a design and analyze it to see if it performs according to the specifications. If it does, we have an acceptable (feasible) design. We may still want to change it to come up with an acceptable system. In both these cases, we must be able to analyze designs to make further decisions. Thus analysis capability must be available in the design process.

1.3.2 Conventional versus Optimum Design Process

It is a challenge for engineers to design efficient and cost-effective systems without compromising their integrity. The conventional design process depends on the designer's intuition, experience and skill. This overwhelming presence of a human element can sometimes lead to dangerous and erroneous results in the synthesis of complex system.

Scarcity and need for efficiency in today's competitive world has forced engineers, to show greater interest in economical and better designs. With recent advances in computer technology affecting various disciplines of engineering; the design process can hardly remain untouched. Recently, the term computer aided design optimization (CADO) has been used for summarizing all computer aids in design. Fig. 1.0 compares the conventional and optimum design process.



(a) **Conventional** (b) **Optimum Design Process**

Fig 1.0 Comparation of Conventional and Optimum Design Process (Jaßbir, (1989))

1.3.3 <u>Role of Computers in Optimum Design</u>

Engineering systems can be analyzed faster and more accurately using computers. This allows us to understand the behaviour of systems more precisely and thus design them more accurately and efficiently. The design process-conventional or optimum-is iterative, requiring the use of the same set of calculations repeatedly. Such repetitive calculations are ideally suited for computer implementation.

1.4 Justification for the Project

During the last three decades, especially with the revolution in computer technology, the developed nation's manufacturing firms have done a lot on machine parts design. In this case computer analysis that involves numerical computations has been used to perform complex calculations needed in few seconds, given all the necessary information about the material components, and the impending external loadings on the machine elements, to design best systems that are cost effective, efficient, and durable.

In the case of Nigeria, the 'Pseudo' technological nature of the nation is clearly evident. She is a nation that has no basis for her foundational technological development yet uses highly sophisticated and complex machines that are imported. But, because a nation cannot successfully by -pass her technological development, it is necessary to evolve a model design base on serious analytical ground coupled with computer aid to intimate our young designers with optimum design procedure to be used in modern machine design. Also, industrial organisations can also adopt it for design practice.

Shafts are important machine elements that the young engineer would have to come accross in day-to-day engineering design decisions. Therefore, the need to choose a type of a shaft - 4-cylinder engine, Crankshaft on which the judgement necessary in the design of shafts for optimum performance can be realistically appraised. This is a type of shaft subjected to fluctuating and shock loading, that implies that a detail analysis for best-cost effective, efficient, reliable and durable system can be achieved.

- Fluctuating loading on a shaft may induce the following effects:
  - (i) Bending moments and torque
  - (ii) Failure:
    - Maximum shear theory of failure
    - distortion energy theory of failure
- shock loading

It is therefore a necessity that the parameters for formulating the problems for the Crankshaft design be understood, also the basic concepts of optimum design and the economic consideration so as to evolve a rigid and safe shaft for optimum performance.

1.5 Objective of the Research Project

The present study is analysing for optimum design of a four-cylinder ICE crankshaft, based on all the design consideration and judgement necessary, using a feasible optimization technique and computer aid in coping with the design calculations involved.

2.1 INTERNAL COMBUSTION ENGINES

Internal combustion engine refers to an engine in which the combustion of a gaseous, liquid or pulverized solid fuel provides heat which gets converted into mechanical work through a piston or turbine (Chopra, 1989).

In piston-type Internal Combustion Engine, the combustion process is assumed to occur at constant volume (Fig 2.0), at constant pressure (fig 2.1a), or by some combination of these (fig 2.1b). The constant-volume process is characteristic of the spark-ignition or otto cycle, the constant-pressure is found only in slow-speed compression - ignition or diessel cycle; with both processes, the cycle is sometimes called a mixed, combination, or limited - pressure cycle. Actually the indicator card obtained from a high-speed, mixed-cycle, compression - ignition engine may be similar to that obtained from a spark-ignition engine.

The fundamental differences are the methods of mixing the air and fuel (before compression in the otto cycle, and usually near the end of compression in the diesel cycle) and the method of ignition.

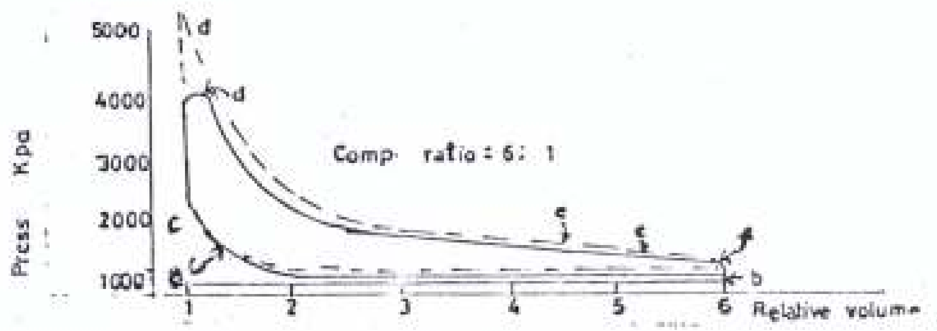


Fig 2.0 Otto cycle (constant volume combination, no excess air). Process of the ideal cycle: a-b, suction stroke,

admission of change; b-c, compression stroke; etc, ignition of the compressed charge, d-e, expansion at e, exhaust valve opens, b-a, exhaust stroke.

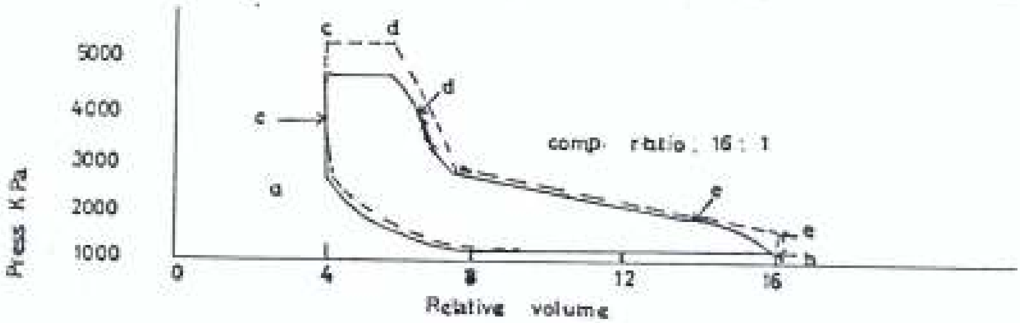


Fig 2.1a Diesel Cycle (constant-pressure combustion, 50-percent excess air). Processes of the ideal cycle: a-b, suction stroke, admission of air; b-c, compression air; c-d injection and combustion of the fuel; d-e, expansion, at e, exhaust valve opens, b-a, exhaust stroke.

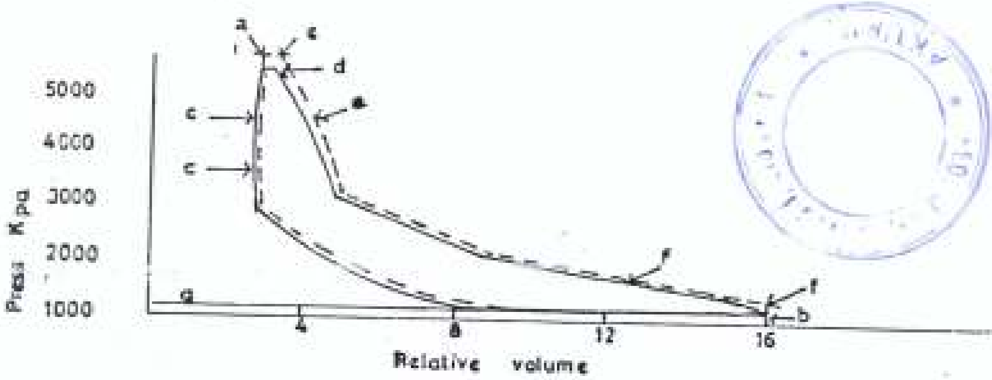


Fig 2.1b Mixed Cycle (constant volume and constant pressure combustion, 50 percent excess air). Processes of the ideal cycle; a-b, suction stroke, admission of air; b-c compression of air, c-d, ignition and constant-volume combustion; e-f, expansion, at f, exhaust begins; b-a, exhaust stroke. Broken lines represent the ideal cycle; solid lines show characteristics of the actual cycle. Primes attached to letters show actual location of events, though some lower-

compression ratio and multiple chamber engines may require starting aids.

2.1.1 <u>The Four-Stroke Cycle (four-cycle)</u>

The Four stroke cycle requires four piston strokes or two crankshaft revolutions per cycle (fig 2.0, 2.1a). This engine cycle is used almost exclusively in automobiles, tractor, and aircraft engines in all types and sizes, and also in other engines of other classification with the exception of most outboard engines (Bolt et al, 1987).

2.1.2 <u>Multi-Cylinder Engines</u>

William et al (1988) declared that almost all automobile engines, whether water-cooled or air-cooled four cycle or two cycle, have more than one cylinder. These multiple cylinders are arranged in-line, opposed or in V-form as shown in Fig 2.1c. Engines for other purposes, such as aviation, are arranged as radial, inverted in-line, inverted V, X-shaped, and other assorted forms.

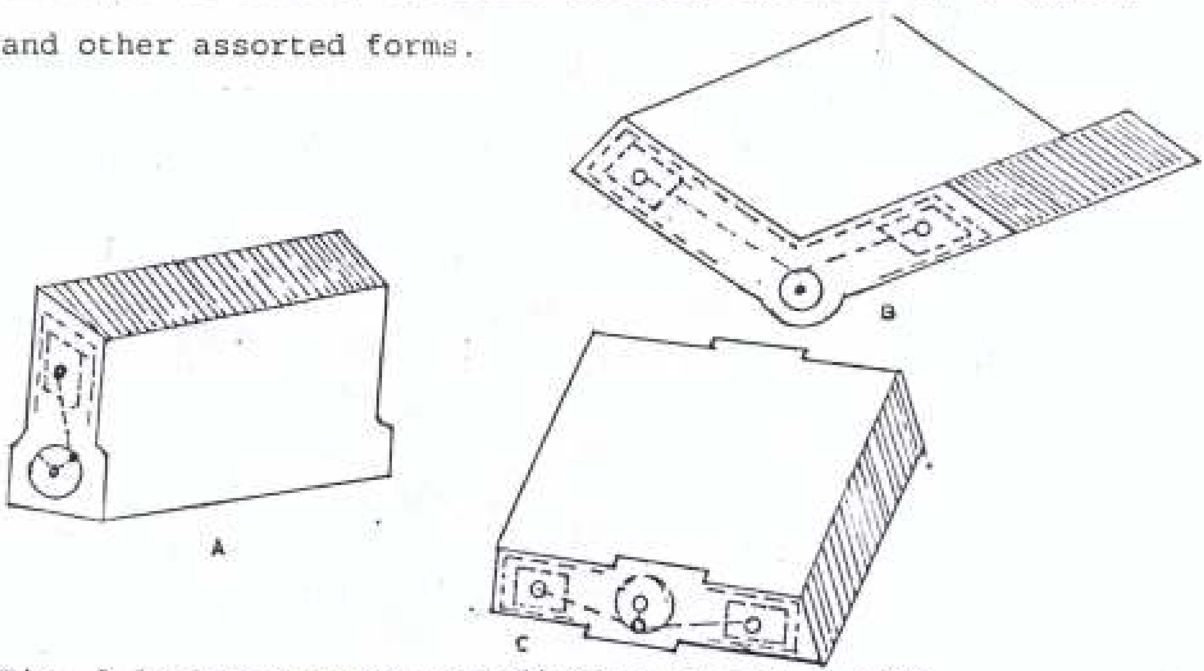


Fig. 2.1c Arrangement of Cylinders: A - In - Line

B - V - type

C - Opposed

2.1.3 <u>Standard Classification of Different Types of Engine (Symmetrical and Unsymmetrical)</u> Fig. 2.1d and Table 2.0

(A) Single-cylinder engines

(B) Double - cylinder engines

(B_1)+ Double - cylinder in-line engine having its cranks directed similarly.

(B_2)+ Double - cylinder engine with cranks at 180°

(C) Four cylinder in-line engine with cranks at 180°

(D) Six cylinder engines.

(D_1)+ Six cylinder in -line engine

(D_2)+ Vee six cylinder engine with 90° V angle and three paired cranks at 120°

(D_3)+ Vee-type six cylinder engine with 60° v-angle of cylinder and six cranks arranged at 60°

(E) Eight-cylinder engine

(E^1)+ Eight cylinder in-line engine

(E_2)+ Eight cylinder vee-type engine

(F) Three, Five etc. cylinder Engine

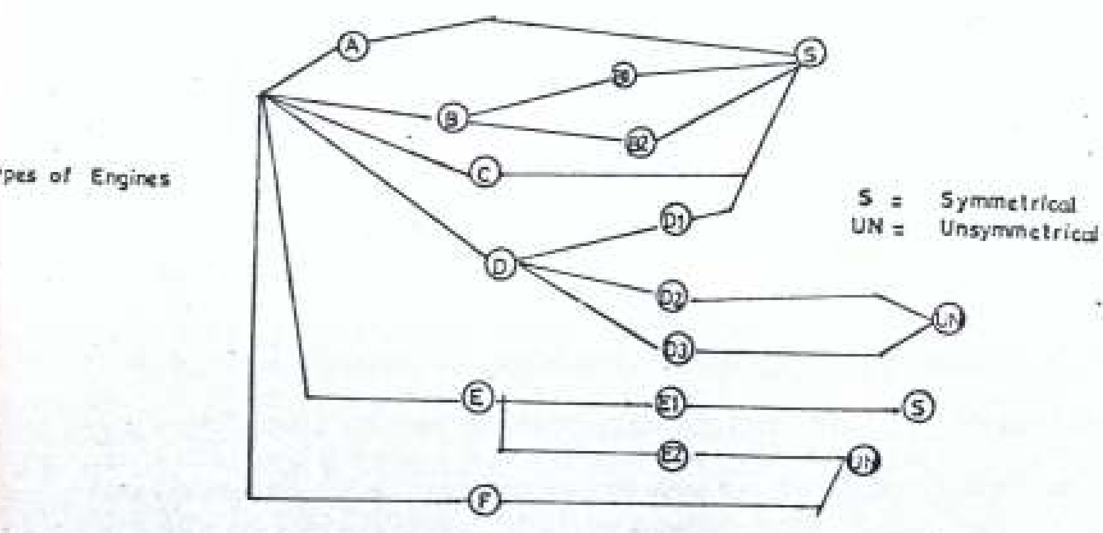
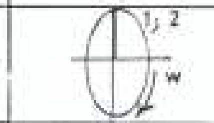
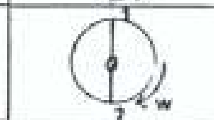
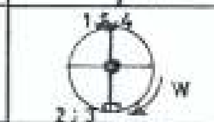
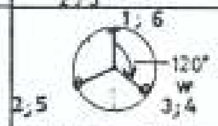
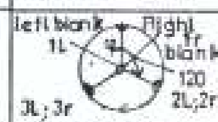
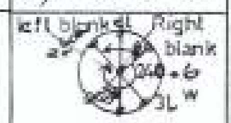
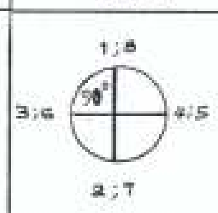
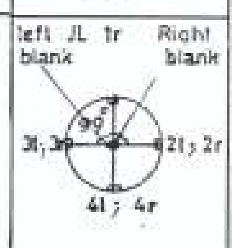


Fig. 2.1d Types of Engine

Table 2.0 Types of Engine and their description

Type of Engine	Firing Order	Firing intervals	Angle at which cranks arranged in crankshaft	Diagrammatic depiction
A	-	-	-	-
B_1	1-2	360°	Cranks directed the same size	
B_2	1-2	Alternate at 180 and 540	-	
C	1-2-4-3 or 1-3-4-2	180°	180°	
D_1	1-5-3-6-2-4 or 1-4-2-6-3-5	120°	120°	
D_2	1l-1r-2l-2r-3l-3r	90° and 150°	120°	
D_3	1l-1r-2l-2r-3l-3r	120°	-	
E_1	1-6-2-5-8-3-7-4	90°	The crankshaft has 8-cranks arranged in two planes square with each other	
E_2	1l-1r-4l-2l-2r-3l-3r-4r	90°	The v-angle of cylinder = 90°, cranks arranged in two planes square with each other	
	l = left r = right			

2.1.4 Purpose of Special Crankshaft Balance Weights

William et al (1988) found out that without special balance weights on the crankshaft, severe vibration would result from:

1. Weight of reciprocating parts

2. Weight of rotating parts
3. Inertia force of reciprocating part
4. Combustion pressures
5. Variation in torque

Fig 2.1e shows the diagramatic display of a portion of crankshaft and exposed view of the counter balance (weight)

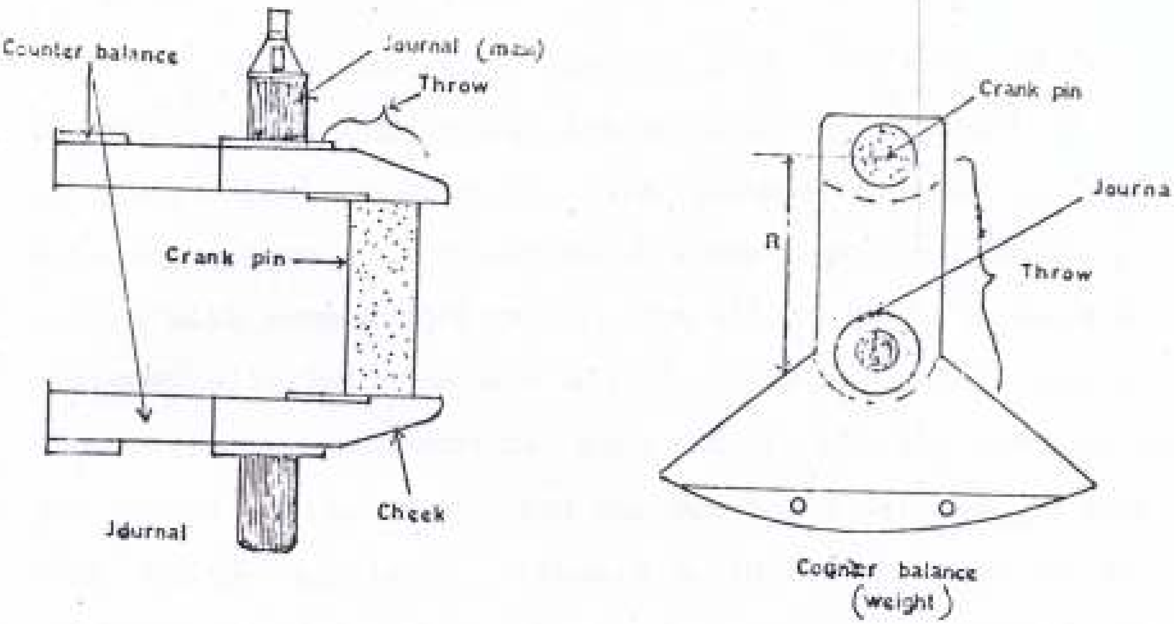


Fig. 2.1e diagrammatic display of One-cylinder Engine Crankshaft and the counter balance. (William et al, 1988).

2.1.5 Balanced Cranks

Fig 2.1f$_a$ shows a method of balancing cranks. When the size of the weight is determined by experiment the counterweight must be detachable as shown in fig 2.1f$_b$, fig 2.1f$_c$ shows crankdisks where the centre of mass of the balance weight does not fall opposite the crankpin.

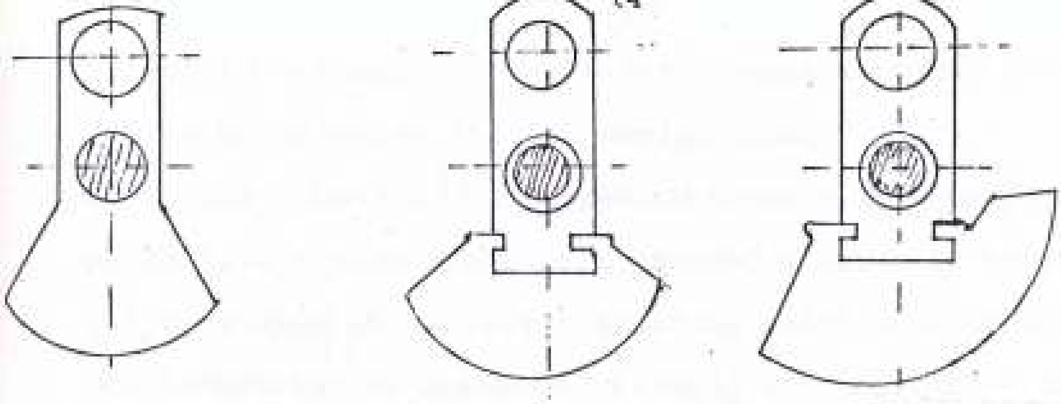


Fig 2.1f (a,b,c). Method of Balancing Cranks. (Kent's, 1950)

2.2 General Description of Cast Nodular Iron Crankshaft Design

The crankshaft will usually fall into one of the following three categories: the straight replacement of an existing steel crankshaft, the update/redesign of an existing engines; the creation of a new engine or family of engine with common components. In all of these cases the designer will be presented with a situation where engine capacity, cost, performance, bore and stroke have been pre-determined. In addition, the maximum crankshaft weight and cost may be specified. Should a family of engines be intended, variations in bore or stroke may provide a sufficient range of capacities using either a common crankshaft or similar shafts with different throw dimension. Additionally, turbo charging may feature as an option which can result in a similar crankshaft but with the addition of fillet rolling of the journal radii to increase the fatigue strength.

The creation of a diesel derivative of an existing engine will, however, require further consideration in respect of balancing and journal hardness. In the event of a common crankshaft replacement, that with the largest (larger) bore and/or the greatest (greater throw) should, of course be selected for the design study.

Over a ten to 15 year period between 1971 and 1986, the automotive engine crankshaft assumed a ratio of proportions which provide an excellent starting point from which a cast crankshaft can be designed to comply with specified criteria such as the overall length and diameter, throw and main journal centres. Fig 2.2 gives the proportions of crankshafts developed in the 80's and they were not substantially dissimilar to those put forward by Barnes-Moss (1975). It is suggested, therefore, that the initial design be drafted on this basis, but also in the light of range of standard bearing shell dimensions - particularly diameters and wall thicknesses - obtainable from the manufacturers. Whilst the projected areas of journal are limited by the maximum acceptable bearing presures, web widths can be reduced by the enhancement of fatigue strength of the casting and main journal areas by the adoption of a greater degree of balance correction.

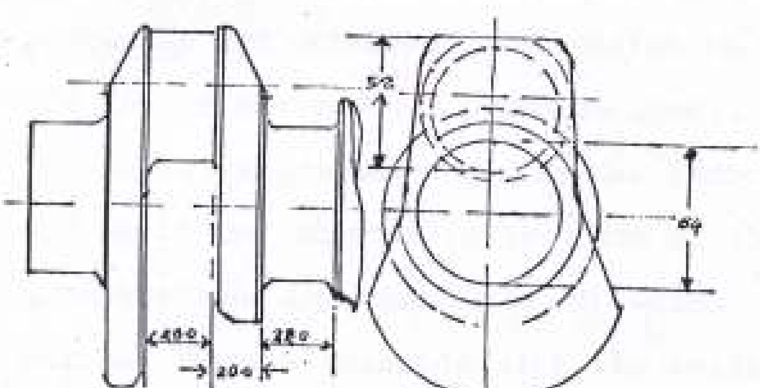


Fig. 2.2 Typical cast crankshaft proportions where the main to main centres are 100·0 mm

(Edward, 1986)

2.3 Crankshaft Design

Edward-Harwell (1986), states that the crankshaft design should be considered from two different angles: the stressing and bending analyses, as well as the finer points of crankshaft design. In his article, he argued that these approaches can be applied to the majority of engine configurations, the four cylinder, five main bearing, 'in-line' engine was selected as representative of current design trends. This type of engine is probably the most economical to produce and makes excellent use of engine compartment space in either the transverse or conventional installation.

2.3.1 Stressing and Bending Analyses

2.3.1.1 Assessment of Bearing Pressures

The maximum pressure on crankpin bearing shells is generally accepted as 380/400 bar based on the projected area of the bearing surface. A method of determining bearing pressures (and subsequently the crankshaft bending stresses) is suggested in the formulae on page 20. However, it is useful to consider the basic causes of these pressures and stresses: that piston load can vary from that due to the maximum gas pressure shortly after ignition, to the negative pressure within the induction stroke. Piston and small end inertia is greatest at the points of maximum acceleration and deceleration which, within the firing stroke, do not coincide with the maximum gas load on the piston crown; big-end assembly inertia forms. Whilst these are constant through 360°, the variation is proportional to the square of the rotational speed; finally it should be remembered that a symmetrical unbalanced four cylinder engine crankshaft appears to be "in balance" as a total unit due to the interaction between the individual crankpins and webs.

The consideration of a single cylinder 'cell' reveals inertia forces due to the centrifugal effect of crankpin and the adjacent webs which, whilst not affecting crankpin bearing loads, have to be considered in relation to the main bearing due to the considerable difference between the dynamic and geometric axis in this section of the crankshaft. The addition of a suitable balance weight or weights can cancel this effect within the crankshaft or, alternatively, can be increased to compensate for the inertia of the big-end assembly. Again, any unbalance remaining will vary as the square of the rotational speed of engine.

2.3.1.1.1 <u>Methods of determining bearing pressures</u>

(i) **Crankpin bearing shells**: The load of the projected area of these shells can be taken as that due to the maximum gas pressure on the piston crown less the maximum reciprocating and rotating inertia forces due to the first two factors above. In engines intended or likely to run at high speeds for prolonged periods or those subject to overrun condition, the forces due to the second and third factors above should be carefully considered as they may impose bearing loads exceeding the gas load.

(ii) **Main bearing shells**: As noted above, the presence of crankshaft 'cell' unbalance has a major influence on the loadings to which these bearings are subjected. The configuration of a five journal crankshaft without balance weights creates differing loads on certain main bearings, and which can result in the unacceptable requirement of varying journal lengths (or diameters).

The partial balancing, therefore, becomes a minimum requirement in order to achieve acceptably equal bearing pressures on bearings of similar length and diameter. The approximated effect of gas load and crankshaft unbalance on individual main bearing pressures is as follows:

1 and 5 main bearings - half of the <u>maximum crankpin load less</u> half of the load due to residual crankshaft unbalance, 2 and 4 main bearings - half of the maximum crankpin load; 3 main bearing - half of the maximum crankpin load less the load due to residual unbalance of one crankpin and its associated webs. It should be noted, that on overrun condition, the two loads can be additive and should, therefore, be considered in the calculation of acceptable bearing pressures.

Main bearing pressures are not fully indicative of the oil film thicknesses and, therefore, safisfactory bearing performance; it is suggested that reference should thus be made to the data available from bearing manufacturers. The need to provide oil feed grooves in four of the five main journal shells has a marked effect in the reduction of oil film thickness due to the creation of two narrow bearing sections. This reduction is in the order of 50 percent. As detailed later the cap half groove can be deleted subject to the cross drilling of the crankshaft journal. The projected area of a bearing is not indicative of the oil film thickness at a given pressure.

A reduction in bearing width accompanied by a corresponding increase in diameter to maintain the projected area will result in reduced oil film thickness. As a broad guide the effect is similar to that on bending fatigue strength by adjustment of web thickness - to the square of the width or thickness.

2.3.1.1.2 Bending Fatigue Strength

Following the assessment of bearing pressures a check should be made on the bending fatigue strength of the crankshaft in its proposed form. Nodular Iron in the 'as cast' or normalised condition (a decision entrusted to the foundry to achieve specification) can tolerate bending stresses of up to plus or minus 10 percent of its ultimate tensile strength and which includes a safety allowance for stress concentration factors on the assumption that the fillet radii are not less than 5 percent of the journal diameters. The use of undercut and radius - rolled radii can increase the fatigue strength towards 20 percent of the ultimate tensile strength, but care should be taken to ensure that the alternative fracture path - collar diameter to collar diameter is adequate - the square of this dimension should be at least twice the distance from fillet radius to fillet radius (primary fracture path).

The bending stress in the webs at the journal overlap section can be determined by a combination of the Kerr Wilson and MIRA formulae:

Stress = 0.75 max load on crankpin X main bearing span/bt^2

where b and t are the breadth and thickness of the web section through the primary fracture path, respectively. The initial results of the above work will enable bearing width, diameter and web thickness to be adjusted to provide acceptable bending stresses and bearing pressures.

In the unusual event of a marginal situation resort may have to be made to fatigue strength enhancement by the use of a higher grade of nodular iron or the employment of deep fillet radius rolling.

2.3.1.1.3 <u>**Fatigue Strength**</u>

Failure by torsional fatigue in four cylinder cast crankshafts can be discounted due to the relative stiffness of this form of crankshaft and the damping capacity of nodular iron compared to crankshaft steels. Attention is, however, drawn to the need to avoid stress raisers on journal surfaces such as poor oil hole blends with journal diameters and to provide a reasonable web breadth (as opposed to thickness) at the point of journal overlap. The use of a basic ovoid profile in conjuction with normal fillet diameters is generally adequate. It is normal practice to fatigue test a number of sample prototype crankshafts on a bending fatigue rig prior to design finalisation. Facilities are also available for torsional fatigue testing should this be considered necessary.

The tests follow normal procedure in the plotting of a S-N curve and are particularly beneficial in the measurements of fatigue strength improvement of deep fillet radius rolled crankshaft.

2.3.2 <u>Detail Design</u>

Based on an awareness of the benefits and limitation of a precision casting, the manufacturing drawing will have been produced in the light of positive decisions in respect of machined and unmachined surfaces together with any special treatment required during the machining operations. Whilst a specialist foundry should be capable of preparing a casting drawing which will 'make' the machined crankshaft, the writer believes that this initiative should lie with the crankshaft designer.

2.3.3 <u>Selection of bearing materials for Internal Combustion Engine</u>

Of prime importance in the selection of bearing materials for internal combustion engine application is the alloy's ability to support the very high loads that originate from the combination of gas and inertia forces. Spike (1971) and Parker (1971), discussed fully the factors affecting the various criteria of plain bearing design. Now, the simplified analysis and assumption to enable the design engineer to easily compute the plain bearing loading is shown in figure 2.3.

A typical example is shown diagramatically in the form of a polar load diagram for a big-end bearing of a four stroke internal combustion engine in Figure 2.3

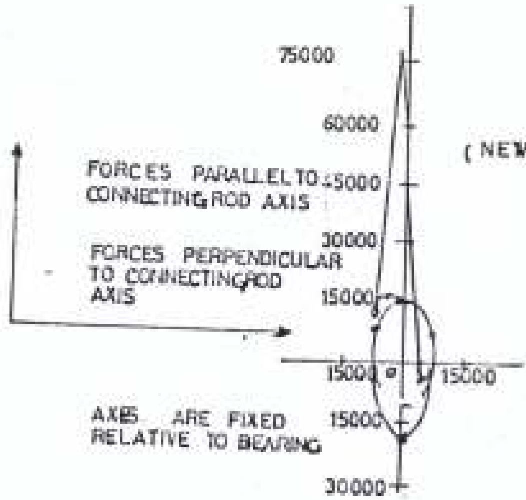
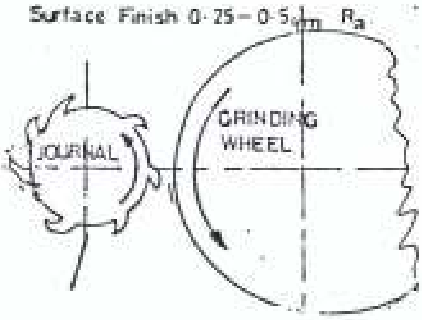


Fig 2.3 Polar load diagram of a big- end bearing for a 4 - stroke engine. (Spike, 1971)

2.3.3.1 <u>Crankshaft finish as it affects material selection for plain bearing wear</u>

Bearing wear cannot be considered in isolation and the role played by the crankshaft journal must also be investigated. Examination of crankshaft journals have shown the type of surface features which cause bearing wear and adequate consideration must be given to the production methods used. Pratt (1973).

To help alleviate problems resulting from crankshaft finish, simple rules on the grinding and polishing directions have been suggested (Figure 2.4). These are: (1) Always include a polishing operation following the grinding operation (2) Always polish in the opposite direction to that of grinding (3) Always polish in the direction of rotation of the journal in the bearing.



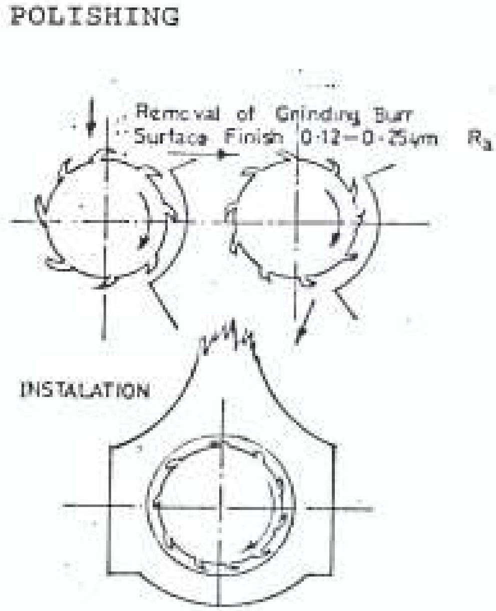
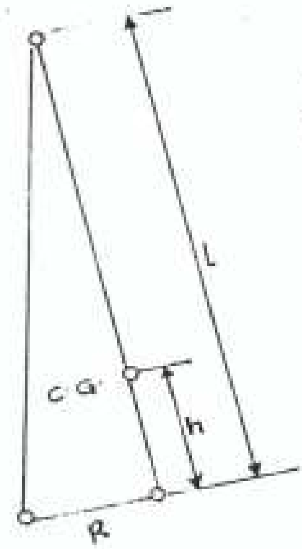


Fig. 2.4 Recommended method of Grinding and Polishing Crankshaft Journals. Pratt, 1973.

2.3.4 Calculation of Crankshaft Stiffness

Cyril (1977) declared that the crankshaft stiffness is the most uncertain element in a torsional vibration calculation. Shaft stiffness can be measured experimentally either by twisting a shaft with a known torque or from the observed values of the critical speeds in a running engine. Figure 2.5a shows the Schematic diagram of a crank and connecting rod, while the figure 2.5b shows the view of a crankweb in a plane normal to the crankshaft axis.



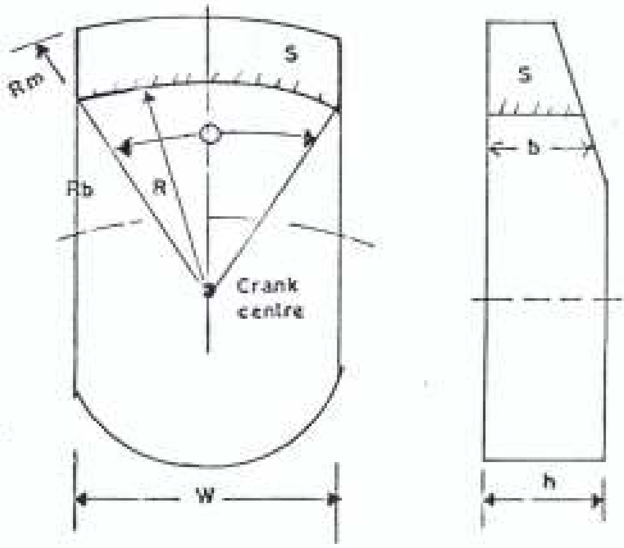


Fig 2.5a Schematic diagram of a crank and connecting rod.

Fig 2.5b View of a crankweb in a plane normal to the crankshaft axis.

<u>Note</u>

h = Fraction of rod length from crankpin to C.G

R = Crank radius; m,

Rm = maximum radius of crankweb, m

Rb = radius of base cylinder, m

If l_e = length of a solid shaft of diameter D_s equal to torsional stiffness to the section of crankshaft between crank centres.

<u>Wilson's formula (Wilson (1942)</u>

$$\frac{l_e}{D_s^4} = \frac{b+0.4ds}{D_s^4-d_s^4} + \frac{a+0.4Dc}{D_c^4-d_c^4} + \frac{r-0.2\,(D_s+D_c)}{hW^3}$$

<u>Ziamanenko's formula (Nestorides, 1958)</u>

$$\frac{l_e}{D_s^4} = \frac{b+0.6hD_s/b}{D_s^4-d_s^4} + \frac{0.8a+0.2(W/r)Dc}{D_c^4-d_c^4} + \frac{r^{\frac{3}{2}}}{hW^3D^{1/2}}$$

Constant's formula (Constant, 1928)

$$\frac{l_e}{D_s^4} = \frac{1}{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\frac{b}{D_s^4-d_s^4} + \frac{a}{D_c^4-d_c^4} + \frac{r^{\frac{3}{2}}}{hW^3}\right)$$

where α_1, $\alpha_2, \alpha_3, \alpha_4$ are modifying factors, determined as follows:

$$\alpha_1 = 1 - \frac{0.0825}{\sqrt{\frac{W_s-d_s}{2W_s} + \frac{W_c-d_c}{2W_c} - 0.32}}$$

If the shaft is solid, assume $\alpha_1 = 0.9$. The factor α_2 is a web-thickness modification determined as follows: If $4h_{/1}$ is greater than 2/3, then $\alpha_2 = 1.666 - 4h/1$. If $4h/1 < 2/3$, assume $\alpha_2 = 1$. The factor α_3 is a modification for web chamfering determined as follows: If the webs are chamfered, estimate α_3 by comparison with the cuts on fig. 2.6. Cut AB and A'B', $\alpha_3 = 1.000$; cut CD alone, $\alpha_3 = 0.965$, cut CD and C'D', $\alpha_3 = 0.930$, cut EF alone, $\alpha_3 = 0.950$; cut EF and E'F', $\alpha_3 = 0.900$; if ends are square, $\alpha_3 = 0.010$. The factor α_4 is a modification for bearing support given by:

$$\alpha_4 = \frac{Al^3W}{D_c^4-d_c^4} + B$$

For marine engine and large stationary engine shafts: A = 0.0029, B = 0.91.

For automobile and aircraft engine shafts: A = 0.0100. B = 0.84.

If α_s is less than 1.0, assume a value of 1.0

The constant's formula is recommended for shafts with large bores and heavy chamfers.

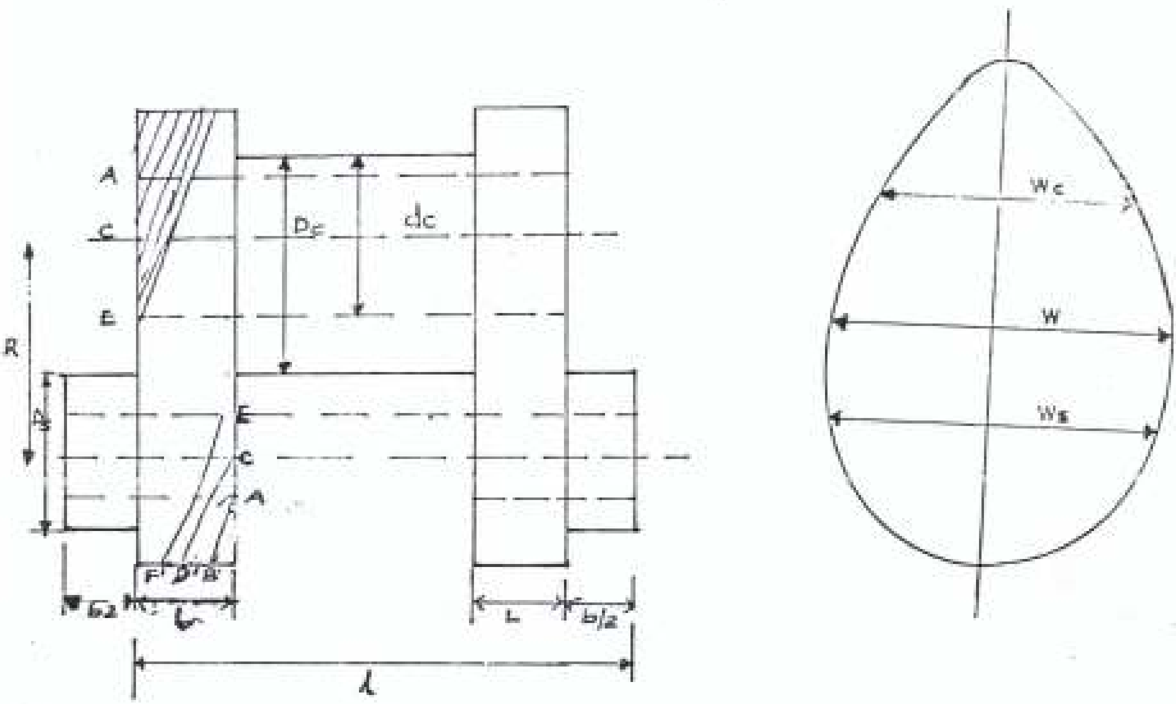


Fig 2.6 Schematic diagram of one Crank of a Crankshaft

The shafting of an engine system may contain elements such as changes of section, collars, shrunk and keyed armatures, etc; which require the exercise of judgement in the assessment of stiffness. For a change of section having a fillet radius equal to 10 percent of the smaller diameter, the stiffness can be estimated by assuming that the smaller shaft is lengthened and the larger shaft is shortened by a length obtained from the curve of fig 2.7. This also may be applied to flanges where D is the bolt diameter. The stiffening effect of collars can be ignored.

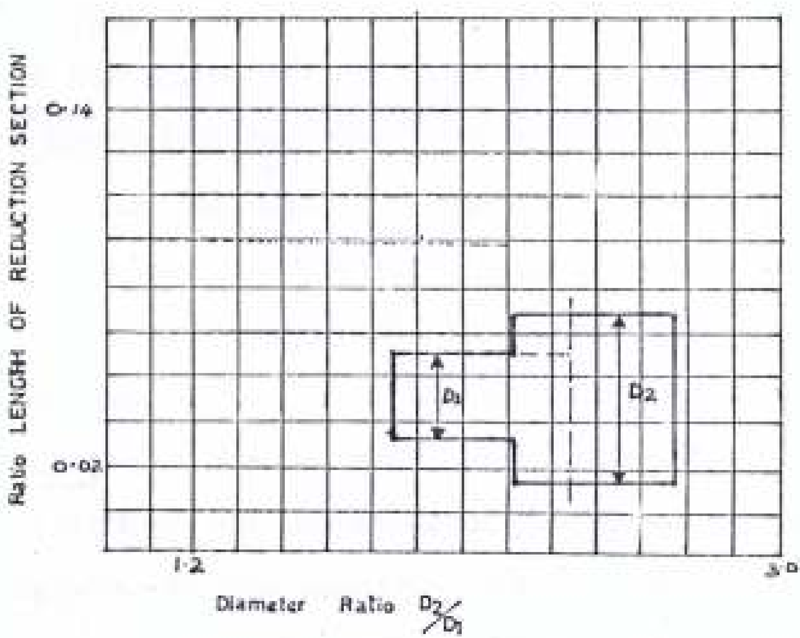


Fig 2.7 **Curve showing the decrease in stiffness resulting from a change in shaft diameter**. (Porter, 1928)

The stiffness of the shaft combination is the same as if the shaft having D_1 is lengthened by λ and the shaft having diameter D_2 is shortened by λ (Porter, 1928).

2.3.5 Critical Speed

Critical speed is a speed of a rotating system that corresponds to a resonant frequency of the system. The significant effect of critical speed may be motion of the journal or flexure of the rotor-depending on the relative magnitudes of the bearing stiffness.

2.3.6 Permissible Amplitudes

Failure caused by torsional vibration invariably initiates in fatigue cracks that start at points of stress concentration - for example at the ends of keyway slots, at fillets where there is a change of shaft size, and particularly at oil holes in a Crankshaft.

At the shaft oil holes the crack begins on lines at 45^0 to the shaft axis and grow in a spiral pattern until failure occurs. Theoretically the stress at the edges of the oil holes is four times the mean shear stress in the shaft, and failure may be expected if this concentrated stress exceeds the fatigue limit of the material. The problem of estimating the stress required to cause failure is further complicated by the presence of the steady stress from the mean driving torque and the variable bending stresses.

In practice the severity of a critical speed is judged by the maximum nominal torsional stress.

$$\tau=\frac{16M_m}{\pi d^3}$$

where M_m is the torque amplitude from torsional vibration and 'd' is the crankpin diameter. This calculated nominal stress is modified to include the effects of increased stress and is compared to the fatigue strength of the material.

2.3.7 United State Military Standard

A United State Military Standard issued by the U.S Navy Department states that the limit of acceptable nominal torsional stress within the operating range is:

$$\tau = \frac{\text{Tonsional Fatigue limit}}{6} \qquad \text{(for cast iron)}$$

$$\tau = \frac{\text{Ultimate tensile strength}}{25} \qquad \text{(for steel)}$$

If the full-scale has been given a fatigue test, then,

$$\tau = \frac{\text{torsional fatigue limit}}{2} \qquad \text{(for either material)}$$

Such tests are rarely, if ever, possible. For critical speeds below the operating range which are passed through in starting and stopping, the nominal torsional strees shall not exceed 1 $^3/_4$ times, the above values.

3.1 KINEMATICS AND DYNAMICS

3.1.1 Kinematics of Slider-Crank Mechanism

In internal combustion engines the reciprocating motion and force of pistons and connecting rods are converted into rotary motion and torque of the crankshaft through the slider-crank (S-C) mechanism.

The two types of S-C mechanism are shown in fig 3.0.

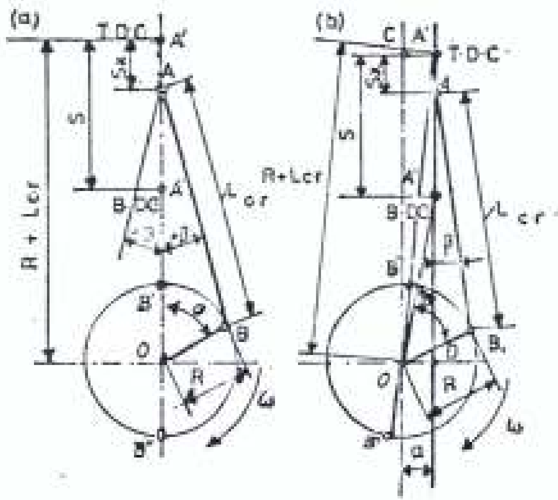


Fig 3.0 - Diagrams of Slider-Crank mechanisms

(a) Central type (b) desaxe type or offset type

For Central Type:

S_x = current travel of the piston ('A' stands for the piston pin axis)

θ = crank angle (OB stands for the crank). counted off from the cylinder axis A'O in the crankshaft rotation direction, clockwise (point O stands for the crankshaft axis, point B - for the crankpin axis, and point A', is

Top Dead Centre (T.D.C)

β = the angle through which the connecting rod axis (AB) diverges from the cylinder axis;

ω = angular velocity of crankshaft rotation

R = OB - radius of crank

S = 2R = A'A" - piston stroke (point A" stands for B.D.C. Bottom Dead Centre)

L.c.r = AB - the connecting rod length

λ = $R/L_{c.r}$ - ratio of the radius of crank to the connecting rod length.

R + L.c.r = A'O - the distance from the crankshaft axis to T.D.C. (Kolchin and Demidov, 1984).

<u>For Offset type</u>

In comparison to the designation for the central crank mechanism, in the offset crank mechanism, angle θ of crank rotation is counted off from straight line CO parallel to the cylinder axis A'D and passing through the crankshaft axis, and S = A'A" = 2R. The desaxe mechanism is evaluated in terms of relative offset K = a/R = 0.05 to 0.15 where a = OD is the cylinder axis offset from the crankshaft axis.

3.1.2 Design of an In-Line Carburettor Engine

For the Kinematic and dynamic analyses, heat analysis and speed characteristics analysis data are needed for any type of engine being analysed.

(a) <u>Kinematics</u> · Table 3.0

The piston travel

$$Sx=R[(1-\cos\theta)+\lambda/4(1-\cos 2\theta)]$$

for a particular value of λ, and variable angle θ

The angular velocity of the crankshaft revolution

$$\omega = \frac{\pi n}{30} \qquad (3)$$

30

n = engine speed in revolution per minute (rpm)

The piston speed

$$V_p \quad = \omega R \; (\sin\theta + {}^\wedge/2 \; \sin 2\theta)$$

The piston Acceleration

$$j = \omega^2 R \; (\cos\theta + \lambda \cos 2\theta) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad 5$$

At $j = 0$, $V_p = \pm V_{max}$. For $\lambda = 0.285$

Table 3.0. Kinematics values

$\theta°$	$((1-\cos\theta + \lambda/4(1-\cos 2\theta))$	Sx, mm	$(\sin\theta + \lambda/2 \times \sin 2\theta)$	V_p, m/s	$(\cos\theta + \lambda \cos 2\theta$	j, m/s²
0°	0.0000	0.0	0.0000	0.0	1.2850	17209
30°	+0.1697	6.0	+0.6234	14.2	1.0085	13506

<u>Dynamics</u>

The dynamic analysis of the crank mechanism consists of determining overall forces and moments caused by pressure of gases and inertia forces which are used to design the main parts for strength and wear and also to determine the non-uniformity of torque and extent of non uniformity in the engine run. During the operation of an engine the crank mechanism parts are acted upon by gas pressure in th cylinder, inertia forces of the reciprocating masses, centrifugal forces, crankcase pressure exerted on the piston (which is about the atmospheric pressure), and (gravity forces are generally ignored in the dynamic analysis).

All forces acting in the engine are taken up by the useful resistance at the crankshaft, friction forces and engine supports.

During each operating cycle (720° for four - and 360° for two- stroke engine), the forces acting in the crank mechanism continuously vary in value and direction. Therefore, to determine changes in these forces versus the crankshaft rotation angle, their values are determined for a number of crankshaft positions taken usually at 10-30 degree intervals.

3.1.3 Gas Pressure Forces

The gas pressure forces exerted on the piston area are replaced with one force acting along the cylinder axis and applied to the piston pin axis in order to make the dynamic analysis easier. It is determined for each moment of time (angle against an indicator diagram obtained on an actual engine or against an indicator diagram plotted on the basis of the heat analysis (usually at the rated power output and corresponding engine speed).

Replotting an indicator diagram into a diagram developed along the crankshaft angle is generally accomplished by the method of Brix .

The piston pressure force

$$P_g = (P'_g - P'_o)\ F_p \ ....(6)$$

where F_p = piston area, m^2

P'_g and P'_o are the gas pressure at any moment of time and the atmospheric pressure respectively.

$\Delta P_g = P_g - P_o$ .... (7)

ΔP_g = excess pressure over the piston

It follows from Eg (7) that the gas pressure curve versus the crank angle vary similarly to the gas pressure curve P_g. In order to determine gas forces P_g against the developed diagram of pressures ΔP_g, the scale must be recomputed. If curve P_g is plotted to the scale M_p(MN per mm) then the scale of the same curve for P_g will be $M_p = M'_p$

F_p(MN per mm).

The Brix correction is

$R\lambda/ (2M_s)$

Where M_s is the piston travel scale.

$M_p = M'_p\ F_p$

3.1.4 **<u>Masses of the parts of the Crank Mechanism</u>**

As to the type of motion the masses of the crank mechanism prts may be divided into reciprocating (the piston group and the connecting rod small end), rotating (the crankshaft and the connecting rod big end), and those performing plane-parallel motion (the connecting rod shank).

The actual crank mechanism is replaced with equivalent system of concentrated masses for the dynamic analysis.

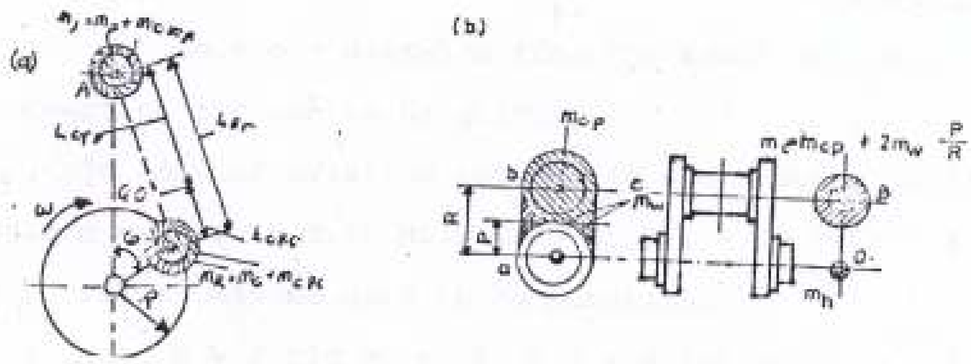


Fig 3.1 System of concentrated masses dynamically equivalent to the crank mechanisms.

(a) System of crank mechanism referred to axes.

(b) referring crank mass.

<u>Notes</u>

(1) The mass of the piston group M'_p is concentrated at the piston pin axis at point A (Fig. 3.1a)

(2) The mass of the connecting rod group mc.r is replaced by two masses one of which (Mc.r.p) is concentrated on the piston pin axis at point A, and the other (Mc.r.c) - on the crank axis, at point B.

The value of these masses (Kg) are:

Mc.r.p = (Lc.r.c/Lc.r) Mc.r..........(8)

Mc.r.c = (Lc.r.c/Lc.r) Mc.r..........(9)

where

Lc.r = connecting rod length

Lc.r.c = distance from the big -end centre to the connecting rod centre of gravity:

Lc.r.p = distance from the small end centre to the connecting rod centre of gravity.

In most of existing automobile and tractor engines Mc.r.p = (0.2 to 0.3) Mc.r. and Mc.r.c = (0.7 to 0.8) Mc.r. Mean values may be used in computations.

Mc.r.p = 0.275 Mc.r, Mc.r.c = 0.725 Mc.r......(10)

The crank mass is replaced with two masses concentrated on the crank axis at point B (Mc) and on the main bearing journal axis at point O (Mc).

Fig 3.1.b. The mass of the main bearing journal plus part of the webs are symmetrical with regard to the axis of rotation is balanced. The mss (kg) concentrated at point B is

Mc = Mc.p + 2 Mw ρ/R..............(11)

Where Mc.p = mass of crankpin with adjacent parts of the webs.

Mw = mass of the web middle part within the outline abcd having its centre of gravity at radius ρ .

In modern short - stroke engines the value of Mw is small compated with Mc.p and may be neglected in most cases. The values of Mc.p and also Mw if necessary, the determined proceeding from the crank size and density of the crankshaft materials.

Therefore, the system of concentrated masses dynamically equivalent to the crank mechanism consists of mass.

$M_j = M_p + Mc.r.p$............................(12)

concentrated at point A, which reciprocates. and mass

$M_R = Mc + Mc.r.c$............................(13)

concentrated at point B, which rotates.

Roughly the values of M'_p, Mc.r and Mc may be determined, using the structural masses

$M' = M/F_p$ (kg/m^2 or g/cm^2)..........................(14)

mass of unbalanced parts of one crankshaft throw without counterweight e.g (for a cast Iron Crankshaft we assume $M_{th} = 140 kg/m^2$)

$M_{th} = M'_{th}\ F_p$..............................(15)

Rotating masses

$M_R = M_{th} + Mc.r.c$.................(16)

3.1.5 **Inertia Forces**

Inertia forces acting in a slider-crank mechanism, in compliance with the type of motion of the driven masses, fall into inertia forces of reciprocating masses P_j and inertia centrifugal forces of rotating masses K_R fig 3.2

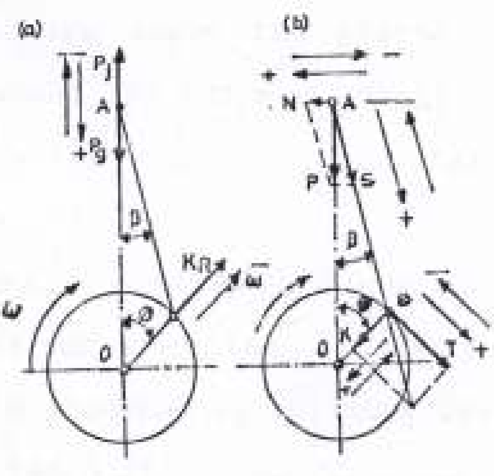


Fig. 3.2 Acting of forces in the Crank mechanism.

(a) Inertia and gas force; (b) total forces)

The inertia force produced by the reciprocating masses

$P_j = M_j\, j = - M_j\, Rw^2 \times (\cos\theta + \lambda\cos 2\theta)$.... (17)

As the case is with piston acceleration, for P_{jI} and secondary P_{jII} forces;

$P_j = P_{jI} + P_{jII}$

$= -(MjRw^2 \cos ß + M_j\, Rw^2\, \lambda\cos 2\theta)$ ....(18)

The centrifugal inertia force of rotating masses

$K_R = - M_R\, Rw^2$..................................(19)

The inertia force of the connecting rod rotating masses

$K_{Rc.r} = - Mc.r.c\ Rw^2$..........................(20)

and the inertia force of the crank rotating masses

$K_{RC} = - M_c\, Rw^2$..................................(21_

3.1.6 Total Forces Acting on Slider Crank Mechanism

The total forces (KN) acting in the slider crank mechanism are determined by algebraically adding the gas pressure forces to the forces of reciprocating masses. (Table 3.1)

$P = \triangle P_g + P_j$..................................(22)

The specific total forces (MPa) are determined by adding the excess pressure above the piston $\triangle pg$ (MPa) and specific inertial forces Pj $(MN/m^2$ = (MPa):

$P = \triangle pg + P'_j$ ....(23)

where

$P'_j = P_j F_p = - (M_j\, Rw^2/F_p)\ (\cos\theta + \lambda\cos 2\theta)$

Force N (KN) normal to the cylinder axis is called the normal force and is absorbed by cylinder walls:

$N = P \tan ß$ ....(25)

Normal force N is known as positive, if the torque it produces with regard to the crankshaft axis opposes the engine shaft rotation.

Force S (KN) directed along the connecting rod acts upon it and is transmitted to the crank. It is known as positive if it compreses the connecting rod, and negative, if it stretches the rod:

$$s = P\ (1/\cos \beta) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (26)$$

Acting upon the crankpin force S produces two component forces Fig. 3.2b:

a force directed along the crank radius (KN)

$$K = P\ \cos\ (\theta + \beta)\ /\mathrm{Cos}\ \beta \ \ldots\ldots\ldots\ldots\ (27b)$$

Force K is known as positive, when it compresses the crank throw webs. Force T is taken as positive, if the torque produced by it coincides with the crankshaft rotation direction.

Table 3.1: Total Forces Acting on Slider Crank Mechanisms

	Sign	Values of tan β at λ of					Sign	
		0.24	0.25	0.26	0.27	0.28		
0	+	0	0	0	0	0	-	360
30	+	0.121	0.126	0.131	0.136	0.141	-	330
60	+	0.211	0.220	0.230	0.239	0.248	-	300
90	+	0.245	0.256	0.267	0.278	0.289	-	270
120	+	0.211	0.220	0.230	0.239	0.248	-	240

	Sign	Values of tan β at λ of					Sign	
		0.24	0.25	0.26	0.27	0.28		
0	+	1	1	1	1	1	+	360
30	+	1.007	1.008	1.009	1.009	1.010	+	330
60	+	1.022	1.024	1.026	1.028	1.030	+	300
90	-	1.030	1.032	1.235	1.038	1.041	+	270
120	-	1.022	1.024	1.026	1.028	1.030	+	240

	Sign	Values of tan β at λ of					Sign	
		0.24	0.25	0.26	0.27	0.28		
0	+	1	1	1	1	1	+	360
30	+	0.806	0.803	0.801	0.798	0.795	+	330
60	+	0.317	0.309	0.301	0.293	0.285	+	300
90	+	0.245	0.256	0.267	0.278	0.289	-	270
120	+	0.683	0.691	0.699	0.707	0.715	-	240

	Sign	Values of tan β at λ of					Sign	
		0.24	0.25	0.26	0.27	0.28		
0	+	0	0	0	0	0	-	360
30	+	0.605	0.609	0.613	0.618	0.622	-	330
60	+	0.972	0.979	0.981	0.985	0.990	-	300
90	+	-	-	-	-	-	-	270
120	+	0.760	0.756	0.751	0.747	0.742	-	240

Table 3.2. Torques on Cylinder

	P_a	j m/s^2	P_j MN/m^2	P MN/m_2	tanβ	P_N MN/m_2	1/cos β	P_s MN/m_2	Pc MN/m_2	Ps MN/m_2	T KN	M_k Nm
0	0.918	17209	-2.43	-2.41	0.00	0.00	1.00	-2.41	-2.41	0	0	0
30	-0.015	13506	-1.91	-1.92	0.14	-0.28	1.01	-1.94	-1.52	-2.19	-5.73	-223
60	-0.15	4798	0.68	-0.69	0.25	0.10	1.03	-0.71	-0.19	-0.69	-3.27	-128
90	0.015	-3817	0.54	0.52	0.30	0.15	1.04	0.55	-.15	-0.52	2.50	-97

using the table overleaf, plot curve of specific forces P_j, P, P, P_N, P_c, P_{ci} versus the crankshaft angle.

3.1.7. **Specific Total Forces**

The specific force (in MN) concentrated on the axis of the piston pin: $P = \Delta p_g + Pj$......................(28a)

The specific rated force (in MPa)

P_N = P tan ß..........................(28b)

The specific force (MN) acting along the connecting rod

P_s = P/ (1/cosß) ....(28c)

The specific force (MN) acting along the crank radius

P_c = P cos (θ + ß) / Cosß.................(28d)

T_m = 2 Pi F_p / (πτ) ....(29)

where T_m is the mean value of the tangential force per cycle, MN;

P_i is the indicated pressure, MN,

F_p = piston area, m^2

τ = number of cycle events

3.1.8 **Torques**

The torsional moment (torque) of one cylinder (MNm) is determined by the value of T given by

$M_{t,\alpha}$ = TR........................930)

The torque variation period of a four stroke engine with equal firing intervals. (Table 3.2)

$\theta = 720/_i$ in a four stroke engine over...........(31)

$\theta = 360^0/_i$ in a two stroke engine over...........(32)

Table 3.2 Torques on cylinders.

θ°	Cylinders								Mt, Nm
	1		2		3		4		
	Crank angle θ	Mt.c, Nm	Crank angle θ	Mt.c N.m	Crank angle θ	Mt.c, Nm	Crank angle θ	Mt.c Nm	
0	0	0	180	0	360	0	540	0	0
30	30	-223.3	210	-94.6	390	176.6	570	-97.4	-238.7
60	60	-127.6	240	-165.2	420	124.8	600	-169.7	-337.7
90	90	97.4	270	-103.9	450	234.3	630	-103.5	124.3
120	120	165.2	300	97.0	480	229.3	660	121.5	612.9

3.1.9 Force Acting on Crankpins

The forces acting upon the crankpin of row are determined analytically.

Row engines - the analytical resulting force acting on the crankpin of a row (in-line) engine fig 3.3 is

(a) (b)

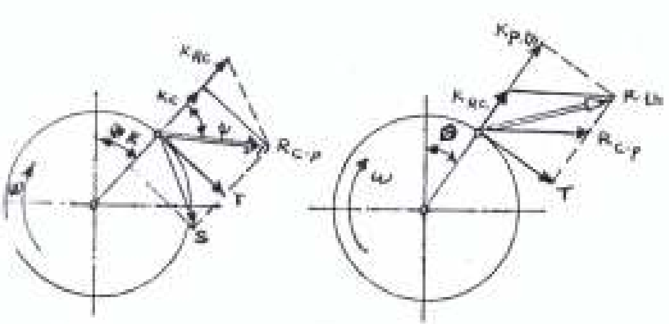


Fig 3.3 Forces loading

(a) Crankpin (b) Crankshaft throw

$$Rc.p = \sqrt{T^2 + P_c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

where $P_c = K + K_{Rc}$ is the force acting on the crankpin by the crank, N.

The direction of resulting force Rc.p in various positions of the crankshaft is determined by angle enclosed between vector Rc.p and the crank axis.

Angle Ψ is found from the relation.

$$\tan \Psi = T/P_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

The mean indicated torque of an engine according to the data of heat analysis

$$M_{t.m} = M_i = M_e/N_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

N_m = Mechanical efficiency of the engine

$$M_{t.m} = \frac{F_1 - F_2}{OA} M_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

CHAPTER FOUR

4.0 PREREQUISITE FOR DESIGN AND DESIGN CONDITIONS

4.1 General

The computations of engine parts with a view to determining stresses and strains occuring in an operating engine are performed by formulae dealing with strength of materials and machine parts.

Causes of the discrepancy between the computed and actual data are: absence of an actual pattern of stresses in the material of the part under design, use of approximate design diagrams showing action of forces and points of their application, presence of alternating loads difficult to take into account and impossibility of determining their actual values; difficulty in determining the operating conditions for any engine parts and their thermal stresses; effects of elastic vibrations that do not lend themselves to accurate analysis, and the impossibility of accurately determining the influence of surface condition, quality of finish (machining and thermal treatment) part size and the like on the intensity of stresses arising.

In view of this, the utilized techniques of surveying allow us to obtain stresses and strains that are nothing more than conventional values characteristic only of relative stress level of the part under design.

Forces caused by gas pressure in the cylinders and inertia of reciprocating and rotating masses, and also loading produced by elastic vibrations and thermal stresses are the main loads on the engine parts.

The loading caused by gas pressure continuously varies during the working cycle and reaches its maximum within a comparatively small portion of the piston stroke. Loading due to inertia forces varies periodically and sometimes reaches in high-speed engines the values exceeding the load due to gas pressure. The above loads are sources of various elastic oscillations dangerous during resonance.

Force due to thermal stresses resulting from heat liberation due to combustion of mixture and friction affect the strength of materials and cause extra stresses in making parts when they a... differently heated and have different linear (or volumetric) expansion) (Kolchin and Demidov, 1984).

4.1.1 Design Conditions

Changes in the basic loads acting on the engine parts are dependent on the operating conditions of the engine. Generally, the engine parts are designed for most severe operating conditions.

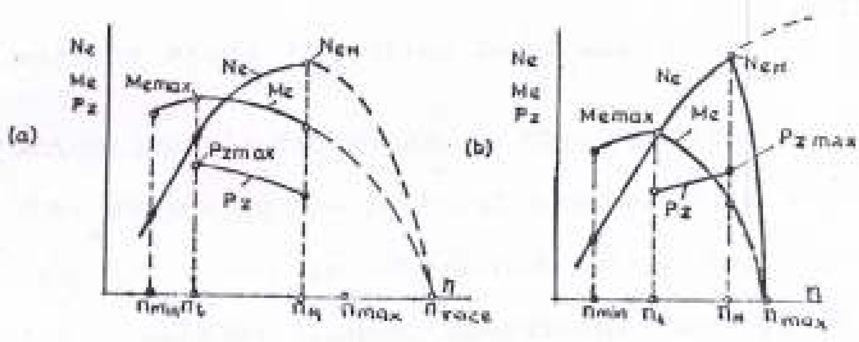


Fig 4.0 Choice of design operating conditions (a) Carburretor engine (b) Supercharged diesel engine

with carburettor engines (fig. 4.0a) the basic designed operating conditions include the following data:

(1) Maximum torque Memax at the engine speed n_m = (0.4 to 0.6) x n_n, when the gas pressure reaches its maximum P_{zmax}, while the inertia forces are comparatively small;

(2) nominal output power N_m at speed n_N, with all analyses of parts are made with taking into account the joint effect of gas and inertia loadings;

(3) maximum speed in idling Πidmas = (1.05 to 1020)n_N, when inertia forces reach their maximum, while the gas pressure is small or even equal to zero. With high-speed diesel engine, Fig. 4.0b, we take the following design operating conditions:

(1) nominal power output N_{eN} at engine speed n_N, when the pressure reaches its maximum $P_z$max, while the parts are designed to challenge the joint effort of gas and inertia loads.

(2) maximum speed in idling Πnid max (1.04 to 1.07) n_N at which inertia forces reach their maximum,

When designing the parts of a carburettor engine maximum gas pressure Pz max is determined by the heat analysis made for the maximum torque operation, or is assumed as approximately equal to the designed (without considering the diagram rounding-off factor) maximum combustion pressure Pz obtained from the heat analysis for the nominal power operation. Inertia forces is the maximum torque computations are neglected.

When making nomimal power operation computations, we assume that gas force Pz acts together with the maximum inertia force at the T.D.C. The value of a maximum gas force is determined by the heat analysis for the nominal power operation taking into account the rounding off factor of the indicator diagram.

The gas pressure is neglected in making the maximum speed computations for idling operation.

4.2 DESIGN OF PARTS WORKING UNDER ALTERNATING LOADS

In practice, all parts of automobile and tractor engines, even under steady state conditions, operate at alternating loads. The influence of maximum loads, and also their variation in time on the service life of automobile and tractor engine parts materially increases with an increase in the engine speed and compression ratio. In this connection, a number of engine parts of importance are designed to meet the requirements for static strength against the action of maximum force and fatigue strength due to the effect of continuously varying loads.

The fatigue strength of parts is dependent on variation of a load causing symmetric, asymmetric or pulsating stresses in the part under design; on fatigue limits σ_{-1}, σ_{-1p} and τ_{-1} (for bending, push-pull and torsional stresses, respectively) and yield strength α_y and τ_y of the part material; on part shapes, size, machining and thermal treatment, and case-hardening.

Depending upon the variation of the acting load, the stresses occurring in the part vary, following a symmetric, assymetric or pulsating cycles. Each cycle is characterized by maximum (α_{max}) and minimum (α_{min}) stresses, mean stress cycle amplitude α_a and cycle assymmetric coefficient r. For the relationship between the above-mentioned characteristics for the cycles, see Table 4.0

Under static loads ultimate strength α_u or yield strength α_y is assumed to be the limit stress. The ultimate strength is utilized in design of parts of made of brittle materials. With plastic materials the dangerous stress is indicated by the yield strength.

Table 4.0 Cycle Characteristics

Cycle Characteristics	Cycle			
	Symmetric	asymetric		Single-sign, pulsating
		Positive, of constant sign	alternating sign	
Maximum stress	$\alpha_{max} = -\alpha_{min} = \alpha_a > 0$	$\alpha_{max} = \alpha_a + \alpha_m > 0$	$\alpha_{max} = \alpha_a + \alpha_m > 0$	$\alpha_{max} = \alpha_{a} + \alpha_{m} = 2\alpha_{a} = 2\alpha_{m} > 0$
Minimum Stress	$\alpha_{min} = -\alpha_{max} = -\alpha_a < 0$	$\alpha_{max} = \alpha_m - \alpha_a > 0$	$\alpha_{min} = \alpha_m - \alpha_a < 0$	$\alpha_{min} = 0$
Mean Stress	$\alpha_m = 0$	$\alpha_m = (\alpha_{max} + \alpha_{min})/2$	$\alpha_m = (\alpha_{max} + \alpha_{min})/2$	$\alpha_m = \alpha_{max}/2$
Stress amplitude	$\alpha_a = \alpha_{max} = -\alpha_{min}$	$\alpha_a = (\alpha_{max} - \alpha_{min})/2$	$\alpha_a = (\alpha_{max} - \alpha_{min})/2$	$\alpha_a = \alpha_{max}/2 = \alpha_m$
Coefficient of asymmetry	$r = \alpha_{min}/\alpha_{max} = -1$	$0 < r < 1$	$-1 < r < 0$	$r = \alpha_{min}/\alpha_{max} = 0$

Under alternating loads the dangerous stress is indicated by fatigue limit α_r($\alpha_r = \alpha_{-1}$ for a symmetric cycle and for a pulsating cycle), or yield strength α_y. In design parts the associated limit is dependent on the stress cycle asymmetry.

When a part is subjected to normal or tangential stresses that meet the condition.

$$\alpha_a/\alpha_m > (\beta_a - \alpha_a')/(1 - \beta_a)$$

or

$$\tau_a/\tau_m > (\beta_\tau - \alpha_\tau)/(1 - \beta_\tau)$$

the computations are made by the fatigue limit.

When a part is under stresses satisfying the condition in Equation τ_a/τ_m

the computationsare made by the yield limit

Where β_σ and β_τ are respectively the ratios of the fatigue limit due to bending and torsional stress to the yield limit.

$\beta_\sigma = \alpha_{-1}/\alpha_y$ and $\beta_\tau = \tau_{-1}/\tau_y$

where α_s and α_τ are the coefficients of reducing an asymmetric cycle to the aquidangerous symmetric cycle under normal and tangential stresses, respectively.
For the values of α'_σ and α'_τ in cast iron materials having different ultimate strength see Table 4.1.

Table 4.1 Values of α'_σ and α'_τ

Ultimate Strength α_u, MPa	Bending α'_σ	Push - Pull α'_σ	Torsion α'_τ
350-450	0.06 - 0.10	0.06 - 0.08	0
450-600	0.08 - 0.13	0.07 - 0.10	0
600-800	0.12 - 0.18	0.09 - 0.14	0 - 0.08
800 - 1000	0.16 - 0.22	0.12 - 0.17	0.06 - 0.10

When there are no data as in Table 4.1 to solve equation, the part safety factor is determind either by the fatigue

limit or by the yield limit. Of the two values thus obtained the part strength is evaluated in terms of a smaller coefficient. To roughly evaluate the fatigue limits under an alternating load, use is made of empirical relationship.:

For steels

$$\sigma_{-1} = 0.40\alpha_u; \quad \sigma_{-1p} = 0.28\alpha_u$$

$$\tau_{-1} = 0.22\tau_u; \quad \sigma_{-1p} = (0.7 - 0.8)\sigma_{-1};$$

$$\tau_{-1} = (0.4 - 0.7)\sigma_{-1};$$

for cast iron

$$\sigma_{-1} = (0.3 - 0.5)\sigma_u; \quad \sigma_{-1p} = (0.6 - 0.7)\sigma_{-1}$$

$$\tau_{-1} = (0.7 - 0.9)\sigma_{-1}; \quad \tau_y = (0.2 - 0.6)\sigma_u$$

For the basic mechanical properties of high duty cast iron, use the appropriate Table 4.2

Table 4.2 Basic Mechanical Properties of Materials

Grade	Mechanical Properties of Steel/Cast-Iron, Mpa					
	α_u	α_y	α_{-1}	α_{-1p}	τ_y	τ_{-1}
BY 45 - 0	450	-	700	-	-	-
BY 40 - 10	400	-	700	-	-	-
BY 60 - 2	600	-	1100	-	-	-

Neglecting the part shape, size and surface finish, the safety factor of engine part is determined from the expressions:

when computing by fatigue limit.

$n_\sigma = \sigma_{-1}/(\sigma_a + \alpha'_\sigma \sigma_m)$

$n_\tau = \tau_{-1}/(\tau_a + \alpha_\tau \tau_m$

when computing by the yield limit

$ny_\sigma = \sigma_y/(\sigma_a + \sigma_m)$

$ny_\tau = \tau_y/(\tau_a + \tau_m)$

The effect of the part shape, size and surface finish on the fatigue strength is allowed for as follows:

(1) by stress concentration factors: theoretical $\alpha_{c\delta}$ and effective $k_\delta(k_\tau)$ accounting for local stress increases due to changes in the part shape (holes, grooves, fillets, threads, etc).

(2) by scale coefficient ϵ_s accounting for the influence of the absolute dimensions of a body on the fatigue limit;

(3) by coefficient of surface sensitivity $\epsilon_{s.s}$ accounting for the effect of surface condition of the part on the yield limit.

By the theoretical stress concentration factor is meant the ratio of the highest local stress to the nominal stress under state loading, neglecting the effect of concentration.

$$\alpha_{c\delta} = \sigma_{max}/\sigma_{nom}$$

The values of $\alpha_{c\delta}$ for a number of most often encountered stress concentrators are given in Table 4.3

The influence of the specimen material as well as the geometry of the stress concentrator on the ultimate strength is accounted for by effective stress concentration factor under variable stresses

$$k_\sigma = \sigma_{-1}/\sigma^c_{-1}$$

Where σ_{-1} and σ^c_{-1} stand for the fatigue limit of a smooth specimen in a symmetric cycle and with a stress concentrator, respectively.

Table 4.3 Type of stress Concentrator

Type of Stress concentrator	$\alpha_{c\sigma}$
Semiconductor groove having the following ratio of the radius to the diameter of the rod	
0.1	2.0
0.5	1.6
1.0	1.2
2.0	1.1
Fillet having the following ratio of the radius to the diameter of the rod	
0.0625	1.75
0.125	1.50
0.25	1.20
0.5	1.10
Square Shoulder	2.0
Cut V-shape groove (thread)	3.0-4.5
Holes having the ratio of the hole diameter to the rod diameter from 0.1 to 0.33	2.0-3.0
Machining marks on the part surface	1.2-1.4

The relationship between factor $\alpha_{c\sigma}$ and k_σ is expressed by the following approximate relationship.

$$k_\sigma = 1 + q(\alpha_{c\sigma} - 1)$$

where q is the coefficient of material sensitivity to stress concentration (it varies within the limits $0 < q < 1$).

The value of q is dependent mainly on the material properties:

Grey cast iron............................0

High duty & malleable cast iron........0.2 - 0.4

Structural steels ....0.6 - 0.8

High duty alloyed steels ....about 1

Besides, coefficient q may be determined against the corresponding curves in Fig 4.1

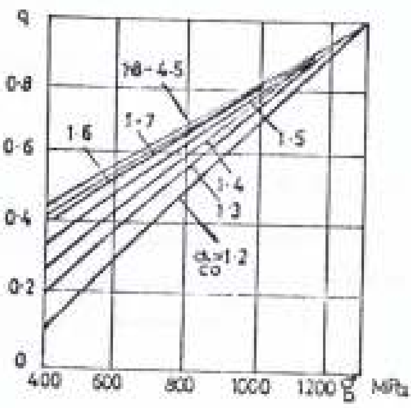


Fig 4.1 Coefficient of steel stress concentration Sensitivity. (Kolchin and Demidov, 1984).

When a part has no abrupt dimensional changes and is properly finished in machining, the only factor causing stress concentrations is the quality of the material internal structure. Then, the effective concentration factor.

$$K_\sigma \quad = \quad 1.2 + 1.8 \times 10^{-4} \, (\alpha_b - 400)$$

where α_b is the ultimate strength, MPa

The relationship between factors K_σ and K_τ can be expressed by the experimental data

$K_{\tau} \quad = \quad (0.4 \text{ to } 0.6)K_{\sigma}$

When designing engine parts the effect of local stresses must be minimized to add to the fatigue strength. This is obtained by increasing the fillet radii in inside corners of the part, by locating the holes in zones of low stresses etc.

By the scale factor ϵ_s is meant the ratio of the ultimate strength of a specimen of diameter d to the ultimate strength of a standard specimen (dst = 10mm). For the values of factor ϵ_s for structural steels and high duty cast irons, see Table 4.4

Table 4.4 Value of Scale Factors

Scale Factors	Part dimensions, mm							
	10*	10-15	15-20	20-30	30-40	40-50	50-100	100-200
ϵ_{σ}	1	1-0.95	0.95-0.90	0.90-0.85	0.85-0.80	0.80-0.75	0.75-0.65	0.65-0.55
ϵ_{τ}	1	1-0.94	0.94-0.88	0.88-0.83	0.83-0.78	0.78-0.72	0.72-0.60	0.60-0.50

* For parts less than 10 mm in size $\epsilon_{s\sigma}$ and $\epsilon_{s\tau}$ may reach 1.1 - 1.2 ($_{s\sigma}$ being ϵ_s in push - pull and bending stresses, while $\epsilon_{s\tau}$ is ϵ_s in torsional stress)

Table 4.5 Surface finish or surface hardening

	Surface finish or surface hardening	$\epsilon_{ss\sigma} = \epsilon_{ss\tau}$	Surface finish or surface hardening	$\epsilon_{ss\sigma} = \epsilon_{ss\tau}$
1	Polishing without surface hardening	1	shot blasting	1.1 - 2.0
2	Grinding without surface hardening	0.97 - 0.85	Rolling	1.1 - 2.2
3	Finish turning without surface hardening	0.94 - 0.80	Carburization Hardening	1.2 - 2.5
4	Rough turning without surface hardening	0.88 - 0.60	Nitriding	1.2 - 3.0
5	With no finishing and surface hardening	0.76 - 0.50		

By the surface sensitivity factor ϵ_{ss} is meant the ratio of the fatigue limit of a specimen having a prescribed surface finish to the fatigue limit of a similar specimen having a polished surface. For the values of factor $\epsilon_{ss\sigma} = \epsilon_{ss\tau}$ for various surface finishes, see Table 4.5.

To increase the fatigue strength, a high surface finish is recommended, especially near the concentrators. The part of importance operating under severe conditions of cyclic stresses are usually ground and polished and sometimes mechanically hardened or heat treated.

With consideration for the effect of stress concentration, dimensions and quality of surface finish, the cycle maximum stress (MPa)

$$\sigma_{max} = \sigma_a k_\sigma / (\epsilon_s \epsilon_{ss}) + \sigma_m$$

or

$$\tau_{max} = \tau_a k_\tau / (\epsilon_s \epsilon_{ss}) + \tau_m \qquad \ldots\ldots 38$$

and the safety factor:

When computing by the fatigue limit

$$n_\sigma = \sigma_{-1}/(\sigma_{a,c} + \alpha_\delta\sigma_m) \quad \text{.........................} 39$$

$$n_\tau = \tau_{-1}/(\tau_{a,c} + \alpha_\tau\tau_m)$$

when computing by the yield limit

$$n_{y\sigma} = \sigma_y/(\sigma_{a,c} + \sigma_m) \quad \text{.........................} 40$$

$$n_{y\tau} = \tau_y/(\tau_{a,c} + \tau_m)$$

where

$$\alpha_{a,c} = \alpha_a k_\delta/(\epsilon_s\epsilon_{ss}) \text{ and}$$

$$\tau_{a,c} = \tau_a k_\tau/(\epsilon_s\epsilon_{ss})$$

when in a complicated stress state the total safety factor of the part jointly affected by tangential and normal stresses

$$n = n_\sigma n_\tau/\sqrt{(n_\sigma^2 + n_\tau^2)} \quad \text{.....................} 41$$

where n_σ and n_τ are particular safety factors.

To determine a minimum total safety factor, the minimum values of n_δ and n_τ should be substituted in the equation 41. Temperature increase affects the fatigue strength in that the yield limit usually drops in smooth specimens and specimens with concentrators.

The value of a permissible safety factor is dependent on the material quality, strain type, operating conditions, construction, acting design and amount of material used are dependent on proper defining of the permissible stress.

4.3 DESIGN OF CRANKSHAFT

4.3.1 General

The crankshaft is a very complicated and strained engine part subjected to cyclic loads due to gas pressure, inertia forces and their couples. The effect of these forces and their moments cause considerable stresses of torsion, bending and tension - compression in the crankshaft material. Apart from this, periodically varying moments cause torsional vibration of the shaft with resultant additional torsional stresses.

Therefore, for the most complicated and severe operating conditions of the crankshaft, high and diverse requirements are imposed on the materials utilized for fabricating crankshafts. The crankshafts material has to feature high strength and toughness, high resistance to wear and fatigue stresses, resistance to impact loads, and hardness. Such properties are possessed by properly machined carbon and alloyed steels and also high duty cast iron. Crankshafts of the Soviet made automobile and tractor engines are made of steels, 40, 45, 45T2, 50 of special cast iron, and those for augmented engines, of high alloy steels, grade 18 XHBA, 40 XHMA and others.

The intricate shape of the crankshaft, a variety of forces and moment loading it, changes in which are dependent on the rigidity of the crankshaft and its bearings and some other causes do not allow the crankshaft strength to be computed precisely. In view of this, various approximate methods are used which allow us to obtain conventional stresses and safety factors for individual elements of a crankshaft. The most popular design diagram of a crankshaft is a diagram of a simply supported beam with one (fig 4.2 a & b) and two (fig 4.2c and d) spans between the supports. (Kolchin and Demidov, 1984).

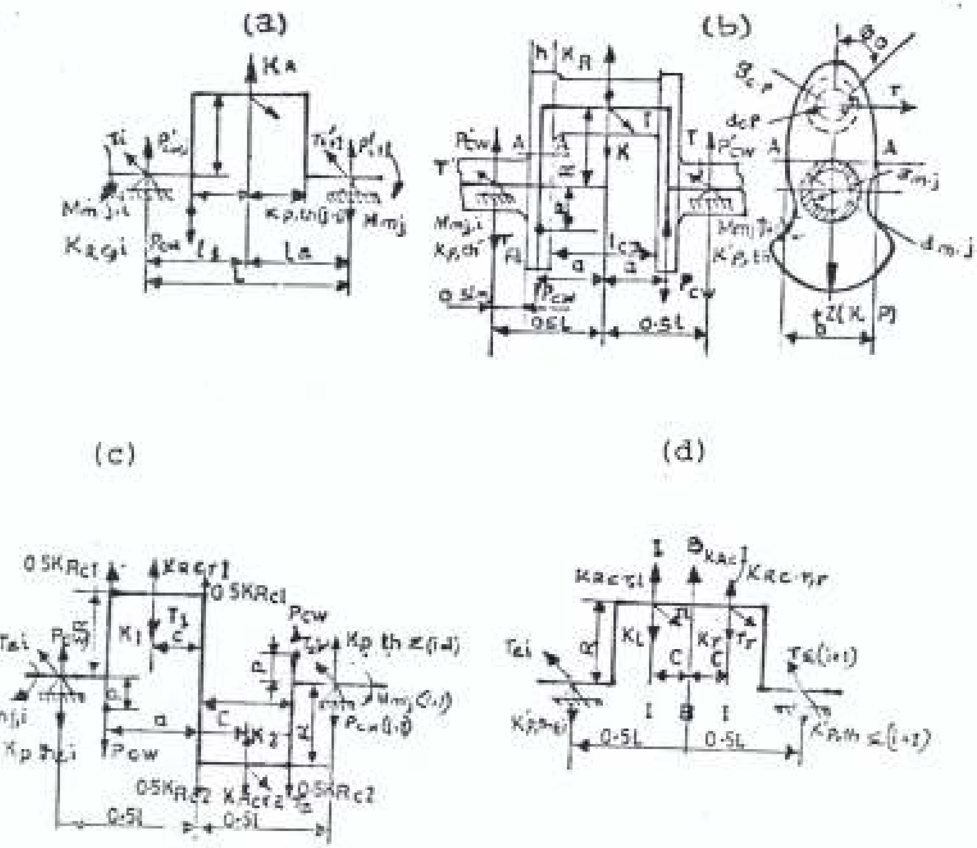


Fig 4.2 Design diagrams of Crankshaft (a), (b) Single span and (c), (d) two span

When designing a crankshaft, we assume that: a crank (or two cranks) are freely supported by supports, the support and force points are in the center planes of the crankpins and journal the entire span (one or two) between the supports represents an ideally rigid beam.

The crankshaft is generally designed for the nominal operation ($n = n_N$), taking into account the action of the following forces and moments (fig 4.2b):

(1) $K_{pth}^{A} = K + K_R = K + K_R$ c.r + K_{Rc} are the forces acting on the crankshaft throw by the crank, neglecting counterweight, where $K = P \cos(\theta + \beta) / \cos\beta$ is the total force directed along the crank radius, $K_R = -M_R R W^2$ is the

centrifugal inertia force of rotating masses; $K_{r\,c.r} = -M_{c.r.c}\,R\omega^2$ is the inertia force of rotating masses of the connecting rod; $K_{Rc} = -M_c\,R\omega^2$ is the inertia force of rotating masses of the cranks;

(2) $Z_c = K_{p'}th + 2P_{cw}$ and is the total force acting in the crank plane, where $P_{cw} = +M_{cw}\,\epsilon\omega^2$ is the centrifugal inertia force of the counterweight located on the web extension.

(3) T is the tangential force acting perpendicularly with the crank plane;

(4) $Z'_c = K'_{p'}th + P'_{cw}$ are the support reactions to the total forces acting in the crank plane where $K'^{p}.th = -0.5\,K_{p'}th$ and $P'_{cw} = -P_{cw}$

(5) T = - 0.5T are the support reactions to the tangential force acting in a plane perpendicular to the crank;

(6) $M_{m.j,i}$ is the accumulated (running on) torque transmitted by the design throw from the crankshaft nose;

(7) $M_{t.c} = TR$ is the torque produced by the tagential force;

(8) $M_{m.j.(i+1)} = M_{m.j,i} + M_{t}.c$ is the diminishing (running off) torque transmitted by the design throw to the next throw.

The basic design relations of the crankshaft elements needed for checking are given in Table 4.6

The dimensions of the crankpins and main journal are chosen, bearing in mind the required shaft strength and rigidity and permissible values of unit area pressure exerted on the bearings. Reducing the length of crankpins and journals and increasing their diameter add to the crankshaft rigidity and decrease the overall dimensions and weight of the engine. Crankpin - and - journal overlapping. (dm.j + dc.p > 2R) also adds to the rigidity of the crankshaft and strength of the webs.

In order to avoid heavy concentrations of stresses, the crankshaft fillet radius should not be less than 2 to 3mm. In practical design it is taken from 0.035 to 0.080 of the journal and crankpin diameter, respectively. Maximum stress concentration occurs when the fillets of crankpin and journal are in one plane.

According to the statistical data, the web width of crankshafts in automobile and tractor engines varies within (1.0 - 1.25) B for carburettor engines and (1.05 - 1.30)B for diesel engines, while the web thickness, within (0.20 - 0.22)B and (0.24 - 0.27)B, respectively.

Table 4.6 Basic Design relations of Crankshaft Element

Engines	1/B	dc.p/B	lc.p/B*	dm.j/B	lm.j/B**
Carburretor engines in - line	1.20-1.28	0.60-0.70	0.45-0.65	0.60-0.80	$\frac{0.45-0.60}{0.74-0.84}$
Vee-type with connecting rods attached to one crankpin	1.25-1.35	0.56-0.66	0.8-1.0	0.63-0.75	$\frac{0.50-0.70}{0.70-0.88}$
Diesel engines in-line	1.25-1.30	0.54-0.75	0.7-1.0	0.70-0.90	$\frac{0.45-0.60}{0.75-0.85}$
Vee-type with connecting rods attached to the crankpin	1.47-1.55	0.65-0.72	0.8-1.0	0.70-0.75	$\frac{0.50-0.65}{0.65-0.86}$

* B (D) is the engine cylinder bore (diameter); lc.p is the full length of a crankpin including fillet.

** The data are for the intermediate and outer (or centre) main journals.

4.3.2 UNIT AREA PRESSURE ON CRANKPINS AND JOURNALS

The value of unit area presssure on the working surface of a crankpin or a main journal determines the condition under which the bearing operates and its service life in the long run. With the bearings in operation measures are taken to prevent the lubricating oil film from being squeezed out, damage to the whitemetal and premature wear of the crankshaft journals and crankpins. The design of crankshaft journals and crankpins is made on the basis of the action of average and maximum resultants of all forces loading the crankpins and journals.

The maximum ($R_{m.j\,max}$ and $R_{c.p\,max}$) and mean ($R_{m.j.m}$ and $R_{c.p.m}$) values of resulting forces are determined from the developed diagrams of the loads on the crankpins and journals.

The mean unit area pressure (in MPa) is:

on the crankpin

$$K_{c.p.m} = R_{c.p.m}/(d_{c.p}\, l'_{c.p}) \quad (42a)$$

on the main journal

$$K_{m.j.m} = R_{m.j.m}/(d_{m.j})(l'_{m.j}) \text{ or}$$

$$K_{m.j.m} = R^{cw}_{m.j.m} / (d_{m.j})(l'_{m.j}) \ldots\ldots\ldots (42b)$$

Where $R_{c.p.m}$, $R_{m.j.m}$ are the resultant forces acting on the crankpin and journal respectively, MN, $R^{cw}_{m.j.m}$ is the resultant force acting on the main journal when use is made of counterweight MN, $d_{c.p}$ and $d_{m.j}$ are the diameters of the crankpin, and main journal, respectively, m; $l'_{c.p}$ and $l'_{m.j}$ are the working width of the crankpin and main journal shells, respectively, m.

The value of the mean unit area pressure attains the following values:

Carburettor engines...................4-12 MPa

Diesel engines........................6-16 MPa

The maximum pressure on the crankpins and journals is determined by the similar formulae due to the action of maximum resultant forces $R_{c.p\,max}$, $R_{m.j\,max}$ or $R''_{m.j\,max}$. The values of maximum unit area pressures on crankpins and journals K_{max} (in MPa) vary within the following limits.

In-line carburettor engines..........7-20

Vee-type carburettor engines.........18-28

Diesel engines 20-42

4.4 DESIGN OF JOURNALS AND CRANKPINS

4.4.1 Design of Main Journals

The main bearing journals are computed only for torsion. The maximum and minimum twisting moment are determined by plotting diagrams or compiling tables (Table 4.7) of accumulated moments reaching in sequence individual journals. To compile such a table use is made of the dynamic analysis data.

Table 4.7: Accumulated moments for journals

0	Mm.j2 (MN.m)	Mm. j3 (MN.m)	Mm. j4 (MN.m)	Mn. j5 (MN.m)
0	0	+910	-440	0
10	-400	+890	-270	-120
20	-560	+860	-190	-260

The order of determining accumulated (running on) moments for in-line engines is shown in Fig 4.3.

The running-on moments and torques of individual cylinders are algebraically summed up following the engine firing order starting with the first cylinder.

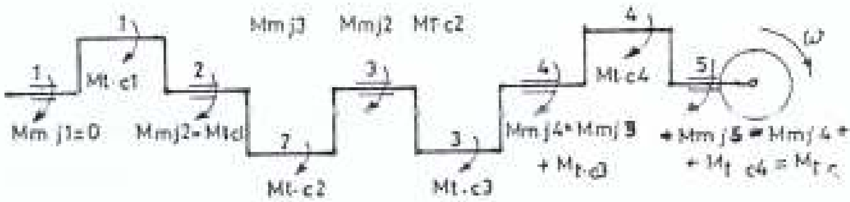


Fig. 4.3 Determination of torques accumulated on main journals in-line engine.

The maximum and minimum tangential stresses (in MPa) of the journal alternating cycle are:

τ_{max} = Mm.j.$i_{max}/W_{\tau mj}$ ....(43a)

τ_{min} = Mm.j.i $min/W_{\tau m.j}$......................(43b)

Where $W_{\tau m.j}$ = $\pi/16d^3$m.j[1-(δm.j/dm.j)^4] is the journal moment resisting to torsion, m^3, dm.j and δm.j are the journal outer and inner diameters, respectively, m.

With τ_{max} and τ_{min} known, we determine the safety factor of the main bearing journal. An effective factor of stress concentration for the design is taken with allowance for an oil hole in the main journal. For rough computations we may assume $K_\tau/(\epsilon_{s\tau}\epsilon_{ss\tau})$ = 2.5

The safety factors of the main bearing journals have the following values:

Carburettor engines......................3 - 5

unsupercharged diesel engines.......... 4 - 5

Supercharged diesel engines............ 2 - 4

4.2.2 <u>Design of Crankpins</u>

Crankpins are computed to determine their bending and torsion stresses. Torsion of a crankpin occurs under the effect of a running on moment Mc.p.i. Its bending is caused by bending moments acting in the crank plane M_x and in a perpendicular plane M_y. Since the maximum values of twisting and bending moments do not coincide in time, the crankpin safety factors to meet twisting and bending stresses are determined separately and then added together to define the total safety margin.

The twisting moment acting on the ith crankpin is:

For one span crankshaft (see fig 5.0a and 5.0b)

$$Mc.\ P,\ i\ = Mm.j.i - T_i;R \ldots\ldots\ldots\ldots\ldots\ldots\ldots(44a)$$

For two span crankshaft (see fig 5.0c and d)

$$Mc.p,i = Mn.j.i - T'_{ii}\ R \ldots\ldots\ldots\ldots\ldots\ldots\ldots(44b)$$

To determine the most loaded crankpin, table in compiled (Table 4.8) shown accumulated moments for each crankpin.

The associated values of Mm.j.i are computed into Table 4.8 from Table 4.7 covering accumulated moments, while values of T'i or T'_{ii} are determined against tables involved in the dynamic analysis.

The values of maximum Mc.p, i max and minimum Mc.p,i min twisting moments for the most loaded crankpin are determined from data of Table 4.8.

Table 4.8 Accumulated moment for Crankpins

	1st Crankpin	2nd Crackpin			3th Crankpin		
	Mc.pi= - T' R	Mn.j2	T' R	Mc.p2= Mm.j2 - T' R	Mn.j.i	T' R	Mc.p.3 = Mm j 3 - T, R
0	- 160	-320	+455	+135	+590	-220	+370
30	- 140	-280	+1335	+1055	-2390	-200	+2190
60	+ 143	+285	+688	+973	+1660	-443	+1217
90	+ 158	+315	+455	+770	-1225	-220	+1005

The extremum of the cycle tangential stresses (in MPa) are

$$\tau_{max} = Mc.p.i\ max\ /W_{\pi.p} \ldots\ldots\ldots\ldots\ldots\ldots(45a)$$

$$\tau_{max} = Mc.p.imin/W_{\pi.p} \ldots\ldots\ldots\ldots\ldots\ldots(45b)$$

where

$W_{\pi.p} = \pi/16d^3m.j[1-(\ \delta m.j/dm.j)^4]$ is the moment resisting crankpin torsion, m^3,dc.p and δc.p are the outer and inner diameters of the crankpin, respectively, m.

The safety factor n, is determined in the same way as in the case of the main journal, bearing in mind the presence of stress concentration due to an oil hole.

Crankpin bending moments are usually determined by a table method (Table 4.9)

Table 4.9 **Crankpin Bending Moment**

Θ	T'	M'	M_TSin Θ	K'p,th	Z'_E	$Z'_{EL/2}$	M_z	M_Z CosΘ	M_i
0	0	0	0	+13706	+3956	+182.0	+483	-181	-182
30	+2863	+131.7	+121	+11595	+1845	+84.9	+386	-145	-23
60	+1636	+75.3	+70	+8419	-1331	-61.2	+240	-90	-20
90	-1249	-57.5	-53	+8323	-1427	56.6	+235	-88	-141
120	-2118	-97.4	-90	+10011	+261	+12.0	+313	-117	-207

The bending moment (Mn) acting on the crankpin in a plane perpendicular to the crankplane

$$Mt = T' \, L/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

where L = (Lm.j + Lc.p + 2h) is the center to center distance of the main journals, m

The bending moment (NM) acting on the crankpin in the crankplane

$$M_z = Z' \, L/2 + P_{cw}a \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

where a = 0.5 (Lc.p + h), m; Z_i = K' Pith + Pcw, Pa

The values of T and K'p, ith, are determined against values of the dynamic analysis and entered in Table 4.9

The total bending moment

$$M_b = \sqrt{(M^2_T + M^2_Z)}$$

Since the most severe stresses in a crankpin occur at the lip of oil hole, the general practice is to determine the bending moment acting in the oil hole axis planes.

$$M_{\theta} \quad = \quad M_1 \sin\theta \ - \ M_2 \cos\theta$$

where α_o is the angle between the axes of the crank and oil hole usually located in the center of the least loaded surface of the crankpin. Angle θ is usually determined against wear diagrams.

Positive moment M_θ generally causes compression at the lip of an oil hole . Tension is caused in this case by negative moment M_θ.

The maximum and minimum values of M_θ are determined against Table 4.9 or by a graphical method directly against the polar diagram of the load on the crankpin.

$$M_{\theta}max \ = \ \frac{-R_c\theta_0 maxL/2}{2}$$

$$= \ \frac{-T'_a\sin\theta \ + \ Kp,th,a'\cos\theta_0}{2} \times \frac{1}{2}$$

$$= \quad M_{Ta'}\sin\theta \ - \ Mca'\cos\theta$$

$$M_{\theta}min \ = \ \frac{-R_c\theta minL/2}{2}$$

$$= \ \frac{-T''_a\sin\theta \ + \ Kp,th.a''\cos\theta}{2} 1/2$$

$$M_{T a''} \sin\theta - M_{c'} a'' \cos\theta$$

where

$M_T a'(a'') = 1/2T'_{a'(a'')} = 1/2(-T_{a'(a'')/2})$ and

$M_c, a'(a'') = 1/2k_p, th, a'(a'') = 1/2(-kp, th, a'(a''))/2$

when use is made of counterweights, moment Mk must be added to the moment occuring due to counterweight inertia force Pcw and due to its reaction Pcw.

By the values of M_4max and M_4min thus obtained determined the extremum values of the bending stresses in the crankpin.

$\sigma max = M_4 max/W_{\delta c.p}$ and $\sigma_{min} = M_{\sigma min}/W_{\delta c.p}$............(49)

where

$W_{\delta c.p} = 0.5W_{rc\ p}$

Bending safety factor n_δ and total safety factor nc.p of the crankpin are determined by the formulae given in **Section 4.2**

safety factor nc.p

Automobile engines...........................2.0 - 3.0

Tractor engines...........................3.0 - 3.5

4.4.3 **DESIGN OF CRANKWEBS**

The crankshaft webs are loaded by complex alternating stresses: tangential due to torsion and normal due to bending and push-pull. Maximum stresses occur where the crankpin fillet joins a crankweb.

Tangential torsion stresses are caused by a twisting moment

$M_{t.w} = T'(0.5(lm.j + h))$..............(50)

The value of T'max and T'min are determined in Table 4·8

The maximum and minimum tangential stresses are determined by the formulae.

$\tau max = M_{t.w}max./W_{tw}$ and

$\tau min = M_{t.w}min/W_{t.w}$

where $W_{tw} = Ubh^2$ is the moment resisting to twisting the rectangular section of the web. The value of factor U is chosen depending on the ratio of the width b of the web design section to its thickness h, Table 4.10

Table 4.10 Web design Section

b/h	1	1.5	1.75	2.0	2.5	3.0	4.0	5.0	10.0	∞
U	0.208	0.231	0.239	0.246	0.258	0.267	0.282	0.291	0.312	0.333

The torsion safety factor n_t of the web and factors K_t, ϵ_s and ϵ_{ss} are to be determined.

In rough computations, $K_t/(\epsilon_s \epsilon_{ss}) = 2$ at the fillets may be taken.

Normal bending and push-pull stresses are caused by bending moment $M_{b.w}$, N (neglecting the bending causing minute stresses in a plane perpendicular to the crank plane) and push or pull force P_w N.

$M_{b.w} = 0.25(K + K_R + 2P_{cw})lm.j$

$P_w = 0.5(K + K_R)$ ....(52)

Extreme values of K are determined from the dynamic analysis table (Kr and Pcw are constant), and maximum and minimum normal stresses are determined by the equation below:

$\alpha_t max = M_{b.w}max/W_{\delta w} + P_w max/F_w$

$\alpha_t min = M_{b.w}min/W_{\delta w} + P_w min/F_w$

where $W_{\delta w} = bh^2/6$ is the moment of web resistance to the bending effect, $F_w = bh$ is area of design section of the web.

While determining the web safety factors at normal stresses n_σ the factor of stress concentration in the fillets is defined from tables or is taken depending on the ratio of the radius of the crankpin - to - web fillet to the web thickness.

Figure 4.4 shows $K_\sigma/(\epsilon_s\epsilon_{ss})$ versus r_{fil}/h. The total safety margin n_w is determined

$$n_w \quad = \quad n_\sigma n_\tau / \sqrt{(n^2_\sigma + n^2_\tau}$$

Automobile engines.................not less than 2.0-3.0

Tractor engines...............................3.0-3.5

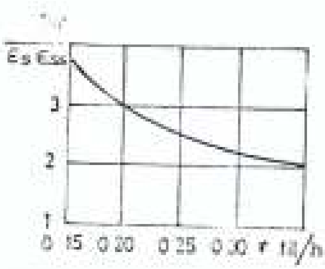


Fig 4.4 $k_\sigma/(\epsilon_s\epsilon_{ss})$ versus r_{fil}/h

CHAPTER FIVE

5.0 PARAMETRIC MODEL SHAPE FOR FOUR CYLINDER ENGINE CRANKSHAFT

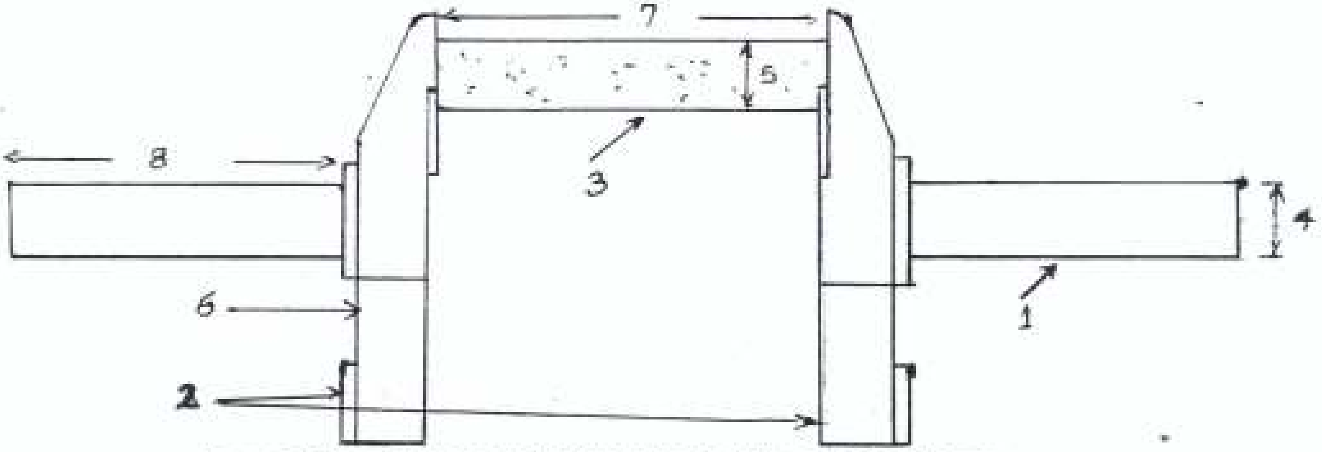


Fig 5.0 **Parametric Crankshaft model shape**

Notes

1. Main Journal
2. Counter weight
3. Crankpin
4. Main-journal diameter (dmj)
5. Crankpin diameter (dcp)
6. Crankweb
7. lcp = Length of crankpin
8. lmj = Length of main journal

5.1 Weight Analyses of each Section of the Crankshaft

(a) One crankpin weight $= \frac{\pi d^2 cp}{4} \times l.c.p \; \rho g$

$$4 \text{ crankpin weight} = \pi d^2 cp \times l.c.p \; \rho g$$

(b) One main journal weight $= \frac{\pi d^2 mj}{4} \; l.m.p \; \rho g$

$$5 \text{ main journals weight} = \frac{5\pi d^2 mj}{4} \; l.m.p \; \rho g$$

(c) There are 2 fillet holes per crankpin

$\therefore$ weight removed from a fillet hole

$$= \pi r^2_{fil} \ \varrho g d c p$$

=====> for 8 fillet holes on the crankpins, weight removed $= 8\pi r^2_{fil} \varrho g d c p$

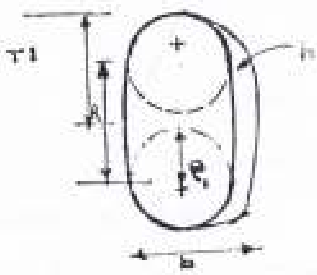
(d) There are 2 fillet holes per main journal

$\therefore$ weight removed from a fillet hole

$= \pi r^2_{fil} d m j \varrho g$

===> For 10 fillet holes on the main journals, weight removed $= 10\pi r^2_{fil} \ d m j \varrho g$

(e) There are four crankwebs without counterweight

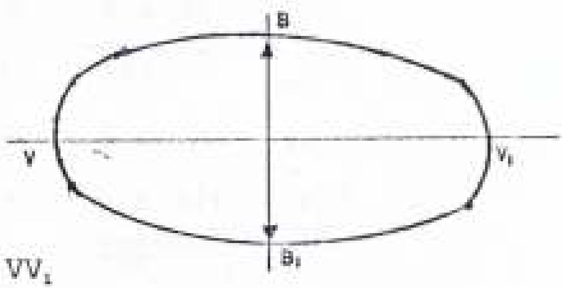


Assumption, Crankweb is an ellipse (Kolchin and Demidov, 1984).

$$\text{Area of ellipse} = \frac{\pi ab}{4}$$

where a = major axis

b = minor axis



Major axis = a VV_1

Minor axis = b = BB_1

Abbott (1976)

In the above

$A = 2R$ (Major axis)

$b = b$ (Minor axis)

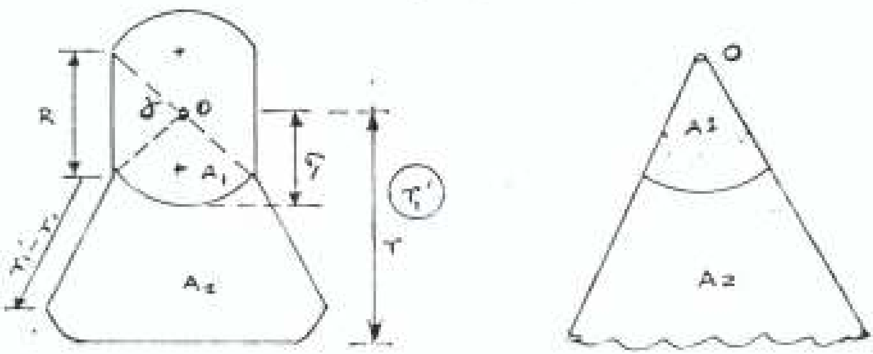
ϱ_1 = Radius of centre of gravity of web

Weight of crankwebs (4) without counterweight

$= (1/4\pi ab)h \times 4$

$= \pi(2R)bh$

(F) There are four crankwebs with counterweight



(Kent's, 1950)

$\therefore$ Area (A) = $A_1 + A_2$

where A_2 = Area of a counterweight

$$A_2 = A - A_1$$

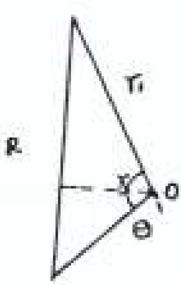
$$A = \frac{\theta \times \pi r^2}{360}$$

$$A_1 = \frac{\theta \times \pi r_1^2}{360}$$

$$A_2 = A - A_1$$

$$= \frac{\theta \times \pi (r^2 - r_1^2)}{360}$$

Now



$$\therefore \sin \alpha/2 = R/2r_1$$

$$\alpha/2 = \sin^{-1}R/2r_1$$

$$\alpha = 2\sin^{-1}R/2r_1$$

$$\theta + \alpha = 180^\circ$$

$$\theta = (180 - 2\sin^{-1}R/2r_1)$$

Total volume for crankweb and the counterweight volume

$$= \frac{(\pi 2R)h \times 4}{4} + \frac{8\pi(180-2\sin^{-1}R/2r_1)(r^2 - r_1^2)}{360}$$

$$\text{Weight} = (\pi 2R)bh + \frac{\pi(180-2\sin^{-1}R/2r_1)(r^2 - r_1^2)}{45}$$

Total weight of Crankshaft

= weight of crankpins + Weight of main journals + Weight of crankwebs and the counterweights.

$$M = \pi d^2 c.p \times l.c.p\varrho g + 5/4\pi d^2 m.j.l.m.j\varrho g$$
$$+ [(2\pi R)bh + \frac{\pi(180-2\sin^{-1}R/2r_1)(r^2 - r_1^2)}{45}]\varrho g$$
$$- 2\pi r_{fil}^2(4d.c.p + 5dmj)\varrho g \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

5.2 <u>Formulation of the Crankshaft Design Problem</u>

To formulate the problem of designing the crankshaft the following notation and data for high duty cast iron B_γ 40-10 are assumed (Kolchin and Demidov, 1984).

1. Maximum unit area pressure on crankpins

   $kc.p\ max = 20$ MPa

2. Maximum resultant forces on crankpins

   $Rc.p\ max = 18,451$ N

3. Maximum Unit area pressure on main journals

$Km.j\ max = .20$ MPa

4. Maximum resultant force on main journal

$Rm.j\ max = 10{,}770N$

5. Maximum bending stress on the crankweb = 90MPa

6. Radius of the crank, R = 39mm

7. Fillet radius, r_{fil} = 3mm

8. Maximum tangential stress of sign alternating cycle,

$\tau\ max = 60MPa$

9. Maximum twisting moments of most loaded crankpin,

$Mc.p\ max = .588Nm$

10. Extreme point of web rotation with counterweight,

r = 75mm

11. Extreme point of web rotation without counterweight,

r_i = 69mm

The six design variables for the problem are defined as

dc.p = Crankpin diameter (m),

dm.j = main journal diameter (m),

lc.p = crankpin length (m),

lm.j = main journal length (m),

b = web breadth (m) and

h = web thickness (m)

For crankshaft in full operation the maximum unit area pressure constraint on each element should be imposed. We have the following design expression for the crankshaft.

Cost function. The problem is to minimize the weight of the crankshaft (mass x acceleration due to gravity) which is deduced as follows:

<u>The Objective or Cost Function for the Crankshaft</u>

Minimize (M) = $\pi d^2 c.p \times l.c.p\rho g + 5/4\pi d^2 m.j \times l.m.j \rho g$

$$+ \quad [(2\pi R)bh + \frac{\pi(180-2\sin^{-1}R/2r_1)(r^2 - r_1^2)]\varrho g}{45}$$

$$- \quad 2\pi r_{fil}^2(4d.c.p + 5dmj)\varrho g$$

Where

M	=	Crankshaft weight
dc.p	=	Crankpin diameter
dm.j	=	main journal diameter
lc.p	=	crankpin length
lm.j	=	main journal length
ϱ	=	material density
g	=	acceleration due to gravity
b	=	web breadth
h	=	web thickness
r	=	extreme point of web rotation with counterweight (75mm)
r_1	=	extreme point of web rotation without counterweight = 69mm
r_{fil}	=	fillet radius

<u>Subject to these constraints on the different elements of the crankshaft</u>

1. $Kc.p\ max \geq \dfrac{Rc.p\ max}{dc.p\ (lc.p - 2r_{fil})}$ Crankpins constraints

$dcp\ min \leq dc.p \leq dc.p\ max$

$lc.p_{min} \leq lc.p \leq lc.p\ max$

Kc.p max = Maximum Unit area pressure on Crankpins

Rc.p max = Maximum resultant forces on Crankpins.

2. $Km.j\ max \geq \dfrac{Rm.j\ max}{dm.j\ (lm.j - 2r_{fil})}$ Main journal constraints

$dm.j\ min \leq dm.j \leq dm.j\ max$

$lm.j_{min} \leq lm.j \leq lm.j\ max$

Km.j max = Maximum Unit area pressure on main journals

Rm.j max = Maximum resultant forces on main journals.

3. $\alpha\ max \geq \dfrac{0.75\ Rc.p\ max \times lm.j}{bh^2}$ Bending fatigue strength crankweb constraints

b = breadth of the web section through the primary fracture path.

h = thickness of the webs section through the primary fracture path

σmax = maximum bending stress on the crankweb.

$bmin \leq b \leq bmax$

$hmin \leq h \leq hmax$

$lm.jmin \leq lm.j \leq lm.jmax$

4.0 $dm.j + dc.p > 2R$ Crankpin and journal overlapping constraints.

R = Radius of the Crank

5.0 $0.035\ dm.j \leq r_{fill} \leq 0.080dc.p$ stress concentration constraint.

6.0 $\tau max \geq \dfrac{16|Mc.p\ max|}{\pi d^3c.p}$ Torsional vibration constraint.

τmax = maximum tangential stress of sign alternating cycle

Mc.p max = Maximumtwisting moments of most loaded crankpin

$dcp\ min \leq dc.p \leq dc.p\ max$

5.3 **Dimensional Limitations**

B = Cylinder Bore stroke

for carburettor in line Engine

B = (60 - 100)mm

Relative dimensions of crankshaft engine are:

(i) $\frac{Lc.p}{B} = (0.45 - 0.65)$

$\Longrightarrow Lc.p = (27 - 65)$ mm

(ii) $\frac{Lm.j}{B} = (0.45 - 0.60)$

$\Longrightarrow Lc.p = (27 - 60)$ mm

(iii) $\frac{b}{B} = (1.0 - 1.25)$

$\Longrightarrow b = (60 - 125)$ mm

(iv) $\frac{h}{B} = (0.20 - 0.22)$

$\Longrightarrow h = (12 - 22)$ mm

(v) $\frac{dc.p}{B} = (0.60 - 0.70)$

$\Longrightarrow dc.p = (36 - 70)$ mm

(i) $\frac{dm.j}{B} = (0.60 - 0.80)$

$\Longrightarrow dm.j = (36 - 80)$ mm

5.4 Parameters available for Crankshaft made of high duty cast Iron BY 40-10 [illegible]

(i) $K_{c.p\ max} \le 20MPa$

$R_{c.p\ max} = 18451N$

(ii) $K_{m.j\ max} \le 20MPa$

$R_{m.j\ max} = 10770N$

(iii) $\sigma_{max} \le 90MPa$

(iv) $R = 39mm$

$\therefore d_{m.j} + d_{c.p} > 0.078m$

(v) $r_{fil} = 3 \times 10^{-3}m$

(vi) $\tau_{max} \le 60MPa$

$M_{c.pmax} = 588Nm$

(vii) $r = 75mm$

(viii) $r_1 = 69mm$

Now let

$d_{c.p} = x_1$ (crankpin diameter)

$d_{m.j} = x_2$ (main journal diameter)

$l_{c.p} = x_3$ (crankpin length)

$l_{m.j} = x_4$ (main journal length)

$b = x_5$ (web breadth)

$h = x_6$ (web thickness)

Substituting the parameters in the optimization problem, we have.

$$\text{Minimize } (M) = \pi\varrho g(d^2c.p \times l.c.p + 5/4\pi d^2m.j \times l.m.j$$

$$+ \quad [(2(2R)bh \quad + \quad \frac{\pi(180-2\sin^{-1}R/2r_1)(r^2 - r_1^2)]}{45}$$

$$- \quad 2r_{fil}^2(4d.c.p + 5dmj))$$

becomes

$$f(X) = (x_1^2x_3 + 5/4x_2^2x_4 + (2(0.079)x_5x_6 +$$

$$(180 - 2\sin^{-1}0.039/0.138)(0.075^2 - 0.06^2)$$

$$- \quad 2(.003)^2(4x_1 + 5x_2)\}$$

$$X = \{x_1, x_2, \ldots\ldots\ldots x_6\}$$

$$f(X) = \{x_1^2x_3 + 1.25x_2^2x_4 + 0.158x_5x_6 - 7.2 \times 10^{-5}x_1$$

$$- 9 \times 10^{-5}x_2 \quad + 0.0028\}$$

subject to the constraints

$$g_1 = \frac{18451}{x_1(x_3 - 0.006)} - 20 \times 10^6 \leq 0$$

$$g_2 = \frac{10770}{x_2(x_4 - 0.006)} - 20 \times 10^6 \leq 0$$

$$g_3 = \frac{13838.25x_4}{x_5(x_6^2)} - 20 \times 10^6 \leq 0$$

$$g_4 = 0.078 - x_1 \quad x_2 \geq 0$$

$$g_5 = -0.003 + 0.035x_3 \leq 0$$

$$g_6 = 0.003 - 0.08x_1 \leq 0$$

$$g_7 = \frac{2995}{x_1^3} - 60 \times 10^6 \leq 0$$

Limitation on the values of the variable

$$0.036 \le x_1 \le 0.070$$

$$0.036 \le x_2 \le 0.080$$

$$0.027 \le x_3 \le 0.065$$

$$0.027 \le x_4 \le 0.060$$

$$0.060 \le x_5 \le 0.125$$

$$0.012 \le x_6 \le 0.022$$

where $x_i \ge 0$.

The solution to this optimization problem is as presented in the Appendix (B).

CHAPTER SIX

6.0 RESULTS AND DISCUSSION

6.1 Analysis of Result

The summarized report revealed the following:

1. The Optimum values for each varibale

X_1 = 0.04216911 m (crankpin diameter)

X_2 = 0.03600000m (main journal diameter)

X_3 = 0.02930594m (crankpin length)

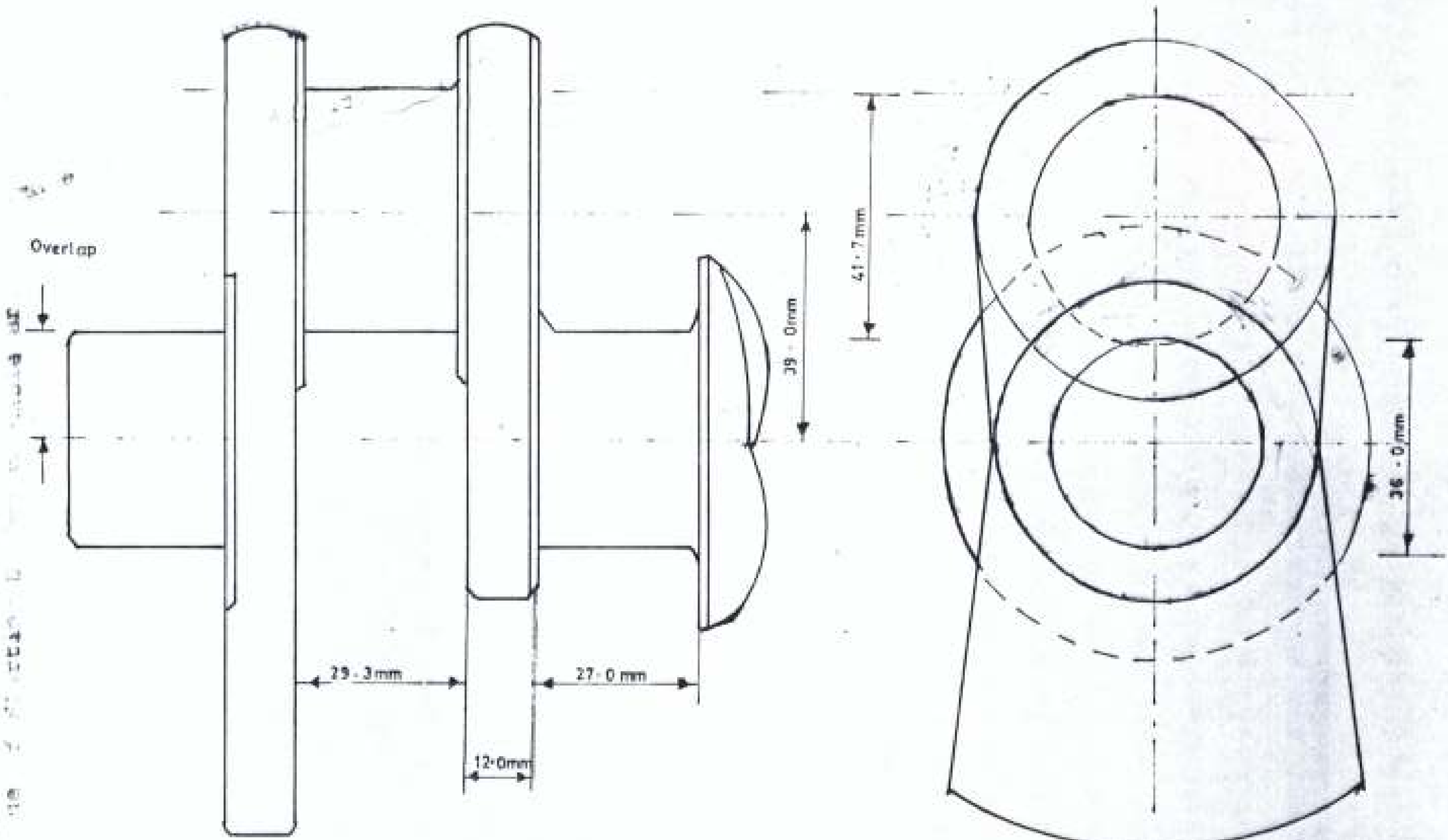
X_4 = 0.02700000m (main journal length)

X_5 = 0.06000000m (web breadth)

X_6 = 0.01200000m (web thickness)

These are the optimum values for the variables used to describe the crankshaft model. The above values still complies with increasing the crankshaft rigidity and decreasing the overall dimensions and weight of the engine through reducing the length of crankpins and journals and increasing their diameter.

*The optimum values enable us draw the crankshaft in standard form.



Typical crankshaft proportions for dhe crank of the optimum design

Scale 1:1

6.1.2. The objective function coefficient for each design variable has the minimum and maximum it can assume and still retain the normal weight level.

6.1.3. Constraints Status

This is a critical decision to the design.

A. Crankpin Constraint

The status of the Crankpin constraint shows that it is active or tight, i.e. it is a binding constraint. Therefore, it must not be altered so as not to affect the functional performance of the crankpin, there is no slack or surplus for this constraint.

B. Main Journal Constraint

The status of the main journal constraint is loose or inactive, and as such it has a surplus of 0.19156003 over and above the computed. This implies that it allows a variation above the value.

C. Crankweb Constraint

The status of this constraint also indicates that it is loose or inactive. Hence it can permit a lesser variation of about 0.58816397 to still achieve our optimum objective.

D. Crankpin and Journal Overlapping Constraints

The status of the crankpin and journal overlapping constraint shows that it is tight or active. Hence.it is strictly binding and hence the surplus or slack indicates zero.

E. Stress Concentration Constraints

1. In respect of the main journal diameter, the status is loose and therefore can tolerate a lesser variation of

up to 0.2049880.

2. In respect of the crankpin diameter, the status is also loose and therefore can accomodate a surplus of up to 0.11177664.

F. Torsional Vibration Constraint

The Status of the constraint shows that it is loose and hence, it can tolerate a surplus of up to 0.31262735.

G. The status of the eighthconstraint is tight because it is an added constant variable of magnitude equal to one.

6.1.4. Shadow Price

This shows how much the value of the cost function would be affected if any of the constraint level is increased by a unit quantity. It is 0.00003021 and 0.00011829 for the binding constraints (1) and (4) and zero for the rest. This indicates that any slight changes in the tight or binding constraint would increase the weight of the crankshaft.

6.1.5. Convergence

An algorithm is said to be convergent if it reaches a minimum point starting from an arbitrary design.

Results of each iteration cost function

Iteration 1

New obj (Min) = 2.799496E-03 + (0.1129451 BIG M)

Iteration 2

New obj. (Min) = 2.801778 E-03 + (5.373595 E -02 BIG M)

Iteration 3

New Obj. (Min.) = 2.827984e -03

Iteration 4

New obj. (Min.) = 2.808126E -03

The result of iterations revealed that it is a convergent algorithm which arrives at the minimum at the fourth iteration.

The formulation of this optimum design has all the constraints either active or inactive. Perhaps, any of the constraint is violated, then the potential strategy constraint approach could have been the most efficient way to handle the problem. Another method is to use the descent method of approach to adjust the violated constraints until a feasible design is arrived at.

CHAPTER SEVEN

7.0 RECOMMENDATIONS AND CONCLUSION

7.1 RECOMMENDATIONS

It is needful to suggest certain recommendations consequent upon the deduction from this research work. Various categories of people would actually benefit from the recommendation while actualising them such as automotive engine design Engineers, local fabricators of crankshaft, local designers, on how to make design decisions that are reliable, cost effective and with adequate strength.

Also, the information would furnish Nigeria's foundry industry's design and drafting engineers with initial information on the preparation of the manufacturing drawing of crankshafts.

Some of these recommendations include:

(a) Approaching the optimum design through stress analysis instead of maximum unit area pressure on the elements.

(b) Trying various other approaches in which some of the constraints could be redundant or violated and employ either the potential constraint strategy or descent function to solve the problem.

(c) Developing a CADD automatic manufacturing drawing that would take care of machined and unmachined surfaces together with any special treatment required during the machining operations for the crankshaft.

(d) Since the factor in 'c' above, that is preparing a casting drawing which will 'make' the machined crankshaft is that of the crankshaft designer, it is therefore believed that the work could be expanded for a PhD work in the future.

Establishing a National and International Link with Automotive Industries, as Well as Designers

It is pertinent to mention that the Institutions downhere especially the Mechanical Engineering Department, FUTA, cannot boast of having enough computer outfit especially those related to automotive Engine design; a link could be made with both national and International automotive industries, as well as automotive engine designers to secure useful information and CADD outfit to use for design purpose.

This proposal could also be a base for the developing Nigerian automotive industry to achieve rapid growth in this area.

Moreso, students/supervisors handling such related projects should be linked with relevant industrial set up for assistance and practical exposure.

7.2 Conclusion

Major effort on this project work was towards studying the analysis of the design of internal combustion engine crankshaft and modelling the design for optimization. Formulating the objective or cost function for the minization problem and establishing the necessary constraints based on the variables and parameters for the problem and employing an effective optimization method to solve the design, using a computer aided approach.

The above was achieved by first identifying the most important variables for the Crankshaft model. These are the crankpin diameter and length, Main-journal diameter and length, web width and thickness.

Then, through search, the parametic modelling was employed.

The crankpin and main journal were taken to be of cylindrical shape, the crankweb was taken to be of elliptical shape bearing in mind the fillet holes effect on the design.

The most teething constraints were identified on first the elements of the crankshaft, that is, the crankpin, main journal and crankweb, using the maximum unit area pressure approach. Also crankpin and journal overlapping has to be in a prescribed range for the strength rigidity of the web. To minimize the stress concentration along the fillet hole, it must have a definite relationship both with the main journal and crankpin diameter.

Moreso, torsional vibration has a significant effect on the crankshaft performance and it was also taken as one of the constraints.

The relative dimensions of the crankshaft engine was sought out, from the cylinder bore stroke for carburettor - in line Engine, and how this related to the other dimensions of the crankshaft elements.

Using a high duty cast iron BY 40-10 crankshaft type for a four cylinder internal combustion engine type, the parameters for our problem were established.

The work continued with the establishment of a necessary optimization procedure for analysing the problem, ranging from the cost function formulation, constraints formulation, dimensional limitations, normalization of the set of constraint equations and then linearization of the problem to arrive at the Linear Programme solvable by any of the already existing software algorithms.

The quantitative system for Business Plus Software Package was employed to solve the LP problem which was developed with the Basic Programming language.

Area of Usefulness of the Above Research Project

(a) It helps to employ optimization method to solve practical engineering problems in the automobile industry.

(b) It provides assistance to the developing automobile industry in Nigeria to achieve design aims, especially in crankshaft design vis-a-vis modern method.

(c) It furnishes local fabricators and crankshaft refurbishing firms with useful information for improved performance.

(d) It aids local designers on how to make design decisions that are reliable, cost effective and with adequate strength.

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APPENDIX A

A_1 Numerical Methods for Constrained Optimum Design

The numerical methods or primal methods to directly solve the original constrained optimization problem take the following procedure.

A_2 Problem Formulation

Find X = (Xi ......, Xn) a design variable vector of dimension n, to minimize a cost function.

$$f = f\ (x) \quad \text{.......................(55.1)}$$

subject to the equality constraints

$$hi\ (x) = 0;\ i = 1 \text{ to } p \quad \text{..........(55.2)}$$

the inequality constraint

$$g_i(x) \leq 0;\ i = 1 \text{ to } m \quad \text{...........(55.3)}$$

and explicit bounds on design variables

$$x_{il} \leq x_i \leq x_{iu};\quad i = 1 \text{ to } n \quad \text{............(55.4)}$$

where x_{il} and x_{iu} are respectively the lower and upper limit values for the ith design variable, x_i

A_3 Basic Concept and Ideas

Numerical methods for constrained problems are based on the following iterative prescription.

Vector form:

$$x^{(k+1)} \doteq x^{(k)} + \Delta x^{(k)}; \quad \text{............(55.5)}$$

$$k = 0,1.2......$$

component form:

$$x_i^{(k+1)} \doteq x_i^{(k)} + \Delta x_i^{(k)}; \quad \text{.................(55.6)}$$

$$k = 0,1,2$$

$$i = 1 \text{ to } n$$

K = iteration number

i = ith design variable

$X^{(0)}$ = starting design estimate

$\Delta x^{(k)}$ = small change in the current design

Now

$\Delta x^{(k)} = \alpha_k d^{(k)}$ ....(55.7)

where $d^{(k)}$ = desirable search direction

α_k = positive scalar called the step size.

A_4 **Search Techniques or direct methods of Optimization**

Step 1: Estimate a reasonable starting design $X^{(0)}$, set the iteration counter K = 0

Step 2: Compute a search direction $d^{(k)}$ in the design space. This calculation generally requires cost function values and its gradients for unconstrained problems, and in addition, constraint functions and their gradients for constrained problems.

Step 3: Check for convergence of the algorithm. If it has converged, terminate the iterative process. Otherwise continue.

Step 4: Calculate a positive step size

Step 5: Calculate the new design as

$x^{(k+1)} = x^{(k)} + \alpha_k d^{(k)}$.........(55.8)

set K = k + 1 and go to step 2.

A_5 **Descent Step Idea**

$f(x^{(k+1)}) < f(x^{(k)})$...............(55.9)

substitute $x^{(k+1)}$ in equation 55.8

$f(x^{(k)}) + \alpha_k d^{(k)} < f(x^{(k)})$...............(55.10)

$f(x^{(k)}) + \alpha_k d^{(k)}$

$$= \quad f(x^{(k)}) + \alpha_k(c^{(k)}.d^{(k)}) < f(x^{(k)}) \ldots\ldots. (55.11)$$

where

$$c^{(k)} = \nabla f(x^{(k)}) \quad \ldots . (55.12)$$

but

$$\alpha_k(c^{(k)}.d^{(k)}) < 0 \ldots\ldots\ldots (55.13)$$

Now, $\alpha_k > 0$

A_6 <u>Convergence of Algorithms</u>

The central idea behind numerical methods of optimization is to search for the optimum point in an iterative manner generating a sequence of designs. It is important to note that the success of a method depends on the guarantee of convergence of the sequence to the optimum point. The property of convergence to a local optimum point irrespective of the starting point is called global convergence of the numerical method.

A_7 <u>Rate of Convergence</u>

In practice, a numerical method may take too many iterations to reach the optimum point. Therefore, it is important to employ methods having a faster rate of convergence. Rate of convergence of an algorithm is usually measured by the number of iterations and function evaluation to obtain an acceptable solution.

A_8 <u>Four basic steps in algorithm analysis</u>

1. Linearization of cost and constraint functions.
2. Definition of a search direction determination subproblem using the linearized cost and constraint

functions.

3. Solution of the subproblem which gives a search direction in the design space

4. Calculation of a step size to minimize an appropriate descent function in the search direction.

A_9 Constraint Status at a Design Point

The precise definitions of the status of a constraint at a design point are needed in the development and discussion of numerical methods.

Active Constraint: An inequality constraint $g_i(x) \le 0$ is said to be active (or tight) at a design point $X^{(k)}$ if it is satisfied as an equality at that point, i.e $g_i(x^{(k)}) = 0$.

Inactive Constraint: An inequality constraint $g_i(x) \le 0$ is said to be inactive at a design $x^{(k)}$ if it has negative value at that point, i.e $g_i(x^{(k)}) < 0$

Violated Constraint: An inequality constraint $g_i(x) \le 0$ is said to be violated at a design point $X^{(k)}$ if it has positive value there, i.e $g_i(x^{(k)}) > 0$. An Equality constraint $h_i(x) = 0$ is violated at a design point $x^{(k)}$ if it has nonzero value there, i.e. $h_i(x^{(k)}) = 0$. Therefore an equality constraint is always either active or violated for any design point.

A_{10} Constraint Normalization

In numerical calculations, it is desirable to normalize all the constraint functions.

For example, consider a stress constraint as

$\sigma \le \sigma_a$ or $\sigma - \sigma_a \le 0$

and a displacement constraint s

$$\delta \leq \delta_a \quad \text{or} \quad \delta - \delta_a \leq 0$$

where

σ = calculated stress in a member

σ_a = an allowable stress (working stress)

δ = calculated deflection at a point

δ_a = an allowable deflection

We can, however, normalize the constraints by dividing them by their respective allowable values to obtain the normalized constraint as

$$R - 1.0 \leq 0$$

where $R = \sigma/\sigma_a$ for stress constraint

and $R = \delta/\delta_a$ for deflection constraint

A_{11} **Linearization of the Constrained Problem**

Let $X^{(k)}$ = design estimate at the k^{th} iteration

$\Delta x^{(k)}$ = change in design

Taylor series expansion for cost and constraint equations are:

Minimize $f(x^{(k)} + \Delta x^{(k)} = f(x^{(k)}) + \nabla f^T(x^{(k)}) \nabla x^{(k)}$

subject to linearized equality constraint

$$h_j(x^{(k)} + \Delta x^{(k)}) = h_j(x^{(k)}) + \nabla h_j^T(x^{(k)}) \Delta x^{(k)} = 0$$

$$j = 1 \text{ to } p$$

and the linearized inequality constraints

$$g_j(x^{(k)} + \Delta x^{(k)}) = g_j(x^{(k)}) + \nabla g_j^T(x^{(k)}) \Delta x^{(k)} \leq 0$$

$$j = 1 \text{ to } m.$$

where ∇f, ∇h_j and ∇g_j are gradients of the cost function, jth equality constraint, and jth inequality constraint respectively.

<u>Some Simplified Notations</u>:

cost function values at the current design point $x^{(k)}$

$$f_k = f(x^{(k)}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13a)$$

negative of the jth equality constraint function value at the current design point $x^{(k)}$

$$e_j = -h_j(x^{(k)}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13b)$$

negative of the jth inequality constraint functions value at the current design point $x^{(k)}$, i.e derivative of the cost function with respect to the ith design variable x_i,

$$c_i = df(x^{(k)})/dx_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13d)$$

ith component of the gradient of the jth equality constraint at the current design point x^k, or derivative of the jth equality constraint function with respect to ith design variable x_i.

$$n_{ij} = dh_j(x^{(k)})/dx_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13e)$$

ith component of the gradient of the jth inequality constraint at the current design point $X^{(k)}$, or derivative of the j^{th} inequality constraint function with respect to ith design variable x_i.

$$a_{ij} = dg_j(x^{(k)})/dx_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13f)$$

d is the n dimensional design change vector or the search direction

$$d_i = \Delta x_i^{(k)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55.13g)$$

Using the above notations, the approximate subproblem is defined as follows:

$$Minimize\ f = \sum_{i}^{n} c_i d_i \quad (f = c^T d) \qquad (55.14)$$

Subject to the linearized equality constraints

$$\sum_{i}^{n} n_{ij} d_i = e_j; \quad j = 1 - p, \quad (N^T d = e) \qquad (55.14a)$$

and the linearized inequality constraints

$$\sum_{i}^{n} a_{ij} d_i \leq b_j; \quad j = 1 \ \ to\ p, \quad (A^T d \leq b) \qquad (55.14b)$$

where the columns of the matrix N (n x p) are the gradients of equality constraints and the columns of the matrix (AA (n x m) are the gradients of the inequality constraints. Note that since f_k is a constant which does not affect solution of the linearized subproblem, it is dropped. Therefore, f represents the linearized change in the original cost function.

Let $n^{(j)}$ and $a^{(j)}$ represent the gradient of the j^{th} equality and j^{th} inequality constraints respectively. Therefore they are given as

$n^{(j)} \quad = \quad (\delta h_j/\delta x_1, \delta h_j/\delta x_2, \ldots\ldots, \delta h_j/\delta x_n)$ ....(55.15)

$a^{(j)} \quad = \quad (\delta g_j/\delta x_1, \delta g_j/\delta x_2, \ldots\ldots, \delta g_j/\delta x_n)$ ....(55.16)

The matrices N and A are also given as

$N \quad = \quad [n^{(j)}]_{(n\ x\ p)}; \qquad A = [a^{(j)}]_{(n\ x\ m)}$ ....(55.18)

APPENDIX B

B_1 Solution to the Formulated Problem

$$f(x) = [x_1^2 x_3 + 1.25 x_2^2 x_4 + 0.158 x_5 x_6 - 7.2 \times 10^{-5} x_1 - 9 \times 10^{-5} x_2 + 0.0028] \ldots\ldots\ldots (55.19)$$

subject to the constraints

$$g_1 = \frac{18451}{x_1 (x_3 - 0.006)} - 20 \times 10^6 \leq 0 \ldots\ldots (55.19a)$$

$$g_2 = \frac{10770}{x_2 (x_4 - 0.006)} - 20 \times 10^6 \leq 0 \ldots\ldots (55.19b)$$

$$g_3 = \frac{13838.25 x_4}{x_5 (x_6^2)} - 90 \times 10^6 \leq 0 \ldots\ldots (55.19c)$$

$$g_4 = 0.078 - x_1 - x_2 \leq 0 \ldots\ldots (55.19d)$$

$$g_5 = -0.003 + 0.035 x_2 \leq 0 \ldots\ldots\ldots\ldots (55.19e)$$

$$g_6 = 0.003 - 0.08 x_1 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots (55.19f)$$

$$g_7 = \frac{2995}{x_1^3} - 60 \times 10^6 \leq 0 \ldots\ldots\ldots\ldots (55.19g)$$

Variable bounds

$$0.036 \leq x_1 \leq 0.080 \ldots\ldots\ldots\ldots\ldots\ldots (55.19h)$$

$$0.036 \leq x_2 \leq 0.070 \ldots\ldots\ldots\ldots\ldots\ldots (55.19i)$$

$$0.027 \leq x_3 \leq 0.065 \ldots\ldots\ldots\ldots\ldots\ldots (55.19j)$$

$$0.027 \leq x_4 \leq 0.060 \ldots\ldots\ldots\ldots\ldots\ldots (55.19k)$$

$$0.060 \leq x_5 \leq 0.125 \ldots\ldots\ldots\ldots\ldots\ldots (55.19l)$$

$$0.012 \leq x_6 \leq 0.022 \ldots\ldots\ldots\ldots\ldots\ldots (55.19m)$$

Normalized Form

$$g_1 = \frac{9.2255 \times 10^{-4}}{x_1(x_3 - 0.006)} - 1 \le 0 \quad \text{(55.19n)}$$

$$g_2 = \frac{5.385 \times 10^{-4}}{x_2(x_4 - 0.006)} - 1 \le 0 \quad \text{(55.19p)}$$

$$g_3 = \frac{1.5376 \times 10^{-4}}{x_2(x_4 - 0.006)} - 1 \le 0 \quad \text{(55.19q)}$$

$$g_4 = -12.82x_1 - 12.82x_3 + 1 \le 0 \quad \text{(55.19r)}$$

$$g_5 = 11.667x_2 - 1 \le 0 \quad \text{(55.19s)}$$

$$g_6 = -26.667x_1 + 1 \le 0 \quad \text{(55.19t)}$$

$$g_7 = \frac{4.99 \times 10^{-4}}{x_1^3} - 1 \le 0 \quad \text{(55.19u)}$$

The gradient of the cost function is as follows:

$$\nabla f = [2x_1x_3 - 7.2 \times 10^{-5},\ 2.50x_2x_4 + 0.158x_6 - 9\times10^{-5},\ x_1^2,\ 1.25x_2^2,\ 0.158x_6,\ 0.158x_5]$$

The gradients of the constraint functions are as follows:

$$\nabla g_1 = \left[\frac{-1.8451 \times 10^{-3}}{x_1^2(x_3 - 0.006)},\ 0,\ \frac{-1.8451 \times 10^{-3}}{x_1(x_3 - 0.006)^2},\ 0,\ 0,\ 0\right] \quad \text{(55.20a)}$$

$$\nabla g_2 = \left[0,\ \frac{-1.077 \times 10^{-3}}{x_2^2(x_4 - 0.006)},\ 0,\ \frac{-1.077 \times 10^{-3}}{x_2(x_4 - 0.006)^2},\ 0,\ 0\right] \quad \text{(55.20b)}$$

$$\nabla g_3 = \left[0,0,0,\ \frac{1.5376 \times 10^{-4}}{x_5^2x_6^2},\ \frac{-3.075 \times 10^{-4}x_4}{x_5^2x_6^2},\ \frac{-4.613\times10^{-4}x_4}{x_5x_6^3}\right] \quad \text{(55.20c)}$$

$$\nabla g_4 = [-12.82, -12.82, 0, 0, 0, 0] \quad (55.20d)$$

$$\nabla g_5 = [0, 11.667, 0, 0, 0, 0] \quad (55.20e)$$

$$\nabla g_6 = [-26.667, 0, 0, 0, 0, 0] \quad (55.20f)$$

$$\nabla g_7 = \left[\frac{-1.996 \times 10^{-4}}{x_1^4}, 0, 0, 0, 0, 0\right] \quad (55.20g)$$

<u>Linearized Form</u>

$$f(x) = f(x^{(0)}) + \nabla f.(x - x^{(0)})$$

Starting Design Point

$$x^{(0)} = (0.0480, 0.0500, 0.0280, 0.0280, 0.076, 0.018)$$

<u>For Cost Function</u>

$$f = 3.160 \times 10^{-3} + [2.616 \times 10^{-3}\ 6.254 \times 10^{-3}\ 2.304 \times 10^{-3}\ 3.125 \times 10^{-3}\ 2.844 \times 10^{-3}\ 1.201 \times 10^{-2}] \begin{bmatrix} x_1 - 0.048 \\ x_2 - 0.050 \\ x_3 - 0.028 \\ x_4 - 0.028 \\ x_5 - 0.076 \\ x_6 - 0.018 \end{bmatrix}$$

The constraints' status

$$f = 3.160 \times 10^{-3} + [2.616 \times 10^{-3}x_1 - 1.256 \times 10^{-4} + 6.254 \times 10^{-3}x_2 - 3.127 \times 10^{-4} + 2.304 \times 10^{-3}x_3 - 6.451 \times 10^{-5}$$
$$+ 3.125 \times 10^{-3}x_4 - 8.75 \times 10^{-5}$$
$$+ 2.844 \times 10^{-3}x_5 - 2.161 \times 10^{-4}$$
$$+ 1.201 \times 10^{-2}x_6 - 2.162 \times 10^{-4}]$$

$f = [2.137 + 2.616x_1 + 6.254x_2 + 2.304x_3 + 3.125x_4 + 2.844x_5 + 1.201x_6] \times 10^{-3}$.

The constraints' status

$b_1 = -g_1(x^{(0)}) = 0.13$ inactive

$b_2 = -g_2(x^{(0)}) = 0.51$ inactive

$b_3 = -g_3(x^{(0)}) = 0.83$ inactive

$b_4 = -g_4(x^{(0)}) = 0.26$ inactive

$b_5 = -g_5(x^{(0)}) = 0.042$ inactive

$b_6 = -g_6(x^{(0)}) = 0.28$ inactive

$b_7 = -g_7(x^{(0)}) = 0.55$ inactive

<u>The gradients of the constraint functions</u>

$\nabla g_1 = (-36.401 \quad 0 \quad -76.244 \quad 0 \quad 0 \quad 0)$

$\nabla g_2 = (0 \quad -19.582 \quad 0 \quad -44.504 \quad 0 \quad 0)$

$\nabla g_3 = (0 \quad 0 \quad 0 \quad 6.244 \quad -4.601 \quad -29.141)$

$\nabla g_4 = (-12.82 \quad -12.82 \quad 0 \quad 0 \quad 0 \quad 0)$

$\nabla g_5 = (0 \quad 11.667 \quad 0 \quad 0 \quad 0 \quad 0)$

$\nabla g_6 = (-26.667 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

$\nabla g_7 = (-37.601 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

<u>Linearized constraint functions</u>

$g(x) = gx(^{(0)}) + \nabla g \cdot (x - x^{(0)})$

(1) $g_1 = -0.13 + (-36.401 \quad 0 \quad -76.244 \quad 0 \quad 0 \quad 0)$

$$\begin{bmatrix} x_1 & - & 0.0480 \\ x_2 & - & 0.0500 \\ x_3 & - & 0.0280 \\ x_4 & - & 0.0280 \\ x_5 & - & 0.0760 \\ x_6 & - & 0.0180 \end{bmatrix}$$

$$g_1 \quad = \quad 36.401x_1 \quad - \quad 76.244x_3 \quad = \quad 3.752 \leq 0$$

(2) $$g_2 \quad = \quad -0.51 + (0 \quad -19.582 \quad 0 \quad -44.504 \quad 0 \quad 0)$$

$$\begin{bmatrix} x_1 & - & 0.0480 \\ x_2 & - & 0.050 \\ x_3 & - & 0.0280 \\ x_4 & - & 0.0280 \\ x_5 & - & 0.076 \\ x_6 & - & 0.018 \end{bmatrix}$$

$$g_2 \quad = \quad -19.582x_2 \quad - \quad 44.504x_4 \quad + \quad 1.715 \quad \leq 0$$

(3) $$g_3 \quad = \quad -0.83 + (0 \quad 0 \quad 0 \quad 6.244 \quad -4.601 \quad -29.141)$$

$$\begin{bmatrix} x_1 & - & 0.048 \\ x_2 & - & 0.050 \\ x_3 & - & 0.028 \\ x_4 & - & 0.028 \\ x_5 & - & 0.076 \\ x_6 & - & 0.018 \end{bmatrix}$$

$$===> g_3 \quad = \quad 6.244x_4 \quad - 4.601x_5 \quad - 29.141x_6 \quad - 0.131 \quad \leq 0$$

(4) $g_4 = -0.26 + (-12.82 \quad -12.82 \quad 0 \quad 0 \quad 0 \quad 0)$

$$\begin{bmatrix} x_1 & - & 0.048 \\ x_2 & - & 0.050 \\ x_3 & - & 0.028 \\ x_4 & - & 0.028 \\ x_5 & - & 0.076 \\ x_6 & - & 0.018 \end{bmatrix}$$

$\Longrightarrow g_4 = -12.82x_1 - 12.82x_2 + 0.996 \leq 0$

(5) $g_5 = -0.042 + (0 \quad 11.667 \quad 0 \quad 0 \quad 0 \quad 0)$

$$\begin{bmatrix} x_1 & - & 0.048 \\ x_2 & - & 0.050 \\ x_3 & - & 0.028 \\ x_4 & - & 0.028 \\ x_5 & - & 0.076 \\ x_6 & - & 0.018 \end{bmatrix}$$

$\Longrightarrow g_5 = 11.667x_2 - 0.625 \leq 0$

(6) $g_6 = -0.28 + (-26.667 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

$$\begin{bmatrix} x_1 & - & 0.048 \\ x_2 & - & 0.050 \\ x_3 & - & 0.028 \\ x_4 & - & 0.028 \\ x_5 & - & 0.076 \\ x_6 & - & 0.018 \end{bmatrix}$$

$\Longrightarrow g_6 = -26.667x_1 + 1.000 \leq 0$

$$(7) \quad g_7 = -0.55 + (-37.601 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) \begin{bmatrix} x_1 - 0.048 \\ x_2 - 0.050 \\ x_3 - 0.028 \\ x_4 - 0.028 \\ x_5 - 0.076 \\ x_6 - 0.018 \end{bmatrix}$$

$$\Longrightarrow g_7 = -37.601x_1 + 1.255 \leq 0$$

<u>Summarized Form</u>

$$f = (2.137 + 2.616x_1 + 6.254x_2 + 2.304x_3 + 3.125x_4 + 2.844x_5 + 1.201x_5) \times 10^{-3}$$

subject to:

$$g_1 = -36.401x_1 - 76.244x_3 + 3.752 \leq 0$$

$$g_2 = -19.582x_2 - 44.504x_4 + 1.715 \leq 0$$

$$g_3 = 6.244x_4 - 4.601x_5 - 29.141x_6 - 0.131 \leq 0$$

$$g_4 = -12.82x_1 - 12.82x_2 + 0.996 \leq 0$$

$$g_5 = 11.667x_2 - 0.625 \leq 0$$

$$g_6 = -26.667x_1 + 1.00 \leq 0$$

$$g_7 = -37.601x_1 + 1.255 \leq 0$$

Variable bound values

$$0.036 \leq x_1 \leq 0.080$$

$$0.036 \leq x_2 \leq 0.070$$

$$0.027 \leq x_3 \leq 0.065$$

$$0.027 \leq x_4 \leq 0.060$$

$$0.060 \leq x_5 \leq 0.125$$

$$0.012 \leq x_6 \leq 0.022$$

The Quantitative System for Business Plus Software Package was employed to solve this LP problem. This was developed by Yih - Long Cheng and Robert S. Sullivan, and was released in 1991. It was developed with the Basic Programming Language, to perform the solution to the following problems.

- Linear Programming Problems
- Quadratic Programming Problems
- Assignment Problems
- Network Modelling
- Project Scheduling - CPM & PERT.
- Dynamic Programming

Note that the constant in the cost function could be deleted since it does not affect our solution, but for our purpose the constant was multiplied with variable X_7 which automatically assume a magnitude of one, i.e $X_7 = 1$. The result as it comes out of the computer is as shown on page 109.

(1)

Input Data of The Problem ayo1   Page: 1

Min +.002616X1 +.006254X2 +.002303X3 +.003125X4 +.002844X5
+.001201X6 +.002137x7
Subject to
(1) +36.4010X1 +0 X2 +76.2440X3 +0 X4 +0 X5
+0 X6 +0 x7 > +3.75200
(2) +0 X1 +19.5820X2 +0 X3 +44.5040X4 +0 X5
+0 X6 +0 x7 > +1.71500
(3) +0 X1 +0 X2 +0 X3 +6.24400X4 -4.60100X5
-29.1410X6 +0 x7 < +.131000
(4) +12.8200X1 +12.8200X2 +0 X3 +0 X4 +0 X5
+0 X6 +0 x7 > +.996000
(5) +0 X1 +11.6670X2 +0 X3 +0 X4 +0 X5
+0 X6 +0 x7 < +.625000
(6) +26.6670X1 +0 X2 +0 X3 +0 X4 +0 X5
+0 X6 +0 x7 > +1.00000
(7) +37.6010X1 +0 X2 +0 X3 +0 X4 +0 X5
+0 X6 +0 x7 > +1.25500
(8) +0 X1 +0 X2 +0 X3 +0 X4 +0 X5
+0 X6 +1.00000x7 = +1.00000

Integrality and Bounds   Page: 1

Var. no.	Name	Integrality(C/I/B)	Lower bound	Upper bound
1	X1	<C>	<+.036000>	<+.070000>
2	X2	<C>	<+.036000>	<+.080000>
3	X3	<C>	<+.027000>	<+.065000>
4	X4	<C>	<+.027000>	<+.060000>
5	X5	<C>	<+.060000>	<+.125000>
6	X6	<C>	<+.012000>	<+.022000>
7	x7	<C>	<+1.00000>	<+1.00000>

Summarized Report for ayo1						Page : 1
Number	Variable	Solution	Opportunity Cost	Objective Coefficient	Minimum Obj. Coeff.	Maximum Obj. Coeff.
1	X1	+.04169111	0	+.00261500	+.00109952	+.00735352
2	X2	+.03600000	+.00473752	+.00625400	+.00151648	+ Infinity
3	X3	+.02930594	0	+.00230300	0	+.00547936
4	X4	+.02700000	+.00312500	+.00312500	+4.657E-11	+ Infinity
5	X5	+.06000000	+.00284400	+.00284400	-4.657E-11	+ Infinity
6	X6	+.01200000	+.00120100	+.00120100	-2.328E-11	+ Infinity
7	x7	+1.0000000	+.00213700	+.00213700	+1.000E+20	+ Infinity

Minimized OBJ = 2.808127E-03 Iteration = 4 Elapsed CPU second = 0

Summarized Report for ayo1						Page : 2
Constr.	Status	RHS	Shadow Price	Slack or Surplus	Minimum RHS	Maximum RHS
1	Tight	>+3.7520001	+.00003021	0	+3.5761862	+ Infinity
2	Loose	>+1.7150000	0	+.19156003	+1.5234400	+ Infinity
3	Loose	<+.13100000	0	+.58816397	-.45716399	+ Infinity
4	Tight	>+.99599999	+.00011829	0	+.94226402	+1.0579195
5	Loose	<+.62500000	0	+.20498600	+.42001200	+ Infinity
6	Loose	>+1.0000000	0	+.11177664	- Infinity	+1.1117766
7	Loose	>+1.2550000	0	+.31262735	+.94237268	+ Infinity
8	Tight	=+1.0000000	0	0	+1.0000000	+ Infinity

Minimized OBJ = 2.808127E-03 Iteration = 4 Elapsed CPU second = 0