THE SIMULATION OF OPIMISATION OF A 3 – TERM CONTROLLER ACTION IN A CRUDE OIL DISTILLATION UNIT.

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THE SIMULATION OF OPTIMISATION OF A 3 – TERM CONTROLLER ACTION IN A CRUDE OIL DISTILLATION UNIT.

BY

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CERTIFICATION

This is to certify that this thesis has been read and approved as meeting the requirement of the Department of Electrical and Electronic Engineering; The Federal University of Technology, Akure for the award of Master of Engineering (Electrical and Electronic) Degree. (Control Engineering Option).

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ABSTRACT

The optimization of a digital computer configured in a direct digital control systems to work as a proportional – plus – integral – plus derivative controller subject to a quadratic constraint in the Warri Refinery and Petrochemical Company is presented.

The design uses state – space techniques, operating on measurements of the plant inputs and outputs.

A multivariable self – tuning controller analysis with inputs and output values in the state – vector is adopted.

Self-tuning control includes two substantial steps, which are executed sequentially in each control cycle. One is parameter identification and the other is the control law construction based on the defined goal which is the optimum gain setting for the controller.

The algorithm developed for the system is based on the transfer function in the Z-domain with Z^{-1} defined as a unity delayer.

The controllability and observability of the control system is investigated and hence the development of a state observer is considered. The observer design criterion is based on the assumption that the transfer function matrix from the input to the observer state be equal to that from the input to the system state.

The discrete–time plant is assumed to be stabilizable (controllable) and detectable (observable), linear and time invariant; hence change of state does not depend on initial time but depends only on the length of time during which the control force is applied. This is the sampling period. The input responses and the output responses obtained at each performance condition are presented. The optimum performance condition is determined by the least square technique. This is shown in the appendix.

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NOMENCLATURE

MV	Output variable of the process
E	Deviation variable of the process
SV	Set point variable of the process
PV	Input variable of the process
K_P	Proportional band constant of the process
K_I	Constant of integral function of process
K_D	Constant of derivative function of process
DE	Change in the Deviational of process
E_{n-1}	Step change in Deviation of process
DMV	Change in output of process
DT	Control period on the process
Z^{-1}	Digital Unit delayer
K	Constant used in Proportional function, Integral function and derivative function.
a	Digital output matrices parameters
b	Digital input matrices parameters
Z	Digital transform
I_t	Integral time.
$I_{t(eff)}$	Effective integral time
Δ_t	Derivative time
$\Delta_{t(eff)}$	Effective Derivative time
T_{PID}	Damped period of PID
$T_{\Delta t}$	Derivative Time period

T_{PT}	Proportional Time Period
PB	Proportional Band
R_R	Slope
μ	Magnitude
I	Integral action
D	Derivative action
QAD	Quarter Amplitude Damping
Δ	Determinant

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

The control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner. One of the most attractive aspects of control system theory is its general applicability to control problem of the most varying engineering types. The control engineer is interested in the dynamics characteristics of a system (OKOCHA & UDOH, 1986). The object making up the system may not be in a state of equilibrium relative to each other and the surrounding.

In principle, it is possible to change the state of a system in any prescribed manner by properly choosing the inputs, at least within reasonable limits. In other words, one may exert influence on the system state by means of intelligent manipulation of its inputs. (ADIELE & ECHEM, 1997). The theory of control is concerned with the mathematical formulation of laws control actions. The control engineer develops the techniques and hardware necessary for the implementation of the control laws to the specific system with intent to maximize or minimize output in a process of optimization.

During the past, optimization methods have taken on extremely important role in all areas of engineering, especially control engineering. Increased competition plus man's desire to perform in an optimum fashion, combined with the fact that the faces an ever – shrinking supply of natural resources, represent some of the reasons for optimum. In modern technology, one encounters with increasing frequency situations where, for various technological reasons, one can accept only those engineering solutions, which are optimal in defined sense. A specific engineering problem does not have a unique solution, indeed it has in most cases an infinite number of possible solutions. If we judge the various solutions on the basis of a specific pay off function (cost, index, value function, index of performance, figure of merit etc) each possible solution is characterized, as a rule by a different value of this function. The one solution associated with the optimum value is referred to as the optimum solution. The word " optimum' may refer to either a maximum or a minimum depending upon the situation at hand. The one solution associated with the optimum value is referred to as the optimum solution. The optimum value may refer to he minimum or minimum, depending on the situation. The most obvious method of optimization is an outright search process – all possible solution in the total set of possible ones are compared and the best one is selected (ELGRED, 1967).

1.2 AIMS AND OBJECTIVES

This project examines, critically the optimisation of conditions of a process control system in an industrial plant and a similar condition with a simulator. The work is directed at collecting data on the different characteristics at different conditions of the various controller actions required to optimize the crude distillation unit of the Warri Refinery and Petroleum Company in Ekpan Delta – State.

The plant' dynamics, $\dot{X} = f(x,u)$ is known and attention limited to a deterministic case. The state x, and the control vector u are constrained. A performance index which is a function of x and u is decided upon and used to determined the proper setting of a compound controller required to bring about the stabilization and consequent optimization of the process system within a short time when the process disturbance exists.

1.3 TYPES OF CONTROLLER ACTION

The six basic industrial automatic controllers action are (OGATA 1970):

(a) Two – position on-off

(b) Proportional control action

(c) Integral control action

(d) Proportional – plus – integral control action

(e) Proportional – plus – derivative control action

(f) Proportional – plus – integral – plus derivative action.

In this project, the general function of the controller to keep the controlled variables near its desired value when a non-zero signal results from a change in command or a disturbance is analysed with the aid of a suitable algorithm. The gains of the controller is computed in light of the performance specifications. The various controller actions highlighted above are investigated with the aid of the responses plotted with the data collected during the process reaction. The results and the characteristic behaviour obtained will be analysed and optimization carried out, with the developed a logarithm, on the process and recommendations is made after comparison of the results of optimization.

1.4 SYSTEM REPRESENTATION

The mathematical relationship of a control system can be represented by block diagrams. These diagrams indicate more realistically the actual processes taking place in the system than purely mathematical representation of the system. (STEFFANO, 1976).

A block diagram consists of a specific configuration of four types of elements; blocks, summing points, take off points, and arrows representing unidirectional signal flow.

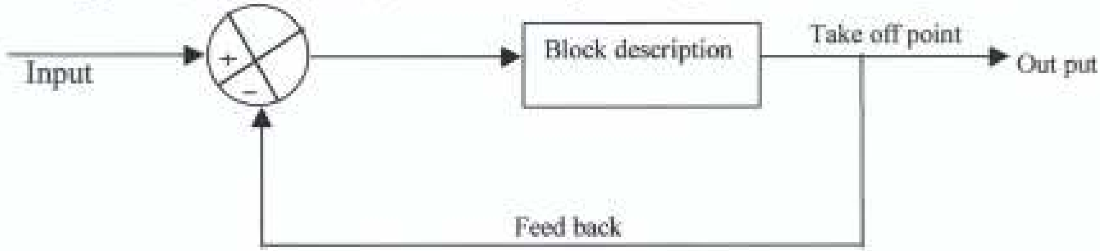


Fig 1.1 BLOCK DIAGRAM DESCRIPTION

The process system in this project is variously illustrated with block diagram.

CHAPTER TWO

LITERATURE REVIEW

Process control using different modes of control have been an age long phenomena. This dated back to the natural regulatory control of some internal physical characteristics which is important or critical to a living organism. For instance the natural regulation of natural quantity such as body temperature, fluid flow rate and many more.

In addition to this natural process. Humans of found it necessary to regulate some of the external physical parameters in their environment to sustain life and thereby initiate artificial process control. This regulation was accomplished by observation of the parameter, comparison with some desired value, and the necessary action to bring the parameter to that desired value. Example of such control are found in fires for light and heat, cooking of food, simple smelting up to complex process involving multi-variables (GRIMBLE 1982).

The adoption of automatic regulatory procedure makes manufacturing of products more effective and this is achieved by what is known as feed back control system. This system is characterized by error. In order to reduce this error, various modes of control were developed which include;

off–on control, proportional control, derivative control, proportional plus integral control, proportional plus derivative control and three–term control also known as PID. In all these modes, optimization is achieved using the PID mode.

In a control process having a number of capacitors in series or having dead time (FINK 1975), there is need to optimize the control action.

Hence PID controller is employed. The frequency response of a PID controller is as shown in Fig 2.1.

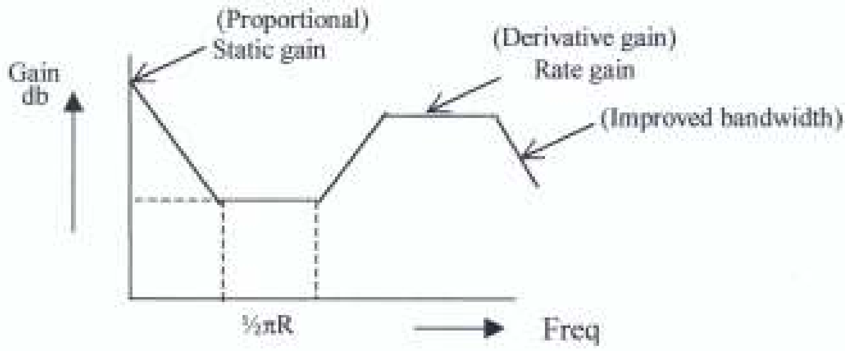


Fig. 2.1 Frequency response of PID controller

For A PID controller, the proportional gain can be increased until the system becomes unstable (ESHBACH 1975), derivative action must be added unit it becomes stable, finally a small amount of integral action is added, enough to insure resetting but not enough to cause instability. This effect is shown by Nyquist plot. Fig 2.2. Integral action will integrate any difference between measurement and set point, and cause the controller's output to change until the difference between measurement and set point is zero.

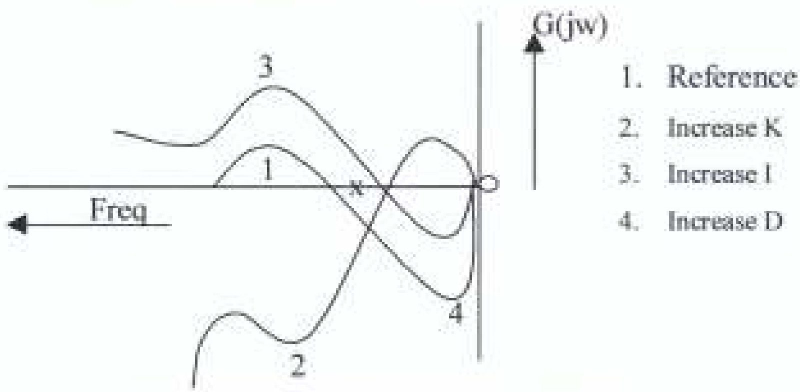


Fig 2.2 Nyquist Plot of a PID controller

PID controller cannot be used in a noisy environment or on process that changes stepwise because the derivative contribution is based on the measurement of rate of change (THOMAS 1984), it can be used on processes that are slow to response and have long periods. Derivative action increase system gain proportionately to the rate of change of the measures value hence the effect of noise will be more pronounced (RAMSAY, 1981) Temperature control being a common example. Derivative action shortens the response period to an upset. This is because the input signal when differentiated is scaled down (KUO, 1966). In physical construction of the controller, interaction occurs between the integral and derivative. This causes the effective values of the modes to differ from their set values. The effective integral time $I_{t(eff)}$ is actually the sum of the two time constants.

$$I_{t(eff)} = I_t + D_t \qquad \text{------------------} \qquad (2.1)$$

The effective Derivative time $D_{t(eff)}$ is

$$D_{t(eff)} = \frac{1}{1/I_t + 1/D_t} \qquad \text{---------------} \qquad (2.2)$$

$$D_{t(eff)} = \frac{1}{1/I_t + I_t}$$

$$= \frac{I_t \cdot D_t}{D_t + I_t} \qquad \text{------------------------} \qquad (2.3)$$

$$I_{t(eff)} = I_t + D_t$$

But at $I_t = D_t$

$$I_{t(eff)} = 2I_t$$

Substitute for D_t in equ (2.3) above

$$D_{teff} = \frac{I^2_t}{2I_t} = \frac{I_t}{2}$$

but $I_{teff} = 2I_t$

$$I_{teff} = \left[\left(\frac{2I_t}{2}\right) \times 2\right]$$

$$= 4\, D_{teff}$$

$$\therefore \quad D_{teff} = I_{teff}/4$$

Two points concerning equation (2.1) and (2.2):

(1) The effective value for derivation time can never be greater than one – fourth the integral time which occurs at $I_t = D_t$.

(2) When $D_t > I_t$ the contribution to each control action is reversed. That is, setting $D_t > I_t$ changes the value for I_t more than for $D_{t(eff)}$. The rule of thumb is that the controller should not be adjusted so that derivative action is greater than integral action.

The performance criteria for a PID controller can be evaluated by measuring the damped periods. Optimum tuning generally result with a QAD – period that is approximately equal to the natural period. The damped period is known as T_{PID} and is equal to (YIH and SHIEH 1981).

Recommendation for response setting are:

Method 1:

Ziegler and Nichols; Open Loop,

$$\frac{100}{PB} = \frac{1.2}{T_{Dt} R_R} \text{ ---------------} \quad (2.3)$$

$$I = 2.0\ T_{DT} \text{ -------------} \quad (2.4)$$

$$D = 0.5\ T_{DT} \text{ -------------} \quad (2.5)$$

Close – Loop,

$$PB = 1.66\ (PB)^* \text{ -----------} \quad (2.6)$$

$$I = 0.5\ T_D \text{ --------------} \quad (2.7)$$

$$D = T/8 \text{ ---------------} \quad (2.8)$$

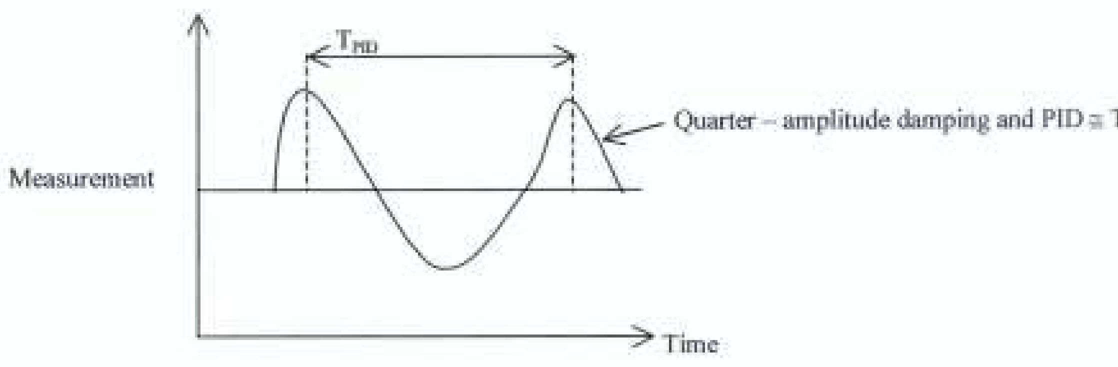


Fig 2.3 Response to derivative action for a PID controller

QAD: This means that the damping period of a derivative action is about a quarter of the damping period in the amplitude of an integral action and the combined effect of varying PID controller action results in Quarter Amplitude Damping.

Method II:

Colen and Coon,

$$\frac{100}{PB} = 1.35\left(\frac{1+\mu/5}{T_{DT}\, R_R}\right) \qquad (2.9)$$

$$I = 2.5\ T_{PT}\left(\frac{1-\mu/5}{1+3\mu/5}\right) \qquad (2.10)$$

$$D = \left(\frac{0.87T_{PT}}{(1+\mu/5}\right) \qquad (2.11)$$

Method III:

Shirskey,

$$PB = 4.0\ (PB)^* \qquad (2.12)$$

$$I = 0.5\ T_D \qquad (2.13)$$

$$D = 0.12\ T_o \qquad (2.14)$$

PID controller operates on an error signal, which is the difference between the measured variable and the set point. For optimization, the controller is to maintain the error within reasonable limits and usually to attain zero error.

The controller achieved this goal by calculating the manipulated variable, this calculation is done as the sum of three modes. The first mode is the proportional control, which is calculated as.

$$M_p(t) = K_c\, e(t) + \text{constant} \qquad (2.15)$$

Where K_c is the proportional gain

And $100/K_c$ = Proportional band

This mode provides instantaneous correction to the manipulated variable for an error signal but cannot reduce the error to zero. Reducing the error involves increasing the gain and leads to instability. This is eliminated by another mode known as integral mode, which is calculated thus,

$$M_i(t) = \frac{K_c}{K_I}\int edt + \text{constant} \quad (2.16)$$

Where:

K_i is the integral time. This mode continuously changes the manipulated variable until the error is zero but it does not respond quickly to the first appearance of non – zero. Additional optimization results from derivative mode, which is calculated as follows

$$M_d(t) = K_cK_D \frac{db(t)}{dt} + \text{constant} \quad (2.17)$$

Where K_D is the derivative time. This mode recognizes the rate of change of the manipulated variable and corrects the manipulated variable appropriately. PID therefore combines the three modes into one logarithm, given in continuous and discrete form.

$$M(t) = K_c\,(e(t) + \frac{1}{K_I}\int e(t)\,dt + K_d \frac{db(t)}{dt} + \text{Constant} \quad (2.18)$$

$$Mn = K_c\,(en + \frac{\Delta t}{K_I} \sum ei + \frac{K_d}{\Delta t}(b_n - b_{n-1}) + \text{constant} \quad (2.19)$$

Where n = execution number

Δt = execution period

The figure 2.4 shows a block diagram of a single – variable control system using PID controller (MARTIN, 1980).

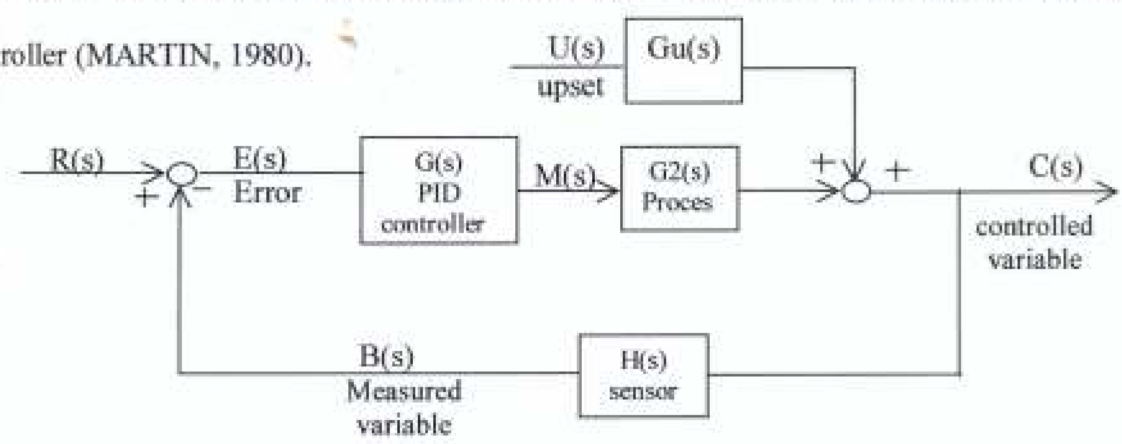


Fig 2.4 AN INDUSTRIAL PROCESS CONTROL SYSTEM

PID controller optimization is also achieved by reducing the error to zero (BELOVE, 1986). Thus an integral term in the error feedback is desirable to maintain steady–state accuracy. However, the oscillatory nature of the system response resulting from an integral term is undesirable.

A solution to this is to constrain the integral term such that it only has effect for small magnitudes of error.

A discrete - term controller has the general form

$$P_n = P_k \left[e_n + T_i \Delta t \sum_{r=0}^{n} e(n-r) + \frac{T_d}{\Delta t}(e_n - e_{n-1})\right] \quad \text{------------------} \quad 2.20$$

where P_n is the controller output at the nth sampling instant,

P_k is the proportional gain constant, e_n is the error at the nth sampling instant, T_i is the integral gain constant, T_d is the derivative gain constant and t is the sampling period

The weighting factor, P_k, T_i and T_d must be chosen in the same manner as the optimal control weighting function Q and R (Q &R are the weighting matrices).

The best response shown in fig 2.5 is obtained from this constrained PID controller (MORRIS & STERLING 1981)

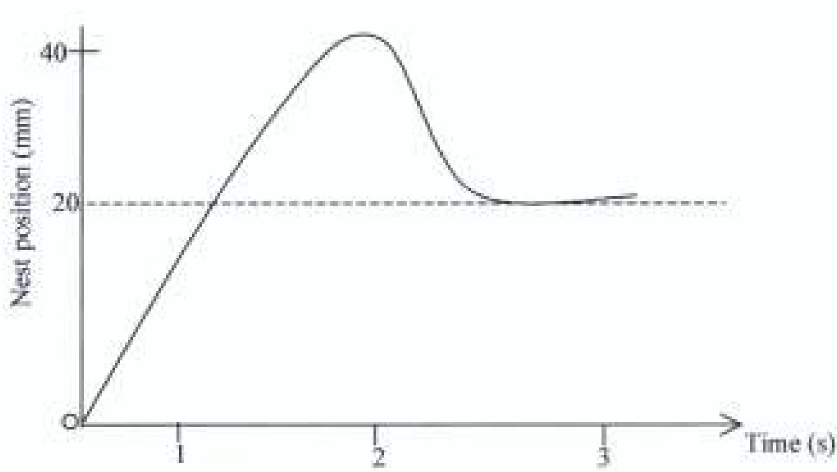


Fig 2.5 Constrained three term controller to 20 mm step disturbance

CHAPTER THREE

THEORETICAL BACKGROUND

3.1 CLASSIFICATION OF CONTROL SYSTEMS

Control system may be classified according to their ability to follow step inputs, ramp inputs, parabolic inputs, etc. The magnitudes of the steady-state errors due to these individual inputs are indicative of the "goodness" of the system.

The open-loop transfer function of a system could be represented with the expression.

$$G(s)H(s) = \frac{K(sT_b + 1)(sT_b + 1) \ldots\ldots (sT_m + 1)}{s^N (sT_1 + 1)(sT_2 + 1) \ldots\ldots (sT_b + 1)} \quad \text{------------} \quad (3.1)$$

It involves the term s^N in the denominator representing a pole of multiplicity N at the origin. A system is called type 0, type 1, type 2, ... If N = 0, N = 1, N = 2, ... respectively. This classification is different from the order of a system . As the type number is increased accuracy is improved; however, increasing the type number aggravates the stability problem. However stability is improved with increase in the index of the denominator. A compromise between steady - state accuracy and relative stability is always necessary.

3.2 STEADY - STATE ERRORS

Error in a control system can be defined simply as a deviation between the out put and the input of the system. This can be attributed to many factors. Changes in the reference input will cause unavoidable errors during transient periods and many also cause steady - state errors. Steady-state error is a measure of the accuracy of a control system. The steady - state performance of a stable control system is generally judged by the steady-state error due to step, ramp, or acceleration inputs. Any physical control system inherently suffers stead - state error in response to certain types of inputs. A system may have no stead-state error to a step input, but the same system may exhibit non-zero steady-state error to a ramp input. Whether or not a given system will exhibit steady-state error for a given type of input depends upon the type of open - loop transfer function of the system.

3.3 CONTROL SYSTEM STRUCTURE

The general block diagram of a control system is shown in Figure 3.1

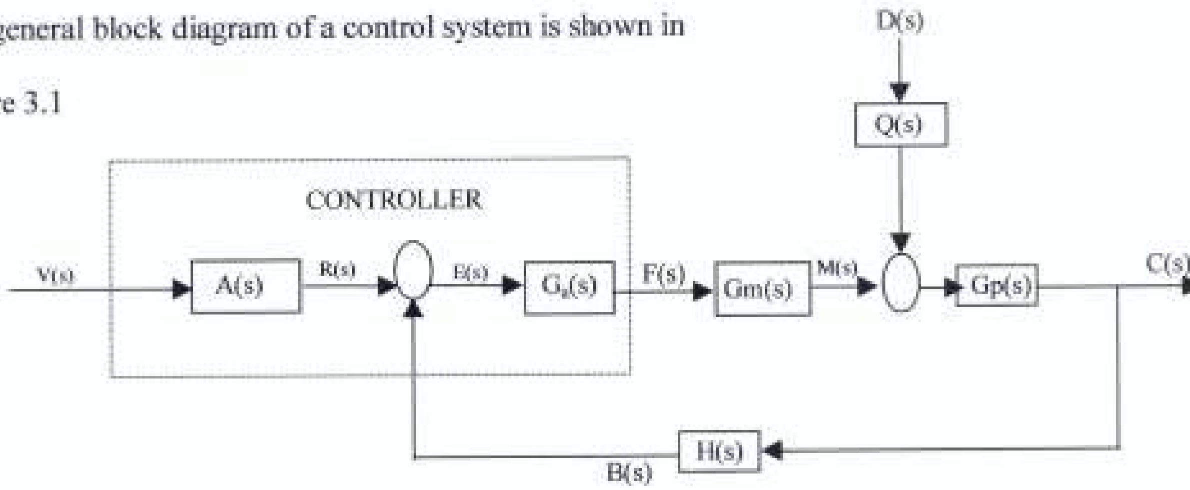


Fig 3.1 CONTROL SYSTEM BLOCK DIAGRAM

$A(s)$	=	Input elements
$G_a(s)$	=	Control logic elements
$G_m(s)$	=	Final control elements
$G_p(s)$	=	Plant elements
$H(s)$	=	Feedback elements
$Q(s)$	=	Disturbance elements
$B(s)$	=	Feedback signal
$C(s)$	=	Controlled variable or output
$D(s)$	=	Disturbance input
$E(s)$	=	Error or actuating signal
$F(s)$	=	Control signal
$M(s)$	=	Manipulated variable
$R(s)$	=	Reference input
$V(s)$	=	Command input

The controller is generally thought of as a logic element that compares the command signal with the measurement of the output and decide what should be done. The input and feedback elements are transducers for converting one type of signal into another type. The object to be controlled is the plant. The manipulated variable is generated by the final control elements for this purpose. The disturbance input also acts on the plant.

3.4 CONTROL LAWS

The control logic element are designed to act on the actuating (error) signal to produce the control signal. The algorithm that is physically implemented for this purpose is the control law or control action. A non-zero error signal results from either a change in command or a disturbance.

The general function of the controller is to keep the controlled variable near its desired value when these occur. (YOUSEFZADEH, 1979)

More specifically, the control objective are:

(i) Minimize the steady state error

(ii) Minimize the settling time

(iii) Achieve other transient specifications, such as minimizing the maximum overshoot.

3.5 TWO - POSITION CONTROL

Two-position control is the most familiar type perhaps because of its use in home thermostats. The control output takes on one of two values. With the on-off controller, the controller output is either on or off. The controller output is determined by the magnitude of the error signal.

3.6 PROPORTIONAL CONTROL

Two-position control is acceptable for many application in which the requirements are not severe. However many situations require finer control.

The proportion control algorithm is one in which the change in the control signal is proportional to the error.

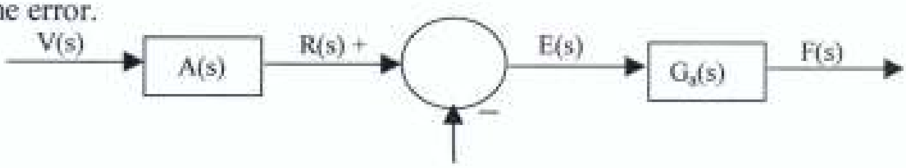


Fig 3.2 CONTROL CONVENTION

From the convention in fig 3.2 an extract from the general structure in fig 3.1 the proportional control is described by

$$F(s) = K_pE(s). \quad (3.2)$$

where F(s) is the deviation in the control signal and K_p is the proportional gain. If the total value displacement is y(t) and the manually created displacement is x, then

$$y(t) = K_pE(t) + x \quad (3.3)$$

The percent change in error needed to move the valve full scale is the proportional band, it is related to the gain as follows:

$$K_p = \frac{100}{\text{Band } \%} \quad (3.4)$$

3.7 INTEGRAL CONTROL

The offset error that occurs with proportional control is a result of the system reaching an equilibrium in which the control signal no longer changes. This allows a constant error to exist. If the controller id modified to produce an increasing signal as long as the error is non-zero, the effect might be eliminated. This is the principle of integral control. In this mode the change in the control signal is proportional to the integral of the error.

From the figure 3.1

$$F(s) = \frac{K_I}{s}E(s), \quad K_I = \frac{1}{T} \quad (3.5)$$

where F(s) is the deviation in the control signal and K_I is the integral gain. T is the integral time setting.

Integral control is useful in improving steady-state performance, but in general it does not improve and may even degrade transient performance. Improperly applied it can produce an unstable control system. It is best used in conjunction with other control modes. This is the basis for using proportional-plus-integral control. The algorithm for this two mode control is

$$F(s) = K_pE(s) + \frac{K_I}{s}E(s) \quad \text{----------------------------} \quad (3.6)$$

The integral action provide an automatic, not manual, reset of the controller in the presence of a disturbance. For this reason it is often called reset action.

The algorithm is sometimes expressed as

$$F(s) = K_p\left(1 + \frac{1}{sT_I}\right)E(s) \quad \text{------------------------------} \quad (3.7)$$

Where T_I is the reset time. The reset time is the time required for the integral action signal to equal that of the proportional term, if a constant error exists. The reciprocal of reset time is expressed as repeats per minute and is the frequency with which the integral action repeats the proportional correction signal.

The proportional control gain must be reduced when used with integral action. The integral term does not react instantaneously to a zero-error signal but continues to correct, which tends to cause oscillations if the designer does not take this effect into account.

3.8 DERIVATIVE CONTROL

Integral action tends to produce a control signal even after the error has vanished, which suggests that the controller be made aware that the error is approaching zero. Integral setting is a function of the dead time associated with the process and other elements in the control loop. The dead time is the delay between two related actions (NORMAAN, 1980).

One way to accomplish this is to design the controller to react to the derivative of the error with the derivative control law.

$$F(s) = K_D sE(s) \qquad (3.8)$$

Where K_D is the derivative gain. This algorithm is also called rate action. It is used to damp out oscillations.

Since it depends only on the error rate, derivative control should never be used alone. By definition, derivative is the time interval by which derivative action will advance the effect of proportional action on the final operator. Derivative action occurs whenever the measurement signal derivative action differentiates the change and maintains a level as long as the measurement continues to change at the given rate, under steady state conditions, the derivative, acts as a 1 do 1 repeaters. It has no influence on a controller's output. When used with proportional action, the PD-control algorithm results.

$$\begin{aligned} F(s) &= (K_p + sK_p)\ E(s) \\ &= K_p(1 + sT_D)\ E(s) \qquad (3.9) \end{aligned}$$

Where T_D is the rate time or derivative time. With integral action included, the proportional - plus integral - plus - derivative (PID) control law is obtained.

$$F(s) = (K_p + K_I/s + sK_D)\ E(s) \qquad (3.10)$$

3.9 DIGITAL IMPLEMENTATION OF CONTROL ALGORITHMS

There are two types of applications of digital computers to control problems. The first is supervisory control in which analogue controllers are directly interfaced the plant to be controlled, while the digital computer provides command signals to the controllers. The second application is direct digital control (DDC) in which the digital device acts at the lowest levels of the system, in direct control of the plant (ASTROM & BJORN 1983). Direct digital control is a system in which the manipulated variables are controlled directly by the computer. A valve is driven according to the result of computer calculation of the control equation or algorithm for that loop. An algorithm is an equation that is repeated many times. Algorthms are named for the control action they provide.

Because digital-control algorithms are not limited by hardware as analogue controllers, the algorithm is essentially limited only by the designer's imagination.

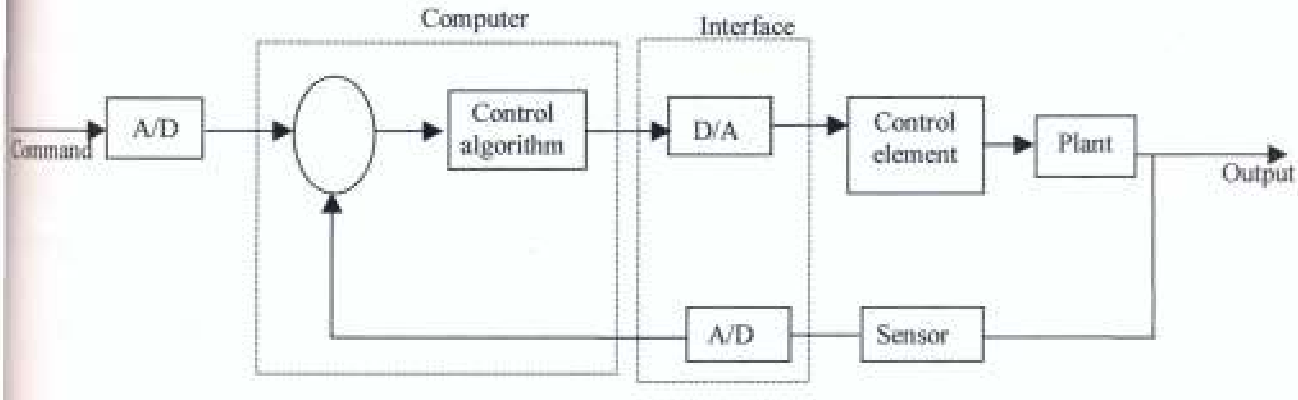


Fig 3.3 Structure of a digital control system

The basic structure of a single-loop DDC controller is as shown in fig 3.3. The computer with its internal clock drives the D/A and A/D converters. It compares the command signal with the feedback signals and generates the control signal to be sent to the final control elements. These control signals are computed from the control algorithm stored in the memory.

When a digital computer is used as a controller for a physical system, the process of signal conversion is essential so that the digital and analogue system can be interfaced in the same system. Sample and Hold (S/H) device is also interfaced quite often.

Analogue/digital and digital/analogue conversions consist of a sequence of data manipulation. A pressure signal (analogue variable) for example is measured and is to be (stored) in the CPU of a digital computer. Subsequently it is desired to read this same function of time out of the CPU, to convert it back to analogue form and to utilizes it for control (FALAKI, 1997). The data transformation expressed by the signal are illustrated in fig 3.4. The following are the steps:

(a) **Sampling:** The instantaneous amplitude of the analogue variable is measured at desired - spaced intervals of time, and attention is limited to the amplitude of the variable at those instants.

(b) **Quantizing:** The amplitude of each sample is rounded up or down to the nearest available discrete level. The numbers of these levels is determined by the number of binary digits, which will be used to represent each quantized data value.

(c) **Coding:** The sample and quantized data are converted into digital code.

(d) **Digital Processing:** The sampled, quantized and coded data are read into the CPU where they are manipulated and stored as prescribed by the digital computer programme.

(e) **Decoding:** A sequence of binary number is read out of the output buffer, and each word is translated into analogue voltage form. That is each binary pulse with an amplitude corresponding to that number.

(f) **Holding and Filtering:** The sequence of pluses is converted into a continuous analogue signal by means of an extrapolating device which "fills in" the gap between successive sample values.

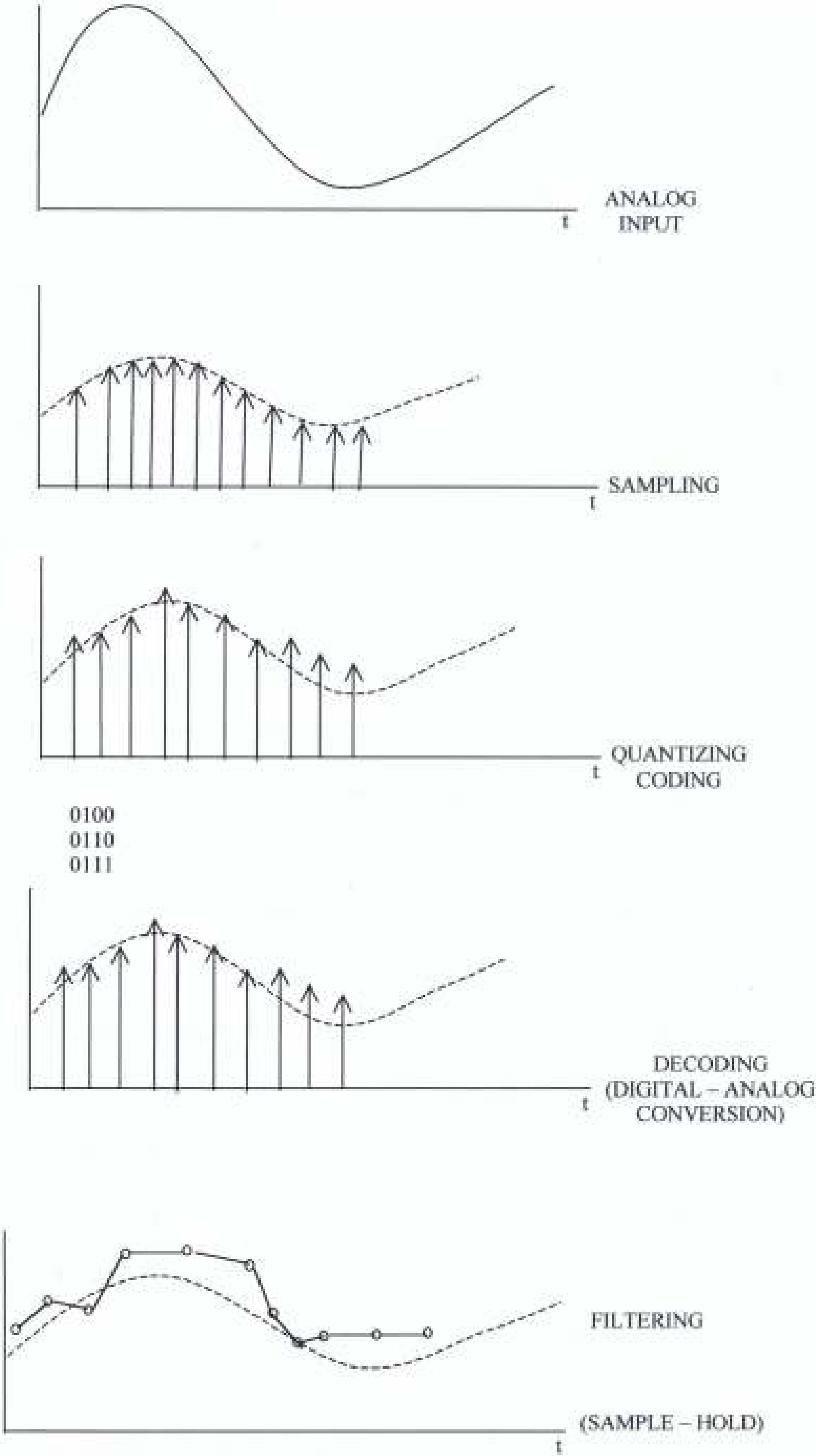


Fig 3.4 Signal Transformation in Analog/Digital Conversion

In actual interfacing system, a number of the functions listed above may be performed by a single hardware unit. For example, sampling, quatilizing, and coding are all performed in the analogue/digital converter. On the other hand, it is usually desired to process more than one analogue variable so that additional hardware is needed to permit the digital computer and portions of the interface hardware to be time-shared by a number of analogue channels. The major hardware components of a closed - loop on - line system are shown in figure 3.5, where it has been assumed that all analogue variables appear as continuos analogue voltages. They include the following:

1. **Multiplexer:** The multiplexer accepts as its input a variety of simultaneously occurring analogue voltages and emits over its single output line a sequence of rectangular pulses each having an amplitude proportional to one of the input voltages. By means of a multiplexer, the analogue outputs to be processed are sequentially connected to the analogue/digital converter input. To reduce the time between successive samples, it is occasionally feasible to use two or more analogue/digital converters, each handling a fraction of the total analogue outputs. In that case, each analogue/digital converter may have an associated multiplexer.
2. **Analogue/Digital Converters:** A device, which translates an analogue voltage pulse into the binary - coded equivalent of the pulse amplitude. The digital output of the analogue/digital converter appears in parallel from as "ones" and "zeros" stored in an array of register cells - one for each bit of the digital word. Once the conversion is complete, the number must be read out of the converter, to make room for the next conversion. Usually the converter emits a pulse as soon as the conversion is complete in order to initiate this read - out.
3. **Digital Hold (Buffer):** A device, which stores a digital number for a limited period of time. Digital computers can usually accept data only a specific instant of time, determined by the clock generator synchronizing data flow within the

computer. It is therefore necessary to make provisions for retaining or holding the analogue/digital converter output until the digital computer is ready. At the same time, it is often necessary to adapt the format of the data to that employed within the digital computer. This may involve the changing of voltage level, negative - number representation, and conversion from Gray to pure binary code.

4. **Demultiplexer (distributor):** A device for distributing data, sequentially generated in the digital computer, to a number of digital registers. This unit has a single input register which accepts a sequence of digital words, and transmits them in turn to an array of output registers, each controlling a separate digital/analogue converter. Demultiplexers therefore permit the time-sharing of the single digital output-channel. Occasionally, analogue demultiplexers are also provided. The output of a single digital/analogue converter can then be applied sequentially to a number of parallel analogue channels.

5. **Digital/Analog Converter:** A device which accepts input data in binary form and generating an analogue voltage proportional to the binary number. Frequently a number of digital/analogue converters work in parallel, each providing an analogue voltage for a different point in the analogue system.

6. **Analogue-hold (sample-hold):** A device, which stores a constant analogue voltage for a limited period of time. In order to convert analogue data in pulse or sampled from into continuous voltages, it is necessary to provide such a memory units. Also, analogue-holds are often used at the input of the multiplexer and at the inputs of the analogue/digital converters.

7. **Timing and Control:** A device which synchronizes and controls the operation of the entire loop. Frequently the timing and control function is vested in the digital computer. The digital computer then emits controls pulses, which activate the multiplexer, the analogue/digital converter, the demultiplexer and the sample-hold units. In other system , a separate clock and control units provided for this purpose.

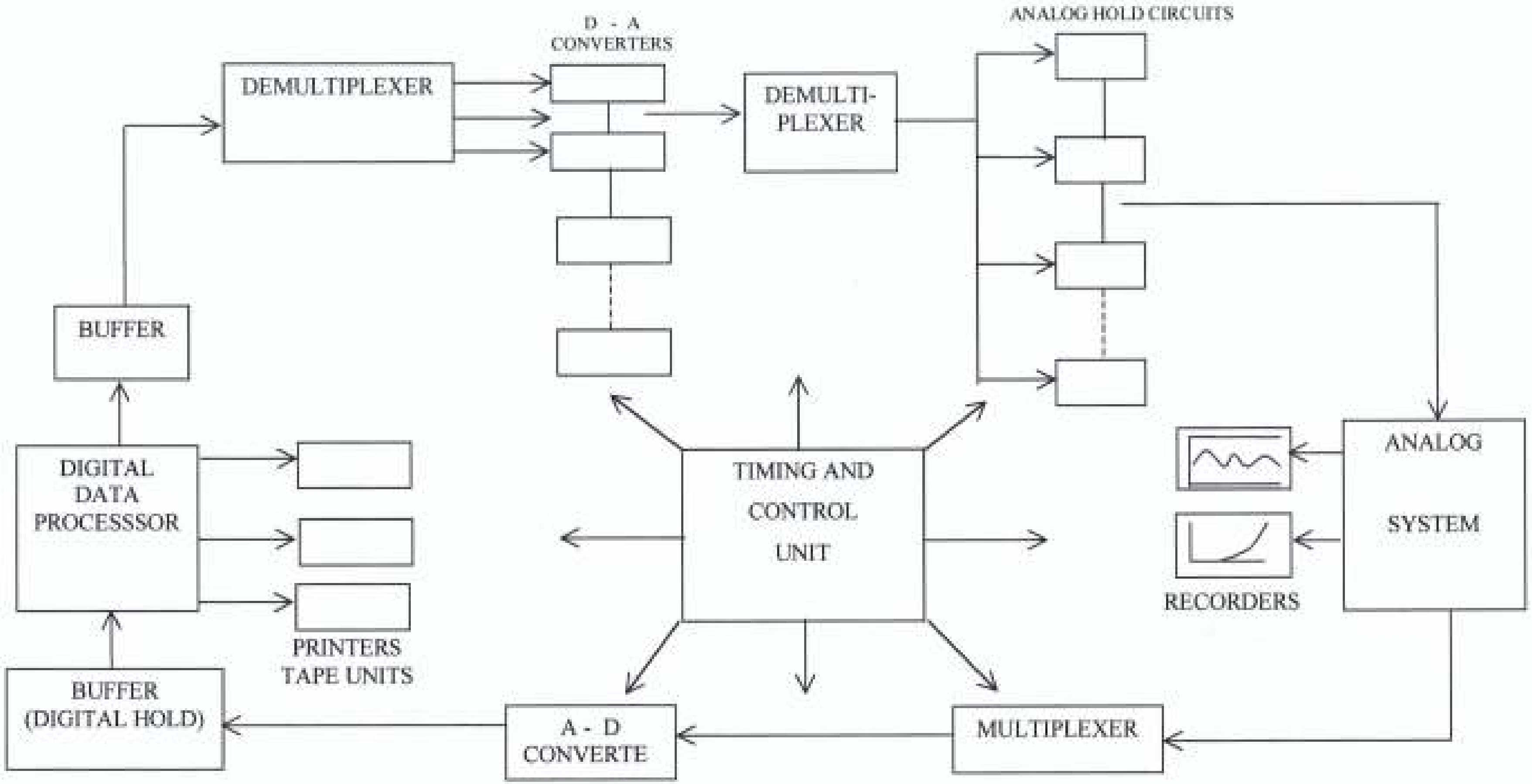


FIG 3.5 CLOSED – LOOP –IN –LINE SYSTEM.

3.10 POSITION AND VELOCITY ALGORITHMS

Two ways to describe the form of a digital control law are

(i) To select a controller transfer function $G_c(s)$ in the S domain and convert it to an equivalent $G_c(z)$ in the Z domain.

(ii) To select $G_c(z)$ directly with this approach, the question now arises as to what are discrete time equivalents of the integral and derivative actions.

Two commonly used forms are the proportional -plus -s um and the proportional - plus - different algorithms. These are

$$F(k) = K_p e(k) + K_I T \sum_{i=0}^{k} e(k) \quad (3.11)$$

$$F(k) = K_p e(k) + \frac{K_D}{T}[e(k) - e(k-1)] \quad (3.12)$$

Where f(k) and e(k) are the control and error signals and T is the sampling period. In transform notation these become

$$F(z) = [K_p + \frac{K_I T z}{z-1}] E(z) \quad (3.13)$$

$$F(z) = [K_p + \frac{K_D}{T}(1-z^{-1})] E(z) \quad (3.14)$$

These algorithms are the position versions of the PI and PD laws

The incremental or velocity versions of the algorithms determine the change in the control signal F(k) - F(k-1). To obtain these, decrement K by 1 in the above equations 3.13 & 3.14 and subtract the results.

This gives

$$F(k) = F(k-1) + K_p [e(k) - e(k-1)] + K_I e(k) \quad (3.15)$$

$$F(k) = F(k-1) + K_p [e(k) - e(k-1)] + K_I [T e(k)] + K_D [T] [e(k) - 2e(k-1) + e(k-2)] \quad (3.16)$$

In transform notation,

$$F(z) = \frac{(K_p + K_I T) z - K_p E(z)}{z-1} \quad (3.16)$$

$$F(z) = \frac{\left(K_p + \frac{K_D}{T}\right) z^2 - \left(K_p + \frac{2K_D}{T}\right) z + \frac{K_D}{T}}{z(z-1)} \quad \text{------------------------} \quad (3.17)$$

Suppose that the control signal F(k) is a valve position. The position version of the algorithm is so named because it specifies the valve position directly as a function of the error signal. The incremental algorithms on the other hand, specifies the change in valve position. The incremental version has the advantage that the valve will maintain the last position in the event of failure or shutdown of the control computer. Also the valve will not "saturate at start up if the controller is not matched to the current valve position.

3.11 DEVELOPMENT OF THE CONTROL LAW FROM THE ANALOGUE FORM.

Three common ways of converting a continuous-time model into a discrete-time form are the Euler method, the Tustin and the Z - Transform Method

The PID algorithm in analogue form is

$$F(k) = K_p e(t) + K_I \int_0^t e(t)\, dt + K_D \frac{de}{dt} \quad \text{---------------------} \quad (3.18)$$

or its equivalent $\frac{F(s)}{E(s)} = K_p + \frac{K_I}{s} K_D s$ ------------------------------------ (3.19)

Differentiating gives

$$\frac{dF}{dt} = K_p \frac{de}{dt} + K_I e + K_D \frac{d^2e}{dt^2} \quad \text{--} \quad (3.20)$$

Application of the Euler method with step size T results in a difference equation of the form.

$$F(k) = F(k-1) + K_p\,[e(k) - e(k-1) + K_I\, Te(k) + \frac{K_D}{T}[e(k) - 2e(k-1) + e(k-2)] \quad \text{--------} \quad (3.21)$$

The error is e(k) = r(k) – c(k) where r and c are the set point and output. The key step is to teat r as a constant temporarily and to write the algorithm in term of r and c. This gives

$$F(k) = F(k-1) + K_p[c(k-1) - c(k) + K_I T\,[r - c(k)] + \frac{K_D}{T}[-c(k) + 2c(k-1) - c(k-2)] \quad \text{----} \quad (3.22)$$

The transfer function is

$$\frac{F(z)}{E(z)} = \frac{a_1Z^2 + a_2Z + a_3}{Z(Z-1)} \quad \text{---} \quad (3.23)$$

$a_1 = K_p + K_iT + a_3$

$a_2 = -(K_p + 2a_3)$

$a_3 = K_D/T$

If no derivative action is present, $K_D = 0$

$$\text{Therefore } \frac{F(z)}{E(z)} = \frac{a_1Z + a_2}{z-1} \quad \text{---} \quad (3.24)$$

With $a_3 = 0$ and $a_2 = -K_p = K_iT - a_1$

The Tustin approximation is

$$\frac{Z(d^ny)}{dt^n} = [D(z)]^n\ Y(z) \quad \text{---} \quad (3.25)$$

$$\text{Where } D(z) = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$$

$$\frac{F(z)}{E(z)} = \frac{a_1Z^2 + a_2Z + a_3}{Z^2-1} \quad \text{---} \quad (3.26)$$

With $K_D = 0$ given a3 = 0

$$a_1 = \frac{K_p + K_iT}{2}$$

$$a_2 = -K_p + \frac{K_iT}{2}$$

for the Z-transform method, with $K_D = 0$

$$\frac{F(z)}{E(z)} = K_p + \frac{K_iZ}{Z-1} \quad \text{---} \quad (3.27)\ A$$

$$= \frac{(K_p + K_i)\,Z - K_p}{Z-1} \quad \text{---} \quad (3.28)B$$

$a_1 = K_p + K_i$

$a_2 = -K_p$

3.12. A STABILITY TEST

The Routh-Hurwrts criterion provides a convenient stability test can be applied to the characteristic polynomial without solving for the roots.

Originally developed to detect the occurrence of a root in the right half of the S-plane, it can be converted for use with discrete models by employing the transformation.

$$Z = \frac{s+1}{s-1} \quad \text{--} \quad (3.29)$$

This transformation maps the inside of the unit circle in the Z-plane onto the entire left half of the Z-plane. Substituting Z into the characteristic equation of S in terms of Z gives a polynomial in S to which the Routh – Hurwitz criterion can be applied. The number of sign changes in the first column of the array indicates the number of roots that lie outside the unit circle.

3.13 BILINEAR TRANSFORMATION

Many analysis and design techniques for continuous-time system such as the Routh-Hurwitz criterion and Bode techniques are based on the property that is in the S-plane the stability boundary is the imaginary axis.

Thus these techniques cannot be applied to discrete-time systems in the Z-plane, since he stability boundary is the unit circle. However through the use of the transformation.

$$Z = \frac{1+(T/2)w}{1-(T/2)\,w} \quad \text{--} \quad (3.30)A$$

or

$$w = \frac{2}{T} \quad \frac{z-1}{z+1} \quad \text{-----------------------------------} \quad (3.30)B$$

The unit circle of the Z-plane transforms into the imaginary axis of the w-plane. This can be seen through the following development.

On the unit circle in the Z-plane

$$Z = e^{jwt} \quad \text{------} \quad (3.30)C$$

$$W = \frac{2}{T}\left.\frac{z-1}{z+1}\right|_{e^{jwT}} = \frac{2}{T}\frac{e^{jwT}-1}{e^{jwT}+1}$$

$$= \frac{2}{T}\frac{e^{jwt/2} - e^{jwT/2}}{e^{jwT/2} + e^{jwT/2}} \quad \text{------} \quad (3.30)D$$

$$W = [j2/T] \tan (wt/2) \quad \text{------} \quad (3.31)$$

Thus, it can be seen that the unit circle of the Z-plane transforms into the imaginary axis of the w-plane.

If jw_w is the imaginary part of w and w_w is the w-plane frequency

$$W_w = \frac{2}{T} \tan \frac{(wT)}{2} \quad \text{------} \quad (3.32)A$$

This expression gives the relationship between frequencies in the s-plane and frequencies in the w-plane.

For small values of real frequency in the S-plane such that wT is small.

$$W_w = \frac{2}{T} \tan\frac{(wT)}{2} = \frac{2}{T}\frac{(wT)}{2} = w \quad \text{------} \quad (3.32)B$$

Thus, the w-plane frequency is approximately equal to the S-plane frequency for this case. Thus approximation is valid for those values of frequency for which tan(wT/2) = wT/2.

$$\text{For} \quad \frac{wT}{2} \leq \frac{\pi}{10} \quad w \leq \frac{2\pi}{10T} = \frac{ws}{10}$$

the error in this approximation is less than 4%

3.14. SELECTION OF CONTROLLER GAINS

Once the form of the control law has been selected, the gains must be computed in light of the performance specifications. In practice, the requirements are usually more detailed. For example, the rise time and maximum overshoot, as well as the other transient response specifications may be encountered. Requirements can be stated in terms of frequency response characteristics, such as bandwidth, resonance frequency and peak amplitude (ONI, 1997). Whatever specific form they take, a complete set of specifications for control system performance generally should include the following considerations for given forms of the command and disturbance inputs:

1. Equilibrium specification
    - (a) Stability
    - (b) Steady – state error
2. Transient specifications
    - (a) Speed of response
    - (b) Form of response
3. Sensitivity specifications
    - (a) Sensitivity to parameter variations
    - (b) Sensitivity to model inaccuracies
    - (c) Noise rejection (bandwidth, etc)
4. Non – linear effects
    - (a) Stability
    - (b) Final control element capabilities

In addition to these performance stipulations, the usual engineering considerations of initial cost, weight, maintainability etc. must be taken into account.

Two approaches exist for designing the controller. The proper one depends on the quality of the analytical description of the plant to be controlled. If an accurate model of the plan is easily developed, the approach is to design a specialized controller for the particular application. The range of adjustment of controller gains in this case can usually be made small because the accurate plant model allows the gains to be pre-computed with confidence. This technique reduces the cost of the controller and can usually be applied to electromechanical system.

The second approach is used when the plant is relatively difficult to model which is often the case in process control. A standard controller with several control modes and wide range of gains is used, and the proper mode and gains settings are obtained by testing the controller on the process in the field. Some guidelines for such tuning are considered. This approach should be considered when the cost of developing an accurate plant model might exceed the cost of controller tuning in the field.

3.15 PERFORMANCE INDICES

A performance index is a number, which indicates the "goodness" of system performance. A control system is considered optimal if the values of the parameters are chosen so that the selected performance index is minimum or maximum. The optimal values of the parameters depend directly upon the performance index selected.

3.16. REQUIREMENT OF PERFORMANCE INDEX

A performance index must offer selectively; that is, an optimal adjustment of the parameters must clearly distinguish non-optimal adjustment of the parameters (GOPAL & NAGRATH, 1993). In addition, a performance index must yield a single positive number or zero, the latter being obtained if and only if the measure of deviation is identically zero. To be useful, a performance index must be a function of the parameters of the systems, and it must exhibit a maximum or minimum.

Finally, to be practical, a performance index must be easily computed. Four common choices for an index are the following:

1. $J = \int_0^\infty |e(t)|\, dt$ - (IAE)

2. $J = \int_0^\infty t\,|e(t)|\, dt$ - (ITAE)

3. $J = \int_0^\infty e^2(t)\, dt$ - (ISE)

4. $J = \int_0^\infty t\, e^2(t)\, dt$ - (ITSE)

where e(t) is the system error. This error usually is the difference between the desired and the actual values of the output which after analysis easily indicate the optimal adjustment of the controller. However, if e(t) does not approach zero as t approach ∞ the preceding indices will not have finite values. In this case, e(t) can be defined as $e(t) = C(\infty) - C(t)$, where C(t) is the output variable. If the index is to be computed numerically or experimentally, the infinite upper limit can be replaced by the limit t_f, where t_f is large enough that e(t) is negligible for $t > t_f$.

The integral – absolute – error (IAE) criterion expresses mathematically that the designer is not concerned with the sign of the error, only its magnitude. The index says nothing about he relative importance of an error occurring late in the response versus an error occurring early. Because of this, the index is not as selective as the integral-of time multiplied absolute error criterion (ITAE). Since the multiplier 't' is small in the early stages of the response, these index weights early error less heavily than later errors. Neither criterion allows highly under-damped or highly over damped system to be optimum. The ITAE criterion usually results in a system whose step responses has a slight overshoot and well damped oscillations.

The integral squared-error (ISE) and the integral-of-time-multiplied squared-error (ITSE) criteria are analogous to the IAE and ITAE criteria, except that the square of the error is employed for three reasons:

1. In some application, the squared error represents the system's power consumption;
2. Squaring the error weights large errors much more heavily than small errors;
3. The squared error is much easier to handle analytically

The derivative of a squared term is easier to compute than that of an absolute value and does not have a discontinuity are e = 0.

The general second-order system with zero steady-state error for a unit step input has the form.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\tau\omega_n s + \omega_n^2} \quad \text{------------------------} \quad (3.33)$$

If a new time scale T is introduced such that $t = \omega_n T$,

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + \tau s + 1} \quad \text{-------------------------------} \quad (3.34)$$

The step response can easily be found in terms of the parameter τ and the various performance indices can be compute. For $e(t) = 1 - C(t)$, the ITAE index and the index.

$$J = \int_0^\infty t|e| + |\bar{e}|)\, dt \quad \text{--} \quad (3.35)$$

have the most sharply defined minima as functions of τ. For these two indices the optimum value of τ occurs at approximately $\tau = 0.7$.

3.17 THE ZIEGLER – NICHOLS RULES

The difficulty of obtaining accurate transfer function models for some process has led to the development of empirically based rules of thumb for computing the optimum gain values for a controller. Commonly used guidelines are the Ziegler-Nicholas rules (WILLIAM, 1983). The rules consists of two separate methods.

The first method requires the open-loop step response, while the second method utilize the results of experiments performed with controller already installed. Using the IAE criterion with a unit response, they found that controllers adjusted according to the following rules usually had a step response that was oscillatory but with enough damping so that the second overshoot was less than 25% of the first (peak) overshoot.

The first method is the process reaction method and relies on the fact that many processes have an open-loop step response like that shown in fig 2.5.

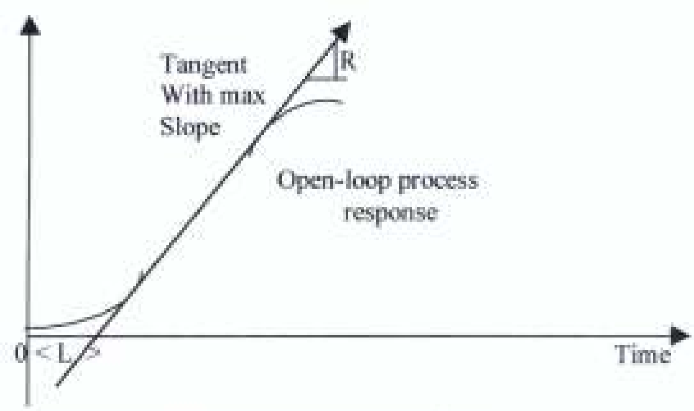


Fig 3.6 Open-loop response Curve

This is the process signature and is characterized by two parameters, R and L. R is he slope of a line tangent to the steepest part of the response curve, and L is the time at which this line intersects the time axis. First-and second-order linear systems do not yield positive values for L, and so the method cannot be applied to such systems.

The second method is he ultimate-cycle method and this utilizes experiments with the controller in place. All control modes except proportional are turned off, and the process is started with proportional gain K_p set at a low value. The gain is slowly increased until the process beings to exhibit sustained oscillations. Denote the period of this oscillation by Pu and the corresponding ultimate gain by K_pu.

The Ziegler-Nichols recommendations are given in table 2.1

TABLE 3.1 THE ZIEGLER – NICHOLS RULES

Controller transfer function $G(s) = K_p (1 + 1/T_is + T_Ds)$		
Control mode	Process reaction method	Ultimate cycle method
P Control	$K_p = 1/RL$	$K_p = 0.5k_pu$
PI Control	$K_p = 0.9/RL$ $T_I = 3.3L$	$K_p = 0.45\ K_pu$ $T_I = 0.83$ pu
PID Control	$K_p = 1.2/RL$ $T_I = 2L$ $T_D = 0.5L$	$K_p = 0.6K_p$ $T_I = 0.5$ Pu $T_D = 0.125$ Pu

CHAPTER FOUR

THE CRUDE DISTILLATION UNIT

4.1 THE PRESSURE CONTROL LOOP

The Warri refining and petrochemical company is a subsidiary company of the Nigerian National Petroleum Corporation, a parastatal of the Federal Ministry of Petroleum Resources. The Company is divided into two sub-units. The Refinery Plant and the Petrochemical Plant. The Refinery plant is involved in the refining of crude while and Petrochemical plant is involved in the production of petrochemical products. The crude distillation unit is a unit in the refinery plant and the control loop for the entire system is as shown in fig 4.1

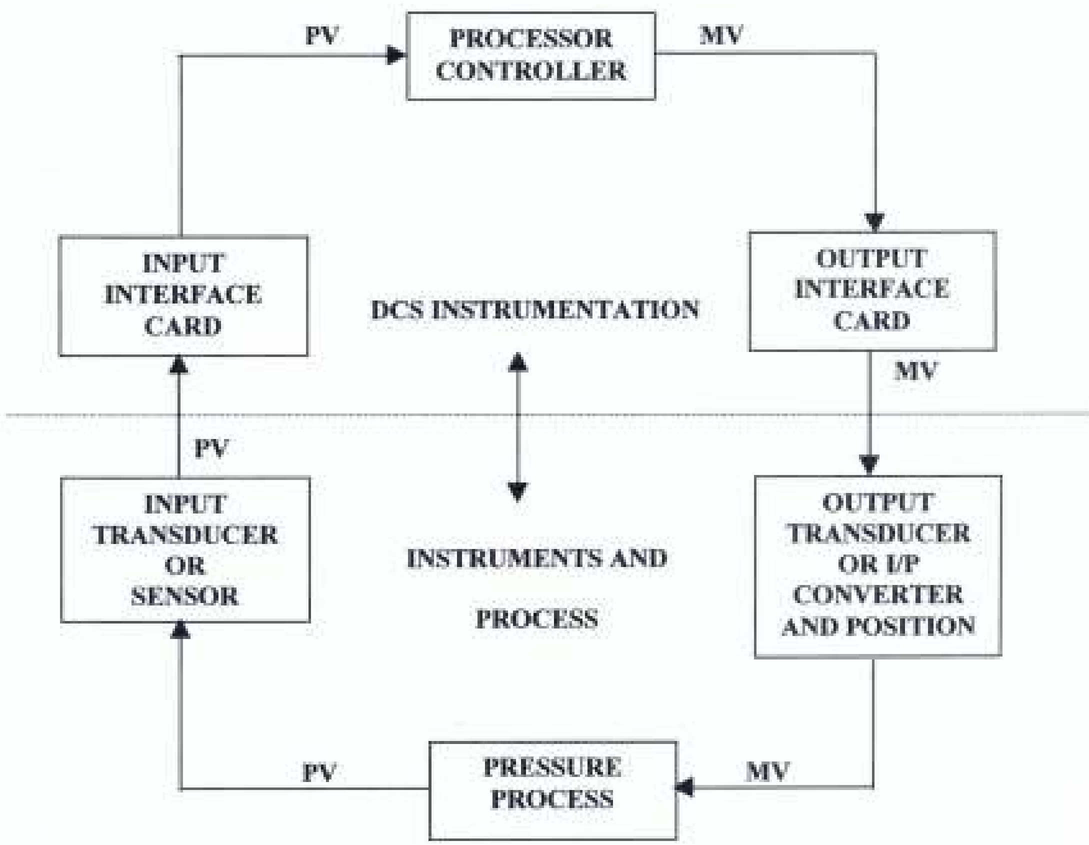


Fig 4.1 Feedback pressure control loop block diagram.

PROCESSOR CONTROLLER: This block is a representation of the component in a field control station (FCS) of the distributed control system (DCS) that performs PID control computation. This component of the direct digital control (DDC) loop is actually a microprocessor in the FCS with many other capabilities such as sequential control and computational functions.

INPUT INTEFACE CARD: This block performs signal conditioning and conversion from analogue to digital signal. It is the representation of some component in the field control station (FCS) of the Distributed Control System (DCS).

OUTPUT INTERFACE CARD: It does output signal conditioning and conversion from digital to analogue signal. It represents some of the components in the FCS of the DCS.

INPUT TRANSDUCER: This include the primary element (sensor), the diaphragm cell, that measure or sense the pressure variations along the process line. It generates a signal and sent the signal to a transmitter. The transmitter conveys the signal in the form of current to the input interface card of the DCS.

OUTPUT TRANSDUCER: This consists of the positioners, control valves and the current-to-pneumatic (I/P) converter. The I/P converter converts current to pneumatic signal. The pneumatic signal, after amplification by the positioner is used to position the stem of the control valve. This block is collectively called the final control element.

PRESSURE PROCESS: This pressure process could be analysed in term of its input and output. The input is the variable that influences the status or conditions of the process which is the manipulated variable (MV) that manipulates the final control element. It is, therefore, a net flow of fluid into the pressure process. The output is the variable which change or can change as a result of the changes in the input variable to the pressure process. This output is the column pressure or line pressure.

4.2 FINAL CONTROL ELEMENT

The final control element consist of the positioner (The positioner receives low level pneumatic signal between 3-15psi from the I/P converter). This signal communicates with the desired position of the control valve and through a feedback linkage, the positioner compares the actual valve's stem position to the desired position and adjust the supply pressure sent to the control valve's actuator until the desired position is attained, the I/P converter and the control valve.

The I/P converter receives a 4~20mA signal from the DCS output interface card and converts it to a 3~15psi signal. Current is used and not voltage because load variation does not easily affect current change; voltage variation results in noise. The internal control valve within the I/P converter is regulated by the 4~20mA signal to deliver corresponding 3~15psi signal to the positioner.

The control valve with the positioner as described above, together with the output of the DCS the manipulated variable (MV) can be viewed as simplified in Figure 4.2

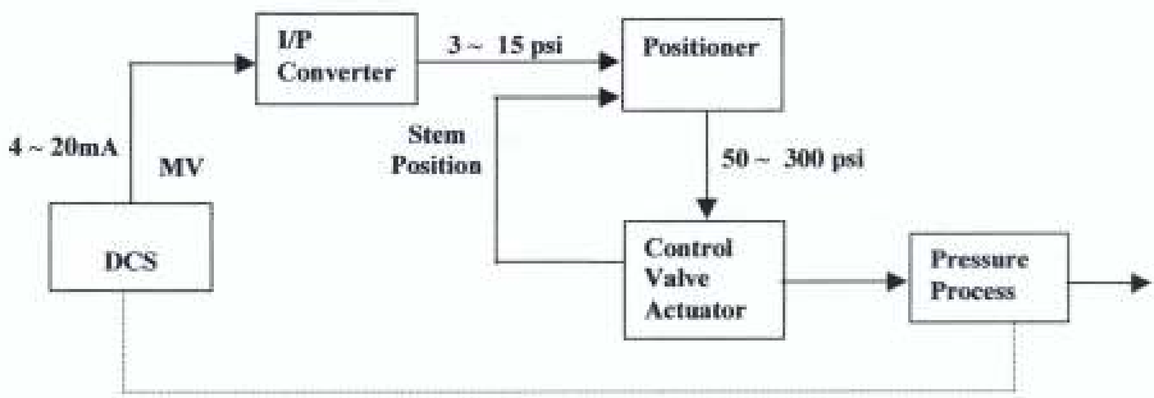


Figure 4.2 Final Control Element

The positioner is the high gain proportional device whose output causes the control valve to move. The final control element as an entity, does contribute to the delay in the pressure control loop because of the inertial of the mechanical parts.

4.3 ACTION OF THE CONTROLLER IN THE PRESSURE PROCESS.

The software PID controller within the DCS microprocessor under the influence of the software programme regulates the pressure of the column to ensure that it is always very close or equal to the value desired by the operator. The operator sets a pressure set point

(SV), the controller compares this compares this value with the process variable (PV) and automatically adjust the manipulated variables (MV) output (adjust opening of valves which controls the gas flow within the lines) so as to minimize the deviation between measured (pressure) process variable and the target value (SV). This is called feedback control.

Figure 4.4 is a representation of the pressure process flow diagram. The relationship between the pressure controller and the two control valves is shown in figure 4.3.

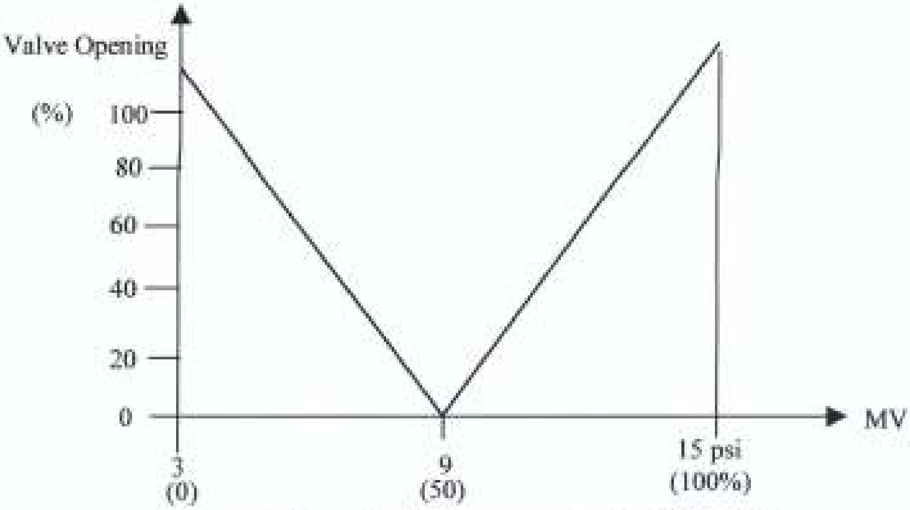


Figure 4.3 Split range control of 10 PC021

The process of figure 4.4 is aimed at separating (C3 ~ C4) from C5 {stabilizing between gas (C3 & C4) and liquid (C5)}. This is being achieved by distillation process inside the column 10C05 under certain temperatures and pressures around the column, which must be maintained constant to have quality products of C3 ~ C4 and C5.

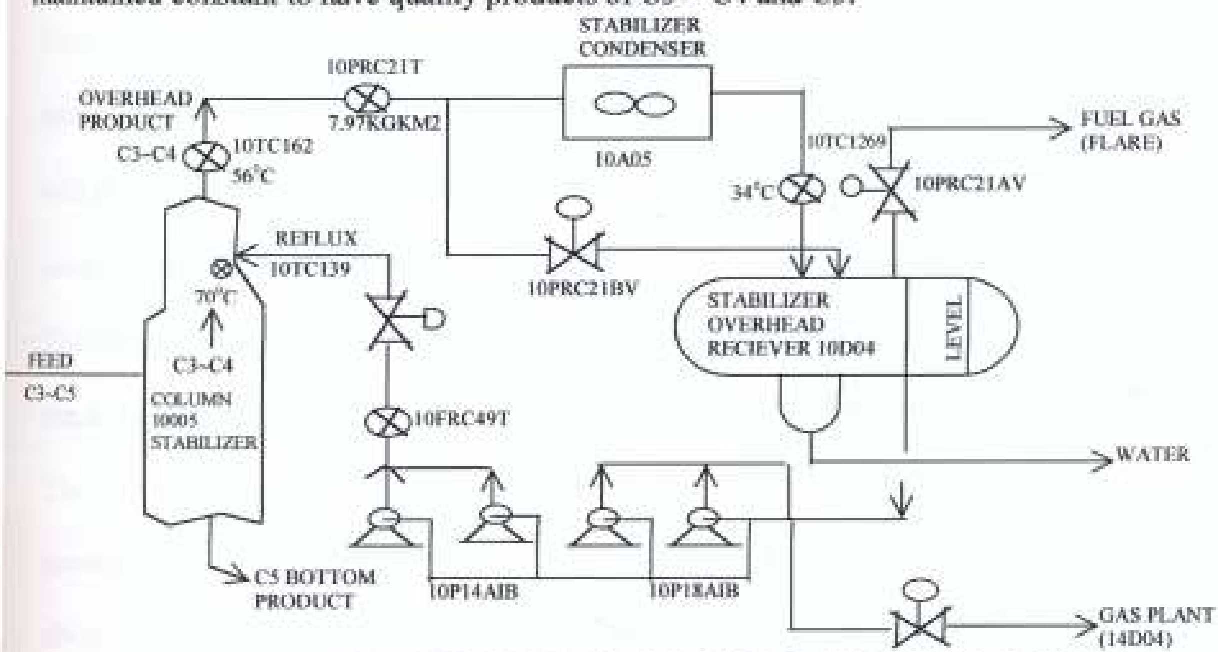


FIG. 4.4. PROCESS FLOW DIAGRAM FOR PRESSURE CONTROLLER 10PC021

To get qualitative C3 ~ C4 at the top of 10C05 the top of the column must be maintained at a pressure and temperature of 7.98kg/cm2 and 70^0C respectively. The reflux flow control via 10FRC49V and the reboiler heat energy control from the bottom of the column maintains the to temperature at 70^0C. The pressure, which is our focus, is maintained at 7.98kg/cm2 by the action of 10PRC21Bv and 10PRC21AV with the complement of the reboiler heat energy and the reflux flow under control.

The C3 ~ C4 gas produced at the top is condensed by 10A05 into liquefied form and sent to 10D04 to build up level and at the same time, the control valve 10PRC21BV permits a fractional portion of it to flow as gas into 10D01. The liquefied gas is then sent to the gas plant (14D01) to separate C3 from the LPG Butane (C4). The control valve 10PRC21AV is expected to open slightly and if possible remain closed so that C3 ~ C4 gas will not go to flare. Those going to the flare are counted as a loss. If the top of 10C0S is at a pressure higher than normal, valve 10PRC21AV opens to bleed it to the flare line to keep the pressure normal.

Note that if the two control valve (10PRC21AV and BV) are closed manually (10PC021 automatically control off) or only 10PRC21AV is made to close the top pressure of 10C05 will rise exponentially with negative effects on the quality or composition of the C3 ~ C4 gas product and if it rises beyond the designed pressure of the system, the column will collapse. The opening of the valves 10PR21AV and BV under the control of 10PC021 is to ensure that the top pressure is always equal to that desired by the plant operations, which is 7.98kg/cm^2. The valve 10PRC21AV will be fully closed and 10PRC21BV will be fully opened if the pressure is too far below the set point and vice versa if otherwise. The pressure does not always stay at 7.98kg/cm^2 due to external disturbances from the feed to 10C05 and also from both the reflux flow and the reboiler heat energy generation. The 10PC021 controller is always controlling as long as pressure deviates from the target value set by the operator. A good control system will always maintain a very small deviation, say $\pm$ 0.1% deviation or say $\pm$ 0.4% in some cases depending on the intensity of the external disturbances. Slight drop in pressure at the top of the 10C05 column can destabilize the system in terms of quality and quantity of the product C3 ~ C4 gas and C5. This in turn will have cascaded effect on other units of the plant.

The disturbance introduced (a decease in set point by 0.5kg/cm^2) which, of course, distablized the whole system was deliberate. The step change was carried out to see the control action of 10PC021 around the pressure process.

4.4 INPUT AND OUTPUT INTERFACE CARDS

For the sake of clarity consider the direct digital control (DDC) loop for the controller shown in figure 4.5

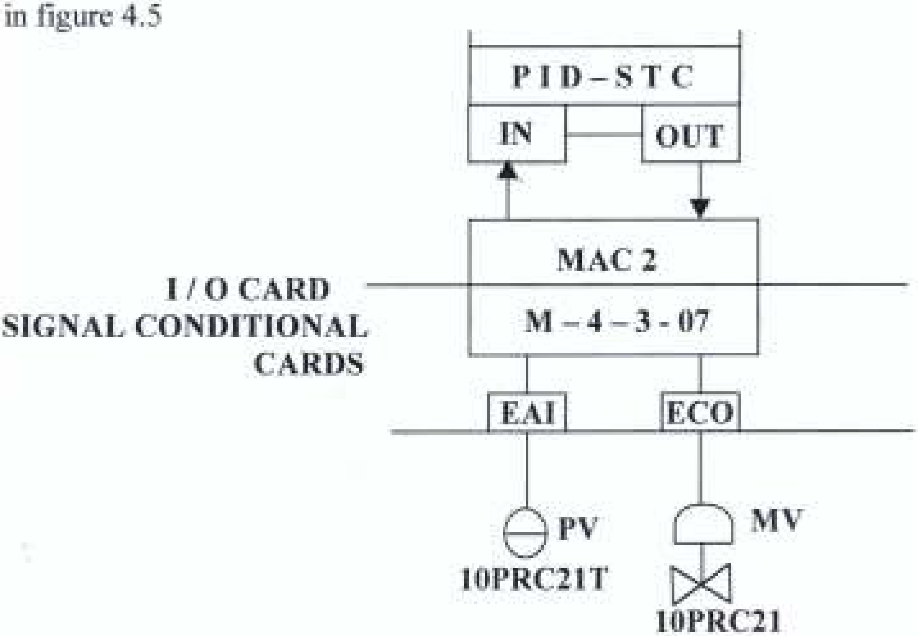


FIG 4.5 10PC021 DDC loop

The EA1 and EC0 cards are the input and output signal conditioner cards respectively. The MAC2 card takes both the input and output signals. It is called the input/output (I/O) card.

The EA1 card supplies DC power to the electronic transmitter 10PRC21T (24V DC) powering the amplifier circuit of the transmitter. At the same time it receives the 4 ~ 20m A DC signal form the transmitter and converts it to 1~ 5V DC signal for MAC2 card to receive . The EA1 card can be viewed to be in the form shown in figure 4.6 without consideration for the 24V DC powering circuit section. It standardize signals from the field or process to 1 ~ 5 DC.

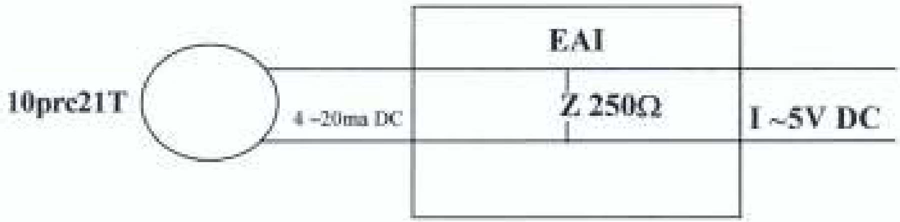


FIG. 4.6. CONVERSION ACTION OF EAI CARD.

The ECO does not supply DC power at all, it only conditions the signal (4 ~ 20mA DC) from the MAC2 card output section to perfectly goes through to the field or process without the field or process condition interfering with the MAC2 card and the rest of FCS performances. Hence there is always a fuse in the circuit or ECO to provide isolation in case of any short circuit.

In summary, the basic functions of the following signal conditioner card are:

* They provide signal isolation by isolating the DCS system from noise from the process plant or field.
* They linearize and/or normalize the signals from the field to 1 ~ 5V DC.

MAC2 card controls 8 analogue loops. There is no isolation between the input and output signals. The major components of the cards are:

(a) NI-bus interface is the circuit linking the MAC2 card 1/0 signals with the FCS processor that performs the PID Computation.

(b) Microprogram control section is that unit that co-ordinate the movement or the data signals to and from the FCS processor in addition to redundancy control co-ordination under the influence of the main FCS processor.

(c) Input/output data sections.

(d) D/A and A/D converters

(e) Input/output circuits, and

(f) Dual redundancy control section.

The internal configuration of the MAC2 card is shown in figure 4.7

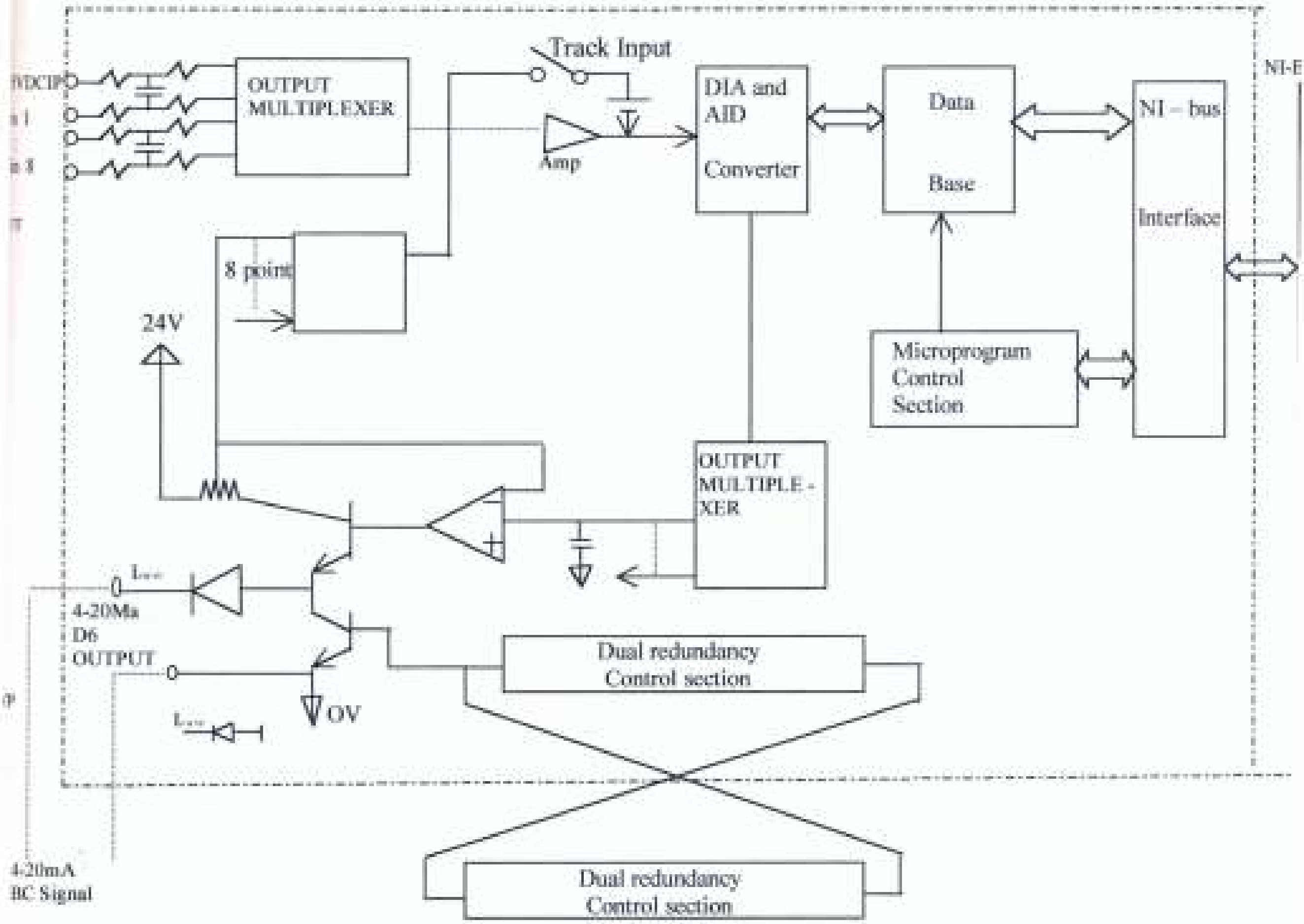


FIG 4.7. MAC2 card internal configuration

There are 8 channels in MAC2 cards for a control loop. According to figure 4.5, the channel number used for 10PC021 control loop is channel 07 obtainable from the address M-4-3-07.

4.5 PROCESSOR CONTROLLER

The processor controller is the microprocessor in field control station (FCS) that performs the PID control computation. This microprocessor, in this case of centum –XL DCS handles a maximum of 255 feedback control instruments for a maximum of 80 control loops. Each loop is processed inorder.

The control for a loop is performed in the sequence:

Red ("Scan") input → Perform Control Computation → Update Control Output.

The time period between these cycles for a loop including the alarm –check is called the "Scan period" of that loop. In this case of centum – XL DCS, the field control station (FCS) for all the 80 loops has a scan period of 'one and second'.

The centum – XL FCS uses a motorola microprocessor MC68030) and it is dual-redundant. The ROM does the actual PID control with the database in the RAM.

The ROM has a size of 64KW while the RAM is 76KW in size where 1W = 16bits.

For better understanding of the processor controller in relation to other blocks, consider a DCS system taken centum – XLDCS as a case study.

4.6 DISTRIBUTED CONTROL SYSTEM (DCS)

The distributed control system is the distribution of control among several processors independent of each other in operation. There is considerable confusion between a computer network and a D.C.S. In a D.C.S, the existence of multiple autonomous computes is transparent to the user. It can type a command to run a programme and it runs. It is up to the operating system to select the best processor, find and transport all the input files to that processor and put the results in the appropriate place. The user of a D.C.S is not aware that there are multiple processors (TANENBAUM 1985).

These instruments are simulated by software as they do not exist physically hence their operation in relation to the process plant is referred to as "CRT Operation". The basic architecture of a DCS consists of an operator station (OPS) where CRT operations is carried out, a control station (Ops) where CRT operation is carried out, a control station (FCS) and a communication system. See figure 4.8a.

The operator station collect data relating to the process plant operations via the data highway called HF-Bus and can display and manipulate these data. The other components or peripherals of the OPS are the printers, hardcopy unit, operator consoles etc.

The field control station (FCS) contains control functions thereby performing process and communication control via the data highway bus. There are limits to the number of control loops a control station can process, in this case 80-loops.

The communication system exchange data at high speed between the operator station (OPS), control station (FCS) and other stations via the data highway called the HF-Bus.

Based on he above, we can conclude that DCS has four interfaces depicted in figure 4.8a.

The process interface is that part linking the DCS with the plant's measurement sensors and final control elements. The DCS receive filed signals (PVS) via the signal conditioners EA1 card through the MAC2 input/output card for the microprocessor of the FCS to handle, checks, computer and update an output value. The update value goes back to the plant final control element via he same MAC2 input/output card and the output signal conditioner ECO card.

The man-machine interface is that between the plan operators and the DCS where central monitoring and CRT operations are carried out.

The sub-system interface is that connecting the DCS with other sub-systems (for example, analyzers, or any other PC based system) for plant control and operational integration.

The Engineering interface permits the communication between an engineer and the DCS for system build-up and maintenance purposes.

The Gateway in figure 4.8 permits the DCS system to be able to communicate with other systems of different communication protocol.

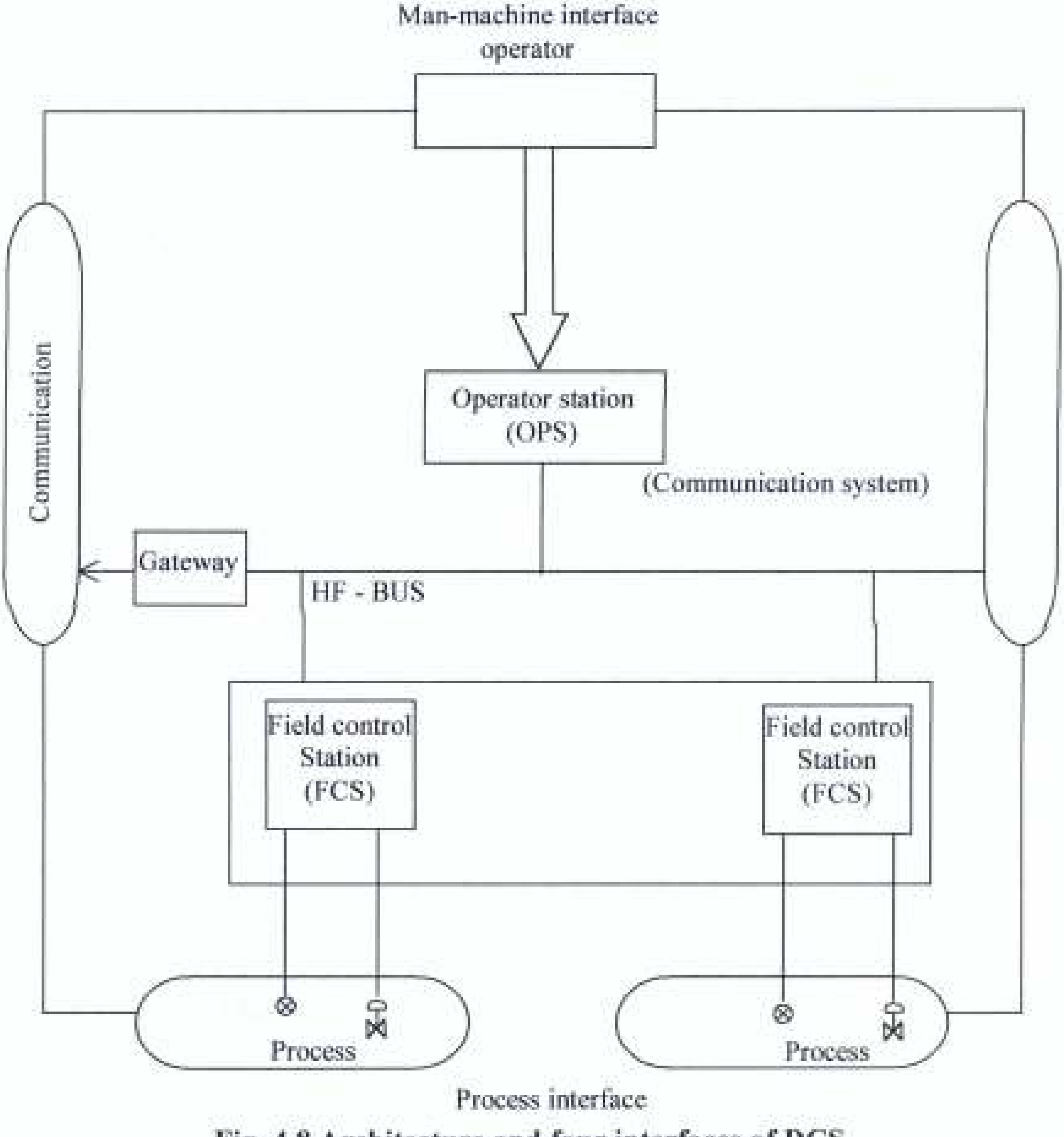


Fig. 4.8 Architecture and four interfaces of DCS

4.7 CONFIGURATION OF A FIELD CONTROL STATION (FCS)

The FCS consists mainly of a station control nest (SCN) the 1/0 nest and the power distribution unit at the front and the signal conditioner nest and their power supply unit at the rear. (Figure 4.9 & figure 4.10)

FRONT

1/0 NEST1	Station Control Nest (SCN)	1/05 NEST
	1	
	2	
	3	
	4	
	5	
	Power distribution Unit	

(a)

Signal	**Power Supply Unit**				12 Signal
Conditioner	1	2	3	4	Conditioner
	5	6	7	8	
Nest 2	9	10	11	12	Nests

REAR

(b)

Signal Conditioners nest 1

EA1	Slot 1
ECO	Slot 2
EA1	Slot 3
:	
	Slot 6

(c)

Figure 4.9 a, b, c The Configuration of a FCS 1

1/0 NEST 1

M A C 2 Slot 1	**M A C 2 Slot 2**				**N C 4**	**N C 4**	**P S 11**	**P S 11**

NC4 = Nest communication card

PUS = Power supply unit

Station Control Nest (SCN) (a)

Left					Right			
PS 33	**SI 11**	**FC 11**	**CP 11**	**DX 11**	**CP 11**	**FC 11**	**SI 11**	**PS 33**

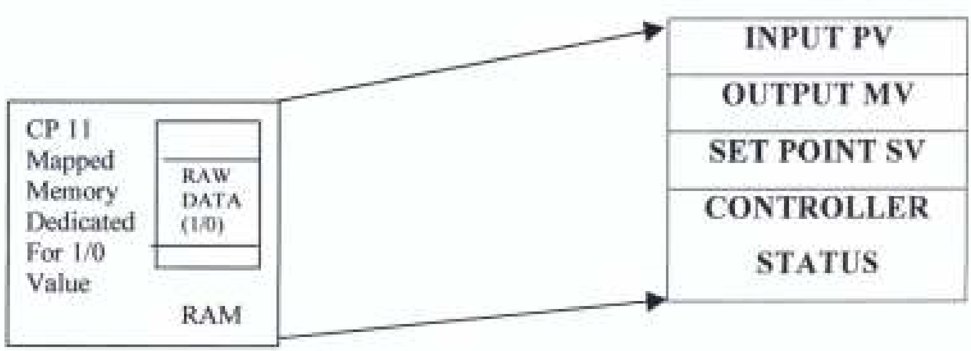
PS 33 = Power Supply Unit

SI 11 = Station Internal Control Card

FC 11 = HF – Bus Controller

CP 11 = Processor Controller

DX 11 = Duplex Control Card



#F8020

Values put in these memory locations are in hexadecimal

Figure 4.10 a, b Configuration of FCS II

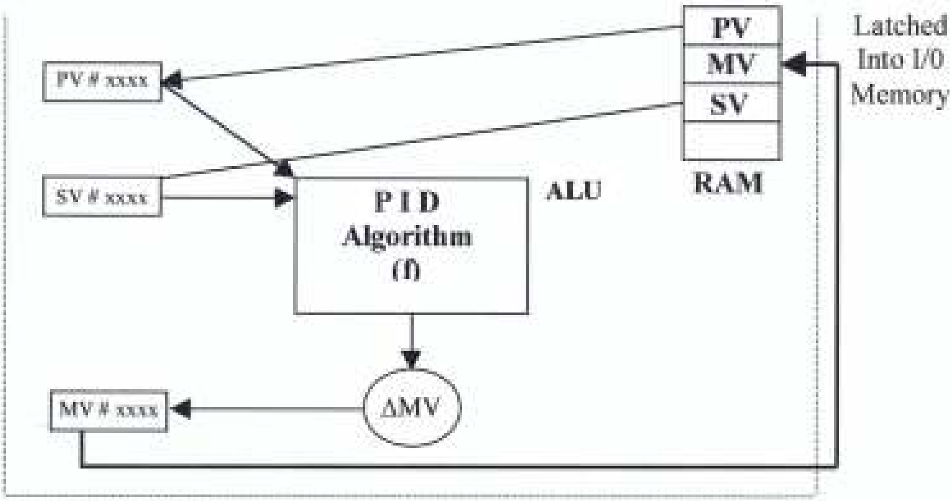


Fig. 4.11. CPU

The CPU fetches PV value from 1/0 memory address. The fetched data is processed within ALU. The result is written or updated into the memory.

Read | Process fetch data | Write result or update

Fetch | WITHN 1 SECOND |

The 1/0 nest contains interfaces cards for input/output which is the MAC2 card handling 8 control loops. It is based on dual redundancy philosophy.

The link between the SCN and 1/0 Nest is through the SI 11 card, NC4 cards and the SI-bus itself that directly links the two cards, permitting communication between the processor controller (CP 11) and the MAC 2 card. The NC4 card is also duplexes.

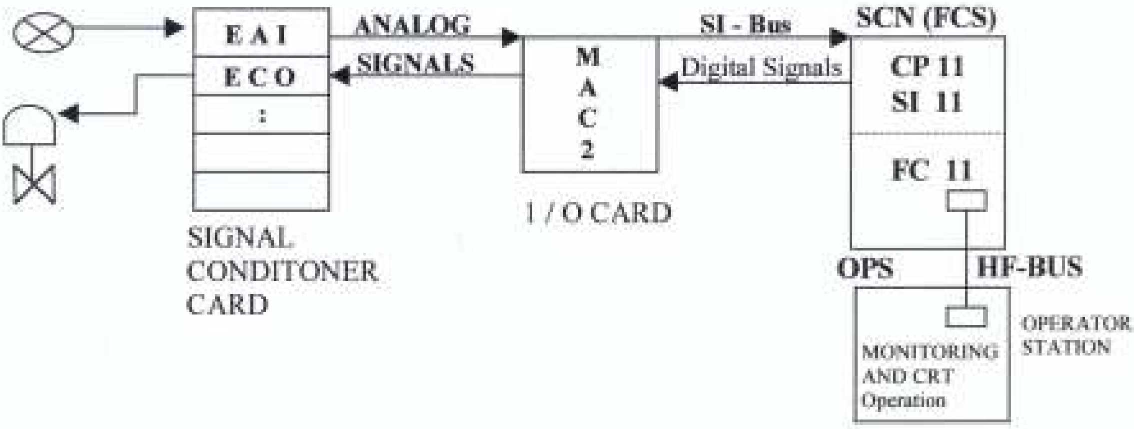


Fig. 4.12. Signal flow diagram of a DCS for process control. (Centum – XL DCS)

The HF-Bus communication rate of 1 Mbps with a transmission distance of 2km. The control transmission method is by baton or token passing method. It is a half-duplex communication method.

4.8 PID ALGORITHM OR THE DCS

The PID control computation within the FCS employs a velocity algorithm to avoid reset windup and bumpless transfer.

Integral action may be thought of as forcing the proportional band to shift, and hence, causing a new controller output for a given relationship between measurement and set point. Integral action will continue to shift the proportional band as long as a difference exist between measurement and set point. Integral action has the same function as an operator adjusting the bias in the proportional only controller.

The width of the proportional band remains constant, and is shifted in a direction opposite to the measurement change. Thus, an increasing measurement signal results in a decreasing output and vise versa.

If the controller is unable to return the measurement to the set point, integral action will drive the proportional band until its lower edge coincides with the set point, hence, a 100 percent output. If the measurement should than start to come back within control range, it must cross the set point the enter the proportioning range, and cause the output to begin throttling. If the system has any substantial capacity, the measurement will overshoot the set point.

This basic limitation is called integral windup (NORMAN, 1980). It is seriously considered on discontinuous or batch process where it is common for the controller output to become saturated and overshoot.

The positional-type algorithm (based on theory) is expressed by:

$$MV(t) = \frac{100}{P}\left\{E(t) + \frac{1}{I}\int E(t)dt + D\frac{dE(t)}{dt}\right\} \text{ ----------------- } \quad (4.01)$$

The transformation and differentiation of equation (1) above for a voltage or current analogue signal sent from a transmitter to the signal conditioner card EA1 and digitized by the input card MAC2 are:

$$\Delta MV_n = \frac{100}{P}\left\{\Delta E_n + \frac{\Delta T}{I} E_n + \frac{D}{\Delta T} \Delta(\Delta PV_n)\right\} \text{ --------- } (4.02)$$

$$\Delta MV_n = \frac{100}{P}\left\{\Delta PV_n + \frac{\Delta T}{1} E_n + \frac{D}{\Delta T} \Delta(\Delta PV_n\right\} \text{ ---------- } (4.03)$$

Equation (4.02) and (4.03) are velocity type algorithm

For this 10PC021. Equation (4.03) was selected or configured on the system. No derivative term was used, I was purely a PI control, so we have

$$\Delta MV_n = \frac{100}{P}\left\{\Delta PV_n + \frac{\Delta T}{I} E_n\right\} \text{ --------------------- } \quad (4.04)$$

Where ΔMV_n : The amount of control output change

E_n	=	SV_n	-	PV_n	:	Deviation
ΔE_n	=	E_n	-	E_{n-1}	:	Change in deviation
P,I, D					:	P,I,D, parameter (proportional, band, integral time and derivative time
ΔT					:	Control period

The control period for this loop 10PC021 was set by the engineer during designing to 1 second P is expressed in % while I and D are expressed in seconds.

CHAPTER FIVE

ANALYSIS AND OPTIMIZATION

5.1 DEVELOPMENT OF THE SYSTEM EQUATION

To develop the algorithm for the controller (i) consider a technique for digital – filter integration. The trapezoidal technique is used to accomplish the digital integration. Another technique is the Simpson's rule.

Let M(kT) be the numerical integral of the signal e(t)

The value of the integral at t = (k + 1) T is equal to the value at kT plus the area added from kT to (k + 1) T.

$$\therefore M\{k+1)T\} = M(kT) + [T/2]\{e(k+1)T\} + e(kT)\} \quad (5.1)$$

Taking the Z – transform,

$$zM(z) = M(z) + [T/2]\{zE(z) + E(z)\}$$

$$\therefore \frac{M(z)}{E(z)} = \frac{T}{2}\frac{[z+1]}{z-1} \quad (5.2)$$

(ii) Now consider a technique for the digital – filter differentiation of the signal e(t). Let the numerical derivative of e(t) at t = kT be M(kT)

$$\therefore M(kT) = \frac{e(kT) - e\{(k-1)T\}}{T} \quad (5.3)$$

$$\therefore \frac{M(z)}{E(z)} = \frac{z-1}{Tz} \quad (5.4)$$

The transfer function of a PID controller using the integration (i) and differentiation (ii) transfer functions developed above is given by

$$D(w) = k_p + \frac{k_I}{w} + k_{pw} \quad (5.5)$$

W is the bilinear transformation from the Z – plane given earlier in the thesis Equation 3.30B.

Or, $$D(z) = k_p + \frac{k_I T}{2}\frac{(z+1)}{z-1} + \frac{k_D}{T}\frac{(z-1)}{z} \qquad (5.6)$$

$$D(z) = \frac{k_p(z-1)(z) + (k_I T/2)\ (Z+1)(z) + (k_D/T)(z-1)(z-1)}{(z-1)(z)}$$

$$D(z) = \frac{k_p(z^2-z) + (k_I T/2)(z^2+z) + (k_D/T)\ (z^2-2z+1)}{z^2-z}$$

$$D(z) = \frac{(k_p + k_I T/2 + k_D/T)z^2 + (-k_p + k_I T/2 - 2k_D/T)z + k_D/T}{z^2-z} \qquad (5.7)$$

$$D(Z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2}}{1 + b_1 Z^{-1} + b_2 Z^{-2}} \qquad (5.8)$$

where $a_0 = k_p + \frac{k_I T}{2} + \frac{k_D}{T}$

$a_1 = -k_p + \frac{k_I T}{2} - \frac{2k_D}{T}$

$a_2 = \frac{k_D}{T}$

$b_0 = 0,\ b_1 = -1,\ b_2 = 0$

5.2 STATE VARIABLES REPRESENTATION

The transfer function of the controller is derived as:

$$D(z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2}}{1 + b_1 Z^{-1} + b_2 Z^{-2}} \qquad (5.9)$$

This expression can be written as:

$$\frac{Y(z)}{U(z)} = D(z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2}}{1 + b_1 z^{-1} + b_2 Z^{-2}}\ \frac{E(z)}{E(z)} \qquad (5.10)$$

Where

$Y(z)$ = Output response

$U(z)$ = Input response

$E(z)$, auxiliary variable = Deviation function

$\therefore Y(z) = (a_0 + a_1Z^{-1} + a_2Z^{-2}) \quad E(z)$ ---------------------------------- (5.11)

$\therefore U(z) = (1 + b_1Z^{-1} + b_2Z^{-2}) \ E(z)$ ---------------------------------- (5.12)

from the real translation theorem of Z – transform,

$Z^0E(z) \rightarrow e(k)$ -- (5.13)a

$Z^1E(z) \rightarrow e(k+1)$ -- (5.13)b

$Z^2E(z) \rightarrow e(k + 2)$ -- (5.13)c

$Z^{-1}E(z) \rightarrow e(k-1)$ -- (5.13)d

$Z^{-2}E(z) \rightarrow e(k-2)$ -- (5.13)e

The state variable can then be defined as

$x_3(k) = e(k)$ -- (5.13)f

$x_2 (k) = e(k-1)$ -- (5.13)g

$\therefore \quad x_1k) = e(k-2)$ -- (5.13)h

$x_2(k) = x_1(k + 1) = e(k-1)$ -------------------- (5.13)i

$x_3(k) = x_2(k+1 = 2(k-1)$ -------------------------- (5.13)j

the state equations of the controller are"

$x(k+1) = x_2(k)$ -- (5.13)k

$x_1(k+1) = x_2(k)$ -- (5.14)

$x_2(k+1) = x_3(k)$ -- 5(.15)

$x_3(k+1) = x_4(k)$ --(5.16)

The matrix representation is

$$\begin{pmatrix} \{x_1 (k+1)\} \\ \{x_2 (k+1)\} \\ \{x_3(k+1)\} \end{pmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{pmatrix} \{0 & 1 & 0\} \\ \{0 & 0 & 1\} \\ \{b_0 & -b_1 & -b_2\} \end{pmatrix} \begin{pmatrix} \{x_1(k)\} \\ \{x_2(k)) \\ (x_3(k)\} \end{pmatrix} \begin{matrix} + \\ + \\ + \end{matrix} \begin{bmatrix} \{0\} \\ \{0\} \\ \{1\} \end{bmatrix} U(k)$$

or $x(k+1) = Ax(k) + BU(k)$ -------------------------------------- (5.17)

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

The output equation obtained is

$$Y(k) = \{a_0 \quad a_1 \quad a_2\} \begin{pmatrix} \{x_1(k)\} \\ \{x_2(k) \\ \{x_3(k\} \end{pmatrix}$$

$$\text{or } Y(k) = C\,x\,(k) \quad \text{------------------------} \quad (5.18)$$

for a given initial state x(0) and the input vector sequence [U(0), U(1), ...], the sequence [X (0), X (1), ...] can be computed from the state equation of the system, iteration. A closed form solution of the state equation is evaluated as follows:

$$X(k+1) = A\,x\,(k) + Bu(k)$$

$$X(1) = A\,x\,(o) + Bu\,(o)$$

$$X(2) = A\,x\,(1) + Bu\,(1)$$

$$= A\,[A\,x\,(o) + Bu\,(o)\,] + Bu\,(1)$$

$$= A^2\,x\,(o) + ABu\,(o) + Bu\,(1)$$

$$X\,(3) = A\,x\,(2) + Bu\,(2)$$

$$= A[A^2\,x\,(o) + ABu\,(o) + Bu\,(1)] + Bu\,(2)$$

$$= A^3\,x\,(o) + A^2\,Bu\,(o) + ABu\,(1) + Bu(2)$$

$$X\,(k) = A^k\,x\,(o) + A^{k-1}\,Bu\,(o) + A^{k-2}\,Bu\,(1) + \ldots + A^0\,Bu\,(k\text{-}1)$$

$$= Ak\,x\,(o) + \sum_{i=0}^{k-1} A^{k-i-1}\,Bu\,(1)\,. \quad \text{------------------------} \quad (5.19)$$

For $k > O$,

Define $\phi\,(k) = A^k$, and

$\phi(o) = I$ = Identity matrix

$$\therefore x(k) = \phi(k)\, x(o) + \sum_{i=0}^{k-1} A\, \phi(k-i-I)\, Bu(i)$$

$\phi(k)$ defines the state transition matrix for the discrete time system.

$$\therefore \phi(k) = Z^{-1}\,[(zI - A)^{-1}z] \qquad (5.20)$$

The state transition matrix is the inverse Z – transform of n x matrix $(z\,I - A)^{-1}z$.

$$(zI - A) = \begin{pmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} z & -1 & 0 \\ 0 & z & -1 \\ 0 & 1 & z \end{pmatrix}$$

$$\Delta(zI - A) = z(z^2 + 1) = \text{determinant} \qquad (5.21)$$

The inverse of nxn matrix $(z\,I - A)$ is given as

$$(zI - A)^{-1} = \frac{\text{adj}\,(zI - A)}{\Delta} \qquad (5.22)$$

where adj $(z\,I - A)$ defines the adjoint of the nxn matrix $(zI - A)$ and Δ is the determinant of the matrix.

$$\therefore \quad \text{adj}\,(z\,I - A) = \begin{pmatrix} z^2 + 1 & z & 1 \\ 0 & z^2 & z \\ 0 & -z & z^2 \end{pmatrix} \qquad (5.22)A$$

$$(z\,I - A)^{-1} = \frac{1}{z(z^2+1)} \begin{pmatrix} z^2+1 & z & 1 \\ 0 & z^2 & z \\ 0 & -z & z^2 \end{pmatrix} \qquad (5.22)B$$

$$\therefore \quad (zI - A)^{-1} z = \frac{1}{(z^2+1)} \begin{pmatrix} z^2+1 & z & 1 \\ 0 & z^2 & z \\ 0 & -z & z^2 \end{pmatrix} \text{ ------------------------ (5.22)C}$$

$\therefore \phi(k) = Z^{-1}[(zI - A)^{-1}z]$ = Inverse Z – transform of the nxn matrix

$$\therefore \quad \phi(k) = \begin{pmatrix} 1 & \sin T & 1-\sin T \\ 0 & \cos T & \sin T \\ 0 & -\sin T & \cos T \end{pmatrix} \text{ ------------------------ (5.22)D}$$

T is the sampling period = 1 second

$$\text{Define statematrix for A} = \phi_A = \begin{pmatrix} 1 & 0.02 & 0.9 \\ 0 & 0.99 & 0.02 \\ 0 & -0.02 & 0.99 \end{pmatrix} \text{ --------------- (5.22)E}$$

$$\text{Define statematrix for B} = \phi_B = \int_0^T \phi_A(k)\, B dt = \begin{pmatrix} 1 & \sin T & (1-\sin T) \\ 0 & \cos T & \sin T \\ 0 & -\sin T & \cos T \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{-----(5.22)F}$$

$$= \begin{pmatrix} \int_0^T (1 - \sin\iota)\, d\iota \\ \int_0^T \sin\iota\, d\iota \\ \int_0^T \cos\iota\, d\iota \end{pmatrix}$$

$$\therefore \phi_B = \begin{bmatrix} 0.99 \\ 0 \\ 0.02 \end{bmatrix} \text{ -------------------- } \quad (5.22)G$$

5.3 TEST FOR CONTROLLABILITY AND OBSERVABILITY

A system is said to be controllable at time t_o if it is possible by means of an unconstrained control vector to transfer the system from any initial state $x(t_o)$ to any other state in a finite interval of time.

A system is said to be observable at time t_o if, with the system in state $x(t_o)$, it is possible to determine the state from the observation of the output over a finite time interval. A discrete time system is describe by

$$x\{(k+1)T\} = Ax(kT) + BU(kT) \text{ ------------------------------- } \quad (5.23)$$

Where

x(kT)	=	state vector (n-dimensional vector)
U(kT)	=	Control signal
A	=	n x n non- singular matrix
B	=	n x 1 matrix
T	=	Sampling period

U(kT) is constant for $kT \leq t < (k+1)T$.

The discrete – time system is state controllable, if there exist a piece –wise constant control signal U(kT) defined over a finite sampling interval $0 \leq kT\, l < nT$ such that, starting from any initial state, the state x(kT) can be made zero for $kT \geq nT$.

The solution to the discrete equation is given by:

$$x(kT) = A^k x(0) + \sum_{j=0}^{k-1} A^{k-j-1} BU(jT) \text{ ------------------------------------ } \quad (5.24)$$

If the system is controllable, then starting from an arbitrary x(0), then x(kT) = 0, for $k \geq n$ by the application of some U(0), U(T) …………., U{(n-1)T}

$$\text{Thus, } A^n x(0) + \sum_{j=0}^{k-1} A^{n-j-1} BU(jT) = 0$$

$$\text{Or } \quad x(0) = -\sum_{j=0}^{k-1} A^{-(j-1)} Bu(jT) \quad \text{---} \quad (5.25)A$$

Since A is an n x n non singular matrix and B is an n x 1 matrix, clearly $A^{-1}B$, $A^{-2}B$, ………, $A^{-n}B$ are n x 1 matrices or column vectors.

Define,

$$A^{-1}B = \begin{pmatrix} [a_{11}] \\ [a_{21}] \\ [a_{n1}] \end{pmatrix} \quad A^{-2}B = \begin{pmatrix} [a_{12}] \\ [a_{22}] \\ [a_{n2}] \end{pmatrix} \quad \ldots\ldots\ldots \quad A^{-n}B = \begin{pmatrix} [a_{1n}] \\ [a_{2n}] \\ [a_{nn}] \end{pmatrix}$$

$$\therefore \quad x_1(0) = -a_{11}U(0) - a_{12}U(T) \ldots\ldots\ldots a_{1n}U\{n-1)T\} \text{------} \quad (5.25)B$$

$$x_2(0) = -a_{21}U(0) - a_{22}U(T) \ldots\ldots\ldots a_{22}U\{n-1)\} \text{-------} \quad (5.25)C$$

$$x_n(0) = -a_{n1}U(0) - a_{n2}U(T) \ldots\ldots\ldots a_{nn}U\{n-1)T) \text{------} \quad (5.25)D$$

If these equations are to give a solution to any set of values $x_1(0)$, $x_2(0)$…, $x_n(0)$ the n x n matrix (a_{ij}) must be non-singular. The condition that n x n matrix (a_{ij}) is non-singular implies that each column of the matrix (a_{ij}) is linearly independent, or the matrix (a_{ij}) is of rank n. Hence, the condition for complete state controllability of the discrete system is possible if and only if the vectors $A^{-1}B$,

$A^{-2}B$, ……… $A^{-n}B$ are linearly independent or then n x n matrix

$$[A^{-1}B| \; A^{-2}B| \ldots\ldots| \; A^{-n}B\} \text{ is of rank n} \text{ ----------------------------------} \quad (5.25)E$$

A discrete-time system is described by

$$x\{k+1)T\} = A\,x(kT) \text{ ---} \quad (5.26)$$

$$y(kT) = C\,x(kT) \text{ ---} \quad (5.27)$$

where

$x(kT)$ = state vector (n – dimensional vector)

$y(kT)$ = output vector (m – dimensional vector)

A = n x n matrix

C = m x n matrix

T = Sampling period

This system is completely observable, if given the output y(kT) for finite sampling periods, it is possible to determine the initial state vector x (0)

The solution to the discrete –time system is given by

$$x(kT) = A^k x(0) \quad (5.28)$$

$$y(kT) = CA^k x(0) \quad (5.29)A$$

complete observeability means that, given y(0), y(T),...., y(NT) then $x_1(0)$, $x_2(0)$, ..., $x_n(0)$ can be determined. To determine n unknowns, only n values of y(kT) is need. Hence, N = n-1.

For a completely observable system, given

$$y(0) = Cx(0) \quad (5.29)B$$

$$y(T) = CAx(0) \quad (5.29)C$$

....

....

$$y\{(n-1)T\} = CA^{n-1}x(0) \quad (5.29)D$$

Then $x_1(0)$, $x_2(0)$, ..., x_n (0) must be determined. Noting that y is an m vector, the proceeding n simultaneous equations yield nm equations, all involving $x_1(0)$, $x_2(0)$ ....$x_n(0)$. To obtain a unique set of solutions $x_1(0)$, $x_2(0)$, ...$x_n(0)$ from these nm equations, then exactly n linearly independent equations must be written among them. This requires that the nm x n matrix.

$$\begin{pmatrix} [\ C\ ] \\ [\ldots\ldots] \\ [\ CA\ ] \\ [\ldots\ldots] \\ [CA^{n-1}] \end{pmatrix}$$

be of rank n

Therefore, the discrete time system is completely observable if and only if the n x nm matrix.

$$[C^*|\ AC^*|\ \ldots\ldots|\ (A)^{n-1}\ C^*] \text{ ----------------------------- } \quad (5.29)E$$

is of rank n, or has n linearly independent column vectors.

The matrix representation of the controller was derived to be

$$\begin{pmatrix} [x_1(k+1)] \\ [x_2(k+1)] \\ [x_3(k+1)] \end{pmatrix} = \begin{pmatrix} [0 & 1 & 0] \\ [0 & 0 & 1] \\ [-b_0 & -b_1 & -b_2] \end{pmatrix} \begin{pmatrix} [x_1(k)] \\ [x_2(x)] \\ [x_3(k)] \end{pmatrix} + \begin{pmatrix} [0] \\ [0] \\ [1] \end{pmatrix} U(k)$$

and

$$y(k) = [a_0 \quad a_1 \quad a_2 \begin{pmatrix} [x_1(k)] \\ [x_2(k)] \\ [x_3(k)] \end{pmatrix}$$

from the matrix representation of the state equation of the controller.

$$A = \begin{pmatrix} [0 & 1 & 0] \\ [0 & 0 & 1] \\ [0 & -1 & 0] \end{pmatrix},\ B = \begin{pmatrix} [0] \\ [0] \\ [1] \end{pmatrix}$$

$$AB = \begin{pmatrix} [0 & 1 & 0] \\ [0 & 0 & 1] \\ [0 & -1 & 0] \end{pmatrix} \begin{pmatrix} [0] \\ [0] \\ [0] \end{pmatrix} = \begin{pmatrix} [0] \\ [1] \\ [0] \end{pmatrix}$$

$$A^2B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore \quad [B \mid AB \mid A^2B] = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$\mathrm{DET} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} = -1 \neq 0$$

$\therefore$ The system is completely controllable since the rank $r = n = 3$

from the output equation, $C = [10 \ -10 \ \ 0]$

$$C^T = \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix} \cdot A^T = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 10 & 1 & 0 \end{pmatrix}$$

$$A^TC^T = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -10 \end{pmatrix}$$

$$(A^T)^2C^T = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 10 \\ -10 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ -10 \\ 10 \end{pmatrix}$$

$$[C^T \quad | A^T C^T \mid (A^T)^2 \; C^T] = \begin{pmatrix} [10 & 0 & -10] \\ [-10 & 0 & -10] \\ [0 & -10 & 10] \end{pmatrix}$$

$$\text{DET} \begin{pmatrix} [10 & 0 & -10] \\ [-10 & 10 & -10] \\ [0 & -10 & 10] \end{pmatrix} = 2 \times 10^3 \neq 0$$

∴ The system is completely observable since the rank r = n = 3.

5.4 DESIGN OF THE STATE – OBSERVER FOR SYSTEM

A significant portion of the optimal control theory of digital control systems rely on the feedback of the state variable to form the control. When the feedback from all the state variables is a required in a given design, and not all the state variables are accessible, it is necessary to "observe" the states from information contained in the output as well as the input variables. The subsystem that performs the observation of the variables based on the information received from the measurement of the input and output is called a state observer or an observer.

The system is described by the equations:

$$x(k+1) = A\,x(k) + BU(k) \quad \text{-----------------------------} \quad (5.30)$$

$$y(k) = C\,x(k) \quad \text{---} \quad (5.31)$$

From (5.30) and (5.13)

$$Z\,x(z) = A\,x(z) + BU(z) \quad \text{---------------------------} \quad (5.32)$$

$$X(z) = (zI-A)^{-1} \quad BU(z) \quad \text{---------------------------} \quad (5.32)A$$

I = Identity matrix

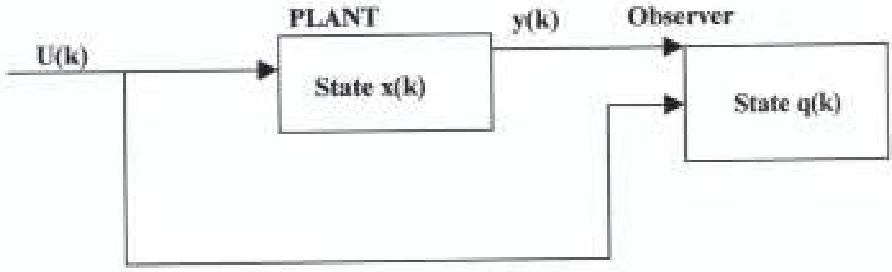


Fig 5.1 The State observer system.

The observe has two inputs, y(k) and U(k). Therefore the observer state equation is

$$\therefore \quad q(k+1) = Fq(k) + Gy(k) + HU(k)$$

Solving

$$Q(z) = (zI - F)^{-1} \{Gy(z) + HU(z)\} \text{ ------------------------ } (5.33)$$

From equation (5.31)

$$y(z) = C\,x(z) \text{ ---------------------------- } (5.34)$$

Substituting (5.34) and (5.32)A in (5.33)

$$Q(z) = (zI - F)^{-1} \{GCx(z) + HU(z)]$$

$$\therefore \quad Q(z) = (zI - F)^{-1} \{GC(zI - A)^{-1} B + H\} U(z) \text{ --------------- } (5.35)$$

The design criterion is that the transfer-function matrix from U(z) to Q(z) be the same as that from U(z) to x (z).

From (5.32)

$$Q(z) = (zI - A)^{-1} BU(z) \quad \text{must be satisfied -------------------- } (5.36)$$

From 5.35 and 5.36

$$(zI - A)^{-1} B = (zI - F)^{-1} GC(zI - A)^{-1} B + (zI - F)^{-1} H \text{ --------------- } (5.37)$$

$$\text{or} \quad \{I - (zI - F)^{-1} GC\} (zI - A)^{-1} B = (zI - F)^{-1} H \text{ ------------------------- } (5.38)$$

$$\therefore \quad (zI\ F)^{-1} \{zI - (F + GC\} (zI - A)^{-1} B = (zI - F)^{-1} H \text{ ---------------------- } (5.39)$$

$$\therefore \quad (zI - A)^{-1} B = \{zI - (F + GC\}^{-1} H \text{ -------------------------------------- } (5.40)$$

This equation is satisfied if H = B and A = F + GC ------------------------------- (5 . 41)

Therefore, the observer equation is

$$q(k+1) = (A - GC)\ q(k) + Gy(k) + BU(k) \quad \text{(5.42)}$$

With the characteristic equation

$$\left| zI - A + GC \right| = 0 \quad \text{(5.43)A}$$

This can also be written as

$$\left| \lambda I - A + GC \right| = 0 \quad \text{(5.43)B}$$

The objective is to design the observe to have a deadbeat response so that q(k) will reach x(k) in one sampling instant. For a deadbeat response, the characteristic equation should be of the form

$$\lambda^2 = 0 \quad \text{(5.44)}$$

$$C = [a_0 \quad a_1 \quad a_2]$$

$$a_0 = k_p + \frac{k_I T}{2} + \frac{k_D}{T}$$

$$a_1 = -K_p + \frac{k_I T}{2} - \frac{2k_D}{T}$$

$$a_2 = \frac{k_D}{T}, \ D = 0, \quad \therefore \quad k_D = 0.00 \quad \therefore a_2 = 0.00$$

$$\therefore a_1 = -k_p + k_I\ T/2$$

Analysis at the installed plant performance condition for the design of the observer is

$$\text{At PB} = 10\%,\ k_p = \frac{100}{PB} = 10$$

And

$$T_I = 65 \text{ Secs},\ k_I = 1/65 = 0.015$$

$$\therefore a_1 = -10 + 0.015 \times ½ = -9.99 \approx -10$$

$$a_0 = 10 + 0.0075 = 10$$

$\therefore C \quad = \quad [a_0 \quad a_1 \quad a_2] \quad = \quad [10 \quad -10 \quad 0]$

$$|\lambda| = \text{Diagonal matrix} = \begin{pmatrix} [\lambda & 0 & 0] \\ [0 & \lambda & 0] \\ [0 & 0 & \lambda] \end{pmatrix}$$

$$I = \text{Identity matrix} = \begin{pmatrix} [1 & 0 & 0] \\ [0 & 1 & 0] \\ [0 & 0 & 1] \end{pmatrix}$$

The observer equation is $[\lambda I \ - (A - GC)]$

The characteristics polynomial is

$$|\lambda I - A| = \begin{pmatrix} [\lambda & 0 & 0] \\ [0 & \lambda & 0] \\ [0 & 0 & \lambda] \end{pmatrix} - \begin{pmatrix} [1 & 0.02 & 0.98] \\ [0 & 0.99 & 0.02] \\ [0 & -0.02 & 0.99] \end{pmatrix}$$

$$= \begin{pmatrix} [\lambda\text{-}1 & -0.02 & -0.98] \\ [0 & \lambda\text{-}0.99 & -0.02] \\ [0 & 0.02 & \lambda\text{-}0.99] \end{pmatrix}$$

$$= (\lambda -1) \begin{pmatrix} [\lambda - 0.99 & -0.02] \\ [0.02 & \lambda\text{-}0.99] \end{pmatrix} + 0.02\,[\,0\,] - 0.98\,[\,0\,]$$

$$= (\lambda - 1\,)\,[\lambda - 0.99\ .\lambda - 0.99) + 0.0004]$$

$$= 0.99\lambda^2 - 0.98\lambda - 0.99\lambda + 0.98$$

$$= 0.99\lambda^2 - 1.97\lambda + 0.98$$

$$= \lambda^2 - 1.99\lambda + 0.99 = \lambda^2 + \alpha_1\lambda + \alpha_2$$

The desired characteristics polynomial is

$$\left|\lambda I - (A^T + C^T G\right| = \lambda^2 + \propto_1\lambda + \propto_2$$

$$A^T = \begin{pmatrix} [1 & 0 & 0] \\ [0.02 & 0.99 & -0.02 \\ [0.98 & 0.02 & 0.99] \end{pmatrix} \quad C^T = \begin{pmatrix} [\,0\,] \\ [-10\,] \\ [\,0\,] \end{pmatrix}$$

$$\text{Let } U = [C^T \;\vdots\; A^T C^T \;\vdots\; (A^T)^2 C^T]$$

$$A^T C^T = \begin{pmatrix} [1 & 0 & 0] \\ [0.02 & 0.99 & -0.02] \\ [0.98 & 0.02 & 0.99] \end{pmatrix} \begin{pmatrix} [\,0\,] \\ [-10] \\ [\,0\,] \end{pmatrix} = \begin{pmatrix} [\,10\,] \\ [\,-9.7] \\ [\,9.6\,] \end{pmatrix}$$

$$(A^T)^2 C^T = \begin{pmatrix} [0 & 0 & 0] \\ [0.02 & 0.98 & -0.04] \\ [1.95 & 0.04 & 0.980 \end{pmatrix} \begin{pmatrix} [\,10\,] \\ [-10] \\ [0] \end{pmatrix} = \begin{pmatrix} [\,10\,] \\ [-9.6] \\ [19] \end{pmatrix}$$

$$\therefore \quad U = \begin{pmatrix} [10 & 10 & 10] \\ [-10 & -9.6 & -9.6] \\ [10 & -9.6 & 19.1] \end{pmatrix}$$

$$\Delta(U) = 10 \quad \{[-185.27 + 92.161\} - 10\,[-191 + 961] + 10\,[96 + 97]$$

$$\Delta(U) = -931 + 950 + 1930 = 1949$$

$$U^1 = 1/(\Delta U) \begin{pmatrix} [-93.1 & -287 & 1] \\ [95 & 91 & -4] \\ [193 & -4 & -3] \end{pmatrix} = \begin{pmatrix} [-0.05 & -0.14 & 0.0015] \\ [0.05 & 0.05 & 0.002] \\ [0.101 & 0.002 & 0.0015] \end{pmatrix}$$

The last row of U^{-1} = [0.10 0.002 0.00151]

$$P = \begin{pmatrix} [P_1\ ] \\ [P_1A^T\ ] \\ [P_1(A^T)^2] \end{pmatrix}$$

$$= \begin{pmatrix} [0.10 & 0.002 & 0.00151] \\ [0.10 & 0.0015 & 0.002] \\ [0.10 & 0.002 & 0.0015] \end{pmatrix}$$

From the characteristic polynomials

$$\overline{K} = [\propto_2 - \propto_2 \mid \propto_1 - \propto_1 \mid \propto_0 - \propto_0]$$

$$= [0.99\ -\ 1.99\ 0\ ]$$

$$\therefore K_0 = \overline{K_P} = \left[0.99 - 1.99 \quad 0\right] \begin{pmatrix} [0.10 & 0.002 & 0.0015] \\ [0.10 & 0.0015 & 0.002] \\ [0.10 & 0.002 & 0.0015] \end{pmatrix}$$

$$\therefore K_0 = [-01 \quad - 0.001 \qquad -0.002]$$

Therefore, the observer parameter is given by $G = K_0^T$

$$G = \begin{pmatrix} [\ -\ 0.1\ ] \\ [\ -0.001\ ] \\ [\ -\ 0.002\ ] \end{pmatrix}$$

5.5 OPTIMIZATION OF CONTROLLER

This is the decision point which here refers to one made on the basis of plant characteristics which have been identified and on the performance index defined.

The optimal design technique for optimal linear regulator control system with quadratic performance index is adopted. This results in a control law of the form

$$U(k) = -K(k) . X(k) \quad \text{--} \quad (5.45)$$

Where k is the constant state.

Feedback gain matrix and the optimal control $U^0(k)$ for k = 0, 1, 2, 3, is determined and hence to minimize.

$$J = (½) \sum_{k=0}^{3} U^2(k) \quad \text{--} \quad (5.46)$$

From the system equation,

$$x(k+1) = A\,x(k) + BU(k) \quad \text{----------------------} \quad (5.47)$$

To find the optimal control $U^0(k)$, k = 0, 1, 2, ..., N – 1 such that the initial state x(0) is driven to the final state x(N) = 0 subject to the control constraint

$$J = (½) \sum_{k=0}^{n-1} U^T(k)\, RU(k) = \text{minimum} \quad \text{--------------------} \quad (5.48)$$

Where R is symmetric and positive definite. R is not equal to r(k).

r(k) is a function in a control law which is not applicable in this case.

The performance index in 5.48 is genrally known as the quadratic form and in this case it represent an energy constraint on the designed system.

Therefore, this represent an alternate not equivalent, constraint on the control U(k)

The solution of the problem begin by defining the Hamiltonian function (KUO, 1987)

$$H\{X(k), P(k+1), U(k)\} = ½ \; [\{(U(k), RU(k)\} + \{P(k+1), x(k+1)\}] \quad \text{--------------------} (5.49)$$

P(k) is a costate (n x 1) vector matrix.

The necessary conditions for J to be a minimum are

$$\frac{\partial H^0(k)}{\partial X^0(k)} = P^0(k) = A^T P^0(k+1) \quad \text{-------------------------------} \quad (5.50)$$

$$\frac{\partial H^0(k)}{\partial P^0(k+1)} = X^0(k+1) = AX^0(k) + BU^0(k) \text{--------------------} \quad (5.51)$$

$$\frac{\partial H^0(k)}{\partial U^0(k)} = RU^0(k) + B^T P^0(k+1) = 0 \text{ ---------------------} \quad (5.52)A$$

where

$$H^0(k) = \tfrac{1}{2}\left[\{(U^0(k), RU^0(k)\} + \{P^0(k+1), AX^0(k) + BU^0(k)\}\right] \text{ ----------} \quad (5.52)B$$

The multiplication by, (½), a constant does not effect the minimization. It only helps in mathematical manipulation of the system.

The optimal control is obtained from (5.52) when $U^0(k) = -R^{-1}B^TP^0(k+1)$ ------- (5.53)

Where the inverse of R exists since it is positive definite.

Equation 5.50 can be solved by itself to give $P^0(k) = (A^{-k})^T P^0(O)$ -------------- (5.54)

Where it is assumed that A has an inverse.

Substitution of equation 5.53 into equation 5.51 gives.

$$X^0(k+1) = AX^0(k) - BR^{-1}B^TP^0(k+1) \text{ --------------------------} \quad (5.55)$$

The solution is

$$X^0(N) = A^N x(O) - \sum_{k=0}^{N-1} A^{N-K-1} B\ R^{-1}\ B^T (A^{-1})^T P^0(k) \text{ ------------------} \quad (5.56)$$

Setting $X^0(N) = O$ and solving for X(O),

$$\therefore X(O) = \sum_{k=0}^{N-1} A^{-K-1}\, B\, R^{-1}\, B^T (A^{-k-1})^T P^0(0) \text{ ------------------------------------} (5.57)$$

$$\therefore U^0(k) = -B^T (A^{-1}\, B\, A^{-2}\, B\, A^{-3}\, B)^{-1}\, X(k) \text{ -------------------------} (5.58)$$

$$U^0(k) = -K \,.\, x(k) \text{ ------------------------------------} (5.59)$$

Which is the optimum control law.

The optimum gain, **K**, is obtained from

$$-\mathbf{K} = -B^T(A^{-1}B \quad A^{-2}B \quad A^{-3}B)^{-1}$$

$P^0(0)$ is equivalent to the deviation from the set point in the data obtained from samples. The plant input U(j), is the pressure signal, PV, from samples.

X (k) is evaluated for each sample from the expression

$$X(k) = \phi(k)\,x(0) + \sum_{i=0}^{k-1} \phi(k-1-i)\,B\,U(i) \qquad (5.60)$$

$U^{o}(k)$ is evaluated for each of the samples from equation (5.59) with X(k) obtained from Equation (5.60) and minimization carried out with the constraint.

$$J = (\tfrac{1}{2}) \sum_{k=0}^{N} U^2(k) \qquad (5.61)$$

The least value of 5.61 will give the condition for the optimum system performance.

CHAPTER SIX

SOFTWARE DEVELOPMENT AND DISCUSSION

6.1 PARAMETERIZATION

In conventional process controllers, the operator can manipulate the set point and the basic parameters such as the gain, integral time, and derivative time in the control law. With computer control, there are many other interesting alternatives. Because of the simplicity of making computations, it is possible to use one parameterization in the control algorithm and another in the operator. The parameters displayed to the operator may then be related to the performance of the system rather than to the details of the control algorithm. The conversion between the parameters is made by an algorithm in the computer.

The parameterization in this project are:

i) PB = 10% T1 = 65 Secs $T_D = 0$

ii) PB = 125% T1 = 25Secs $T_D = 0$

iii) PB = 185% T1 = 10Secs $T_D = 0$

iv) PB = 220% T1 = 15Secs $T_D = 0$

PB = PROPORTIONAL BAND TI = INTEGRAL TIME,

T_D = DERIVATIVE TIME.

The sampling period is 1 second with T = Sampling period – 1 Sec.

K_p = Proportion gain = $\frac{100}{PB}$

K_I = Integral gain = 1/T1

K_D = Derivative gain = 0

$a_0 = K_p + K_I T/2 + K_D$

$a_1 \quad = \quad -K_p + K_I T/2 - 2K_D/T$

$a_2 \quad = \quad \dfrac{K_D}{T}$

$b_0 \quad = \quad 0, \qquad b_1 \quad = \quad -1, \qquad b_2 \quad = \quad 0$

6.2 PROGRAMMING

Programming is an important aspect of the implementation of a control system, both with respect to the efficiency of the system and the time required for the implementation.

The effort required and the approaches used depends on the available software and the nature of the control problem. There are many high level languages that may be used but the language adopted in this project is the C^{++}. The purpose of the programming is to

(i) Test Controllability of the system from the determinant of the matrix

$$[B \mid AB \mid A^2B]$$

(ii) Test observability of the system from the determinant of the matrix

$$[C^T \mid A^TC^T \mid (A^T)^2(C^T)]$$

(iii) By taking $|\lambda|$ to be a diagonal matrix and $|I|$ to be an identity matrix with, $\lambda^2 = 0$, deadbeat response, to find an (n x 1) matrix for the observer from the matrix expression

$$|\lambda| - A + GC] = 0$$

(iv) By taking the feedback gain setting for the system to be $\mathbf{K} = B^T \; [A^{-1}B \;\; A^{-2}B \;\; A^{-3}B]^{-1}$, to formulate an optimum control law of the form $U^0(k) = - K\, x\,(k)$ for the system.

(v) To find the optimum performances by the least square method for the system from the expression (BERNARD, 1987).

$$J = (\tfrac{1}{2}) \sum_{K=0}^{N} \{U^0(k)\}^2$$

The flow chart for the project is as shown in fig 6.1. From the parameterization as indicated above:

(I) PB = 10%, T1 =65sec, $T_D = 0$.

This is the plant's set performance condition and the sample data obtained at this condition is shown in appendix I. The analysis of the data is illustrated in table 6.1. The system output response is shown in fig 6.2 and input response is shown in fig 6.3.

(II) PB = 125% T1 = 25 sec, $T_D = 0$

This is the first simulated performance condition on the plant and the sample data obtained at this condition is shown in appendix II. The analysis of the data is illustrated in table 6.2. The system output response is shown in fig 6.4 and the input response is shown in fig 6.5

(III) PB = 185%, T1 = 10sec, $T_D = 0$

This is the second simulated performance condition on the plant and the sample data is illustrated in table 6.3. The system output response is shown in fig 6.6 and the input response is shown in fig 6.7.

(IV) PB = 220%, T1 = 15 sec, $T_D = 0$.

This is the third simulated performance condition on the plant and the sample data obtained at this condition is shown in appendix IV. The analysis of the data is illustrated in table 6.4. The system output response is shown in fig 6.8 and the input response is shown in fig 6.9

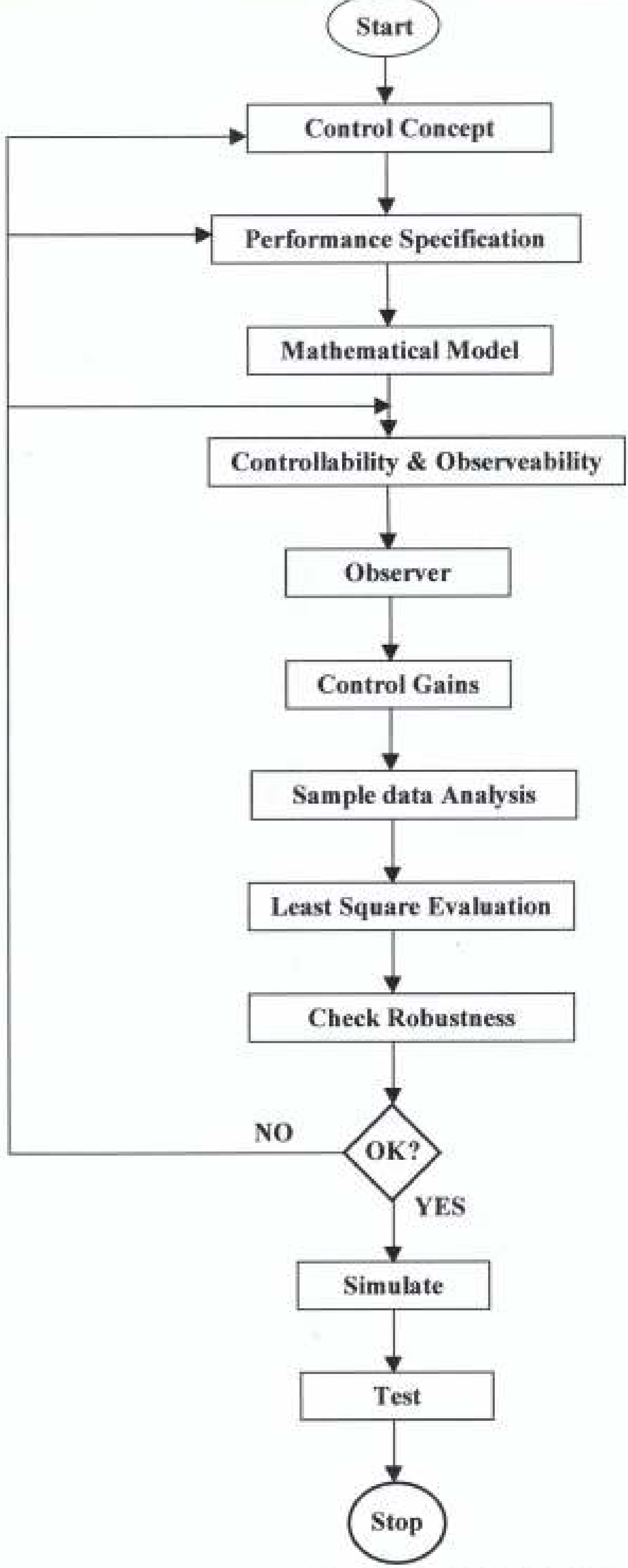


Fig 6.1A THE DESIGN PROCESS FLOW CHART

TABLE 6.1

PLANT DATA ANALYSIS

At PB = 10% TI = 65 SEC, $T_D = 0$

U (i)	ϕ(k)	X (k)	U^0 (k)	$\{U^0(k)\}^2$
7.98	0.50	11.23	-2627.82	6905437.95
7.98	0.50	11.23	-2627.82	6905437.95
7.98	0.50	11.23	-2627.82	6905437.95
7.98	0.50	11.23	-2627.82	6905437.95
7.98	0.50	11.23	-2627.82	6905437.95
7.98	0.50	11.23	-2627.82	6905437.95
7.97	0.49	11.16	-2611.44	6819618.87
7.97	0.49	11.16	-2611.44	6819618.87
7.96	0.48	11.08	-2592.72	6722197.00
7.96	0.48	11.08	-2592.72	6722197.00
7.95	0.47	11.01	-2576.34	6637527.80
7.83	0.35	10.11	-2365.74	5596725.75
7.75	0.37	9.51	-2225.34	5952138.12
7.73	0.25	9.36	-2190.24	4797151.26
7.65	0.17	8.76	-2049.84	4201844.03
7.63	0.15	8.61	-2014.74	405177.27
7.57	0.09	8.16	-1909.44	3645961.11
7.50	0.02	7.63	-1785.42	3187724.58
7.49	0.01	7.56	-1769.04	3129520.52
7.49	0.01	7.56	-1769.04	3129520.52
7.48	0.00	7.48	-1750.32	3063620.10
7.48	0.00	7.48	-1750.32	3063620.10
7.48	0.00	7.48	-1750.32	3063620.10
7.47	-0.01	7.41	-1733.94	3006547.92
7.47	-0.01	7.41	-1733.94	3006547.92
7.47	-0.01	7.41	-1733.94	3006547.92
7.48	0.00	7.48	-1750.32	3063620.10
7.48	0.00	7.48	-1750.32	3063620.10
7.49	0.01	7.56	-1769.04	3129502.52
7.49	0.01	7.56	-1769.04	3129502.52
7.49	0.01	7.56	-1769.04	3129502.52
7.48	0.00	7.48	-1750.32	3063620.10

J = 72887193.16

TABLE 6.2

ANALYSIS FROM SAMPLE ONE SIMULATION

At PB = 125%, T1= 25sec, T_D = 0

U (i)	ϕ(k)	X (k)	U^0 (k)	$\{U^0(k)\}^2$
6.50	0.00	6.50	-1521.00	2313441.00
6.50	-1.00	5.58	-1368.90	1873887.21
6.50	-1.00	5.58	-1368.90	1873887.21
6.54	-0.96	0.30	-70.20	4928.04
6.57	-0.93	0.53	-124.208	15380.96
6.59	-0.91	0.68	-159.12	25319.17
6.63	-0.87	0.98	-229.32	52587.66
6.71	-0.79	1.58	-369.72	136692.88
6.83	-0.67	2.48	-580.32	336771.30
6.93	-0.52	3.23	-755.82	571263.87
6.98	-0.52	3.60	-842.40	709637.76
7.01	-0.49	3.83	-896.22	803210.29
7.01	-0.49	3.83	-896.22	803210.29
7.03	-0.46	4.04	-945.36	893705.53
7.03	-0.46	4.04	-945.36	893705.53
7.15	-0.35	4.88	-1141.92	1303981.29
7.18	-0.32	5.10	-1193.40	1424203.56
7.21	-0.29	5.33	-1247.22	1555557.73
7.25	-0.25	5.63	-1317.42	1735595.46
7.25	-0.25	5.63	-1317.42	1735595.46
7.29	-0.21	5.93	-1387.62	1925489.26
7.31	-0.19	6.08	-1422.72	2024132.20
7.35	-0.15	6.38	-1492.92	2228810.13
7.36	-0.14	6.45	-1509.30	2277986.49
7.39	-0.11	6.68	-1563.12	2443344.13
7.39	-0.11	6.68	-1563.12	2443344.13
7.40	-0.10	6.75	-1579.50	2494820.25
7.40	-0.10	6.75	-1579.50	2494820.25
7.41	-0.09	6.83	-1598.22	2554307.17
7.41	-0.09	6.83	-1598.22	2554307.17
7.43	-0.07	6.98	-1633.32	2667734.22
7.45	-0.05	7.13	-1668.42	2783625.30
7.48	-0.02	7.35	-1719.90	2958056.01
7.48	-0.02	7.35	-1719.90	2958056.01

J = 25347995.19

TABLE 6.3

ANALYSIS FROM SAMPLE TWO SIMULATION

At PB = 185%, T1 = 10 sec, $T_D = 0$

U (i)	$\phi(k)$	X (k)	U^0 (k)	$\{U^0(k)\}^2$
6.50	0.00	6.50	-1521.00	2312441.00
6.50	-1.00	5.58	-1368.90	1873887.21
6.50	-1.00	5.58	-1368.90	1873887.21
6.55	-0.95	0.38	-88.93	7906.77
6.63	-0.87	0.98	-229.32	52587.66
6.74	-0.75	1.88	-439.92	193529.61
6.92	-0.58	3.15	-737.10	543316.41
7.17	-0.33	5.03	-1177.02	1385376.08
7.33	-0.17	6.23	-1457.82	2125239.15
7.40	-0.10	6.75	-1579.50	2494820.25
7.40	-0.10	6.75	-1579.50	2494820.25
7.41	-0.09	6.83	-1598.22	2554307.17
7.39	-0.11	6.68	-1563.12	2443344.13
7.38	-0.12	6.60	-1544.40	2385171.36
7.38	-0.12	6.60	-1544.40	2385171.36
7.40	-0.10	6.75	-1579.50	2494820.25
7.44	-0.06	7.05	-1649.70	2721510.09
7.46	-0.04	7.20	-1684.80	283855.04
7.48	-0.02	7.35	-1719.90	2958056.01
7.50	0.00	7.50	-1755.00	3080015.00
7.50	0.00	7.50	-1755.00	3080015.00
7.50	0.00	7.50	-1755.00	3080015.00

J = 23273558.76

TABLE 6.4

ANALYSIS FROM SAMPLE THREE SIMULATION

At PB = 220%, T1 = 15 sec, T_D = 0

U (i)	φ(k)	X (k)	U^0 (k)	$\{U^0(k)\}^2$
6.50	0.00	6.50	-1521.00	2313441.00
6.50	-1.00	5.58	-1368.90	1873887.21
6.50	-1.00	5.58	-1368.90	1873887.21
6.53	-0.97	0.23	-53.82	2896.59
6.54	-0.96	0.30	-70.20	4928.04
6.58	0.92	0.60	-140.40	19712.16
6.75	-0.75	1.89	-442.26	195593.91
6.99	-0.51	3.68	-861.12	741527.65
7.06	-0.44	4.20	-17.64	311.17
7.08	-0.42	4.35	-1017.90	1036120.41
7.16	-0.34	4.95	-1158.30	1341658.89
7.19	-0.31	5.18	-1212.12	1469234.8
7.20	-0.30	5.25	-1228.50	1509212.25
7.21	-0.29	5.33	-1247.22	1555557.73
7.25	-0.25	5.63	-1317.42	1735595.46
7.27	-0.23	5.78	-1352.52	1829310.35
7.28	-0.22	5.85	-1368.90	1873887.21
7.29	-0.21	5.93	-1387.62	1925489.26
7.32	-0.18	6.15	-1439.10	2071008.81
7.35	-0.15	6.38	-1492.92	2228810.13
7.35	-0.15	6.38	-1492.92	2228810.13
7.38	-0.12	6.60	-1544.40	2385171.36
7.38	-0.12	6.60	-1544.40	2385171.36
7.41	-0.09	6.83	-1598.22	2554307.17
7.42	-0.08	6.90	-1614.60	2606933.16
7.43	-0.07	6.98	-1633.32	2667734.22
7.43	-0.07	6.98	-1633.32	2667734.22
7.46	-0.04	7.20	-1684.80	2838551.04
7.46	-0.04	7.20	-1684.80	2838551.04
7.47	-0.03	7.28	-1703.52	2901980.39
7.48	-0.02	7.35	-1719.90	2958056.01
7.49	-0.01	7.43	-1738.62	3022799.50
7.49	-0.01	7.43	-1738.62	3022799.50
7.49	-0.01	7.43	-1738.62	3022799.50
7.50	0.00	7.50	-1755.00	3080025.00
7.50	0.00	7.50	-1755.00	3080025.00

J = 56443523.95

6.3 DISCUSSION OF RESULTS

The system has been shown to be controllable and observable and the design of an observer has been implemented.

The derivative action is neglected, thus, effectively making the controlled run as a proportional-plus-integral controller i.e. a phase-lag compensator. This is with the intent to increase the low-frequency gain and thus reduce the steady-state errors in the system. Although, the derivative action added would have helped in improving system stability and increasing system bandwidth, the aim of optimization is to reduce the steady-state errors, hence neglecting the derivative action will not affect system performance.

The system response at each condition is illustrated. The performance condition is shown with each response curve. The optimum gain setting is obtained as

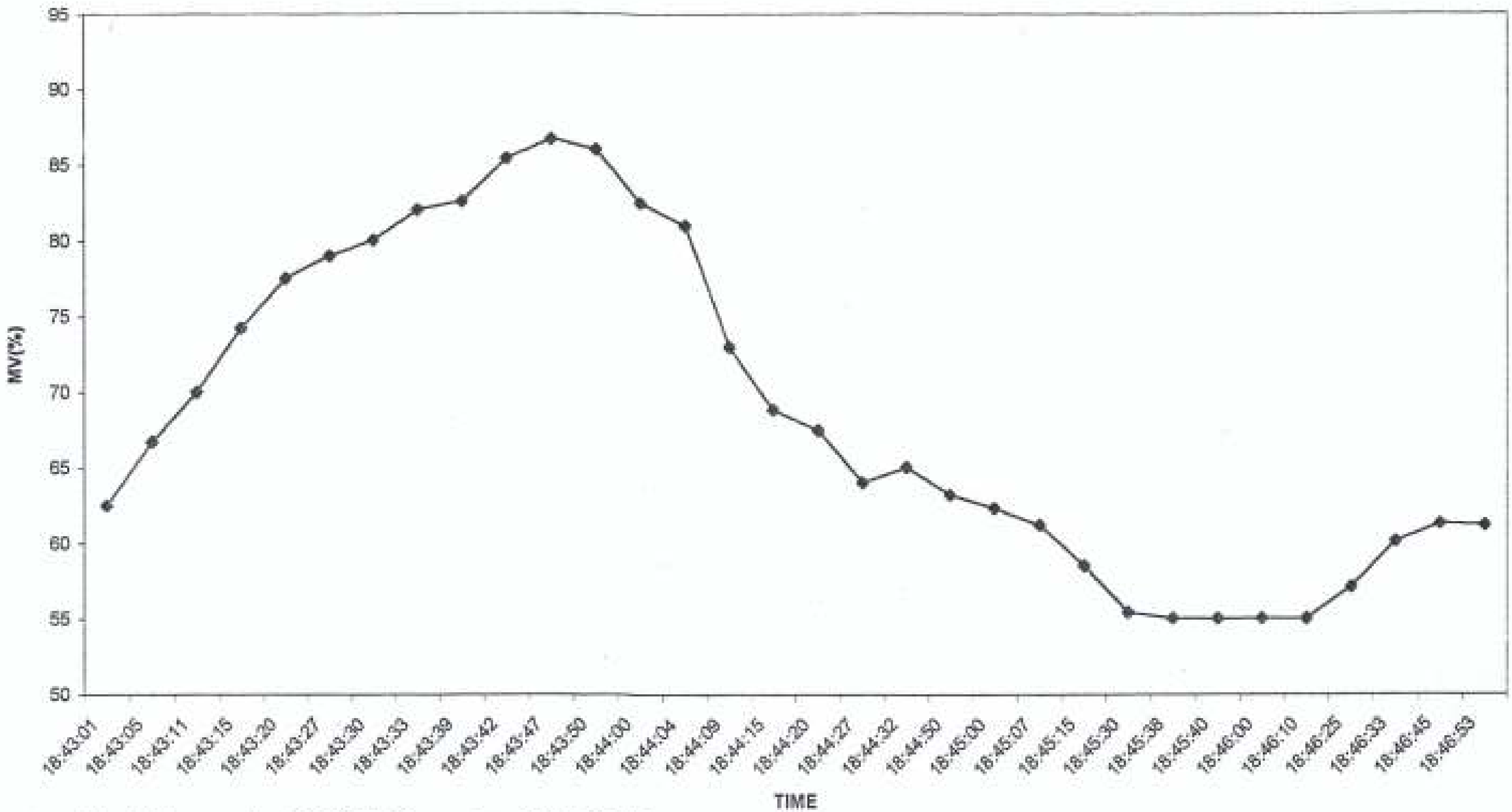
K = (2.99 2300.75 234.14). the optimum control law is formulated. The optimum performance at each operating condition is given as:

$J_1 = 2534.7$ $J_2 = 2327.3$, $J_3 = 5644.3$

The optimum performance on the installed system is J = 7288.7 This value shows that the system is not operating at optimum. The optimum performance, as seen from the values obtained in the analysis will be at $J_2 = 2327.3$ The performance condition for the controller to achieve this is

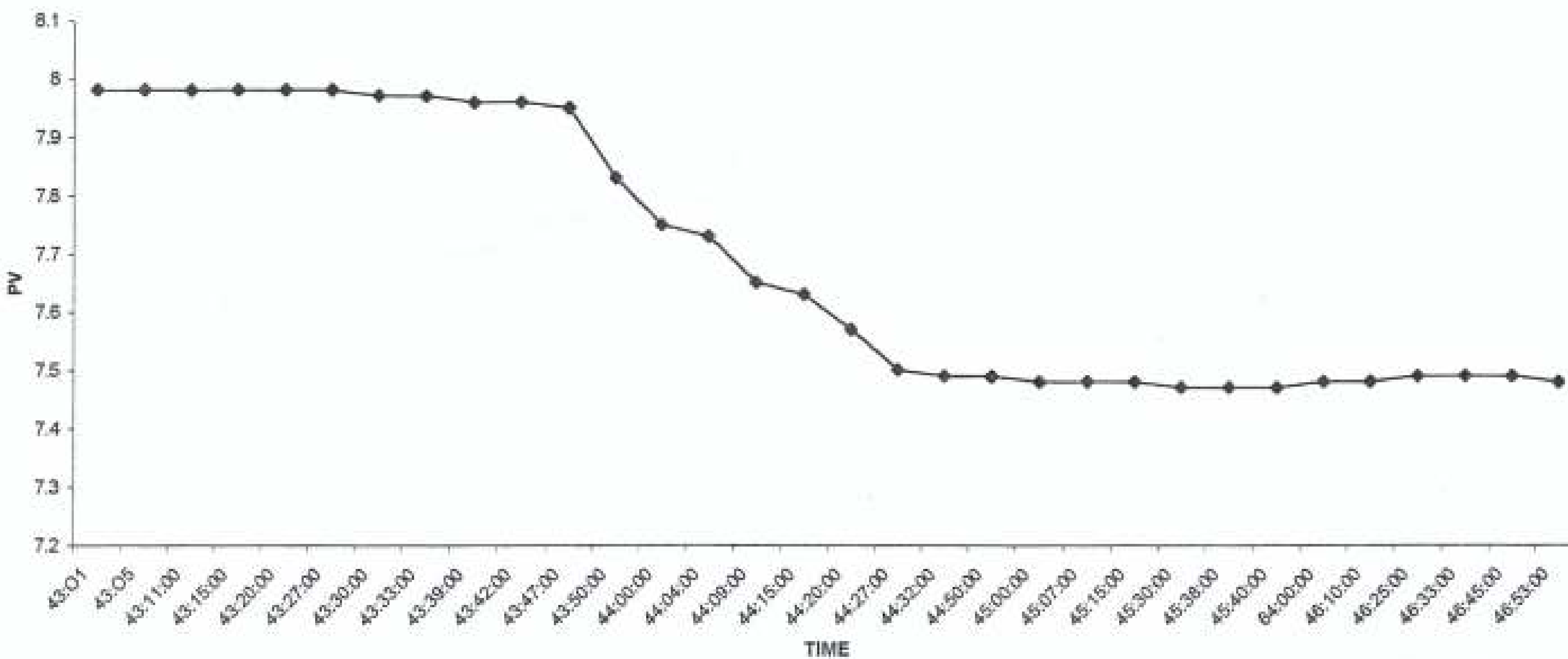
PB = 185%, TI = 10 secs.

At this condition, the rise time is small and the maximum overshoot, which is a measure of the relative stability of the system is low. The setting time is almost constant at 0.02 Seconds.



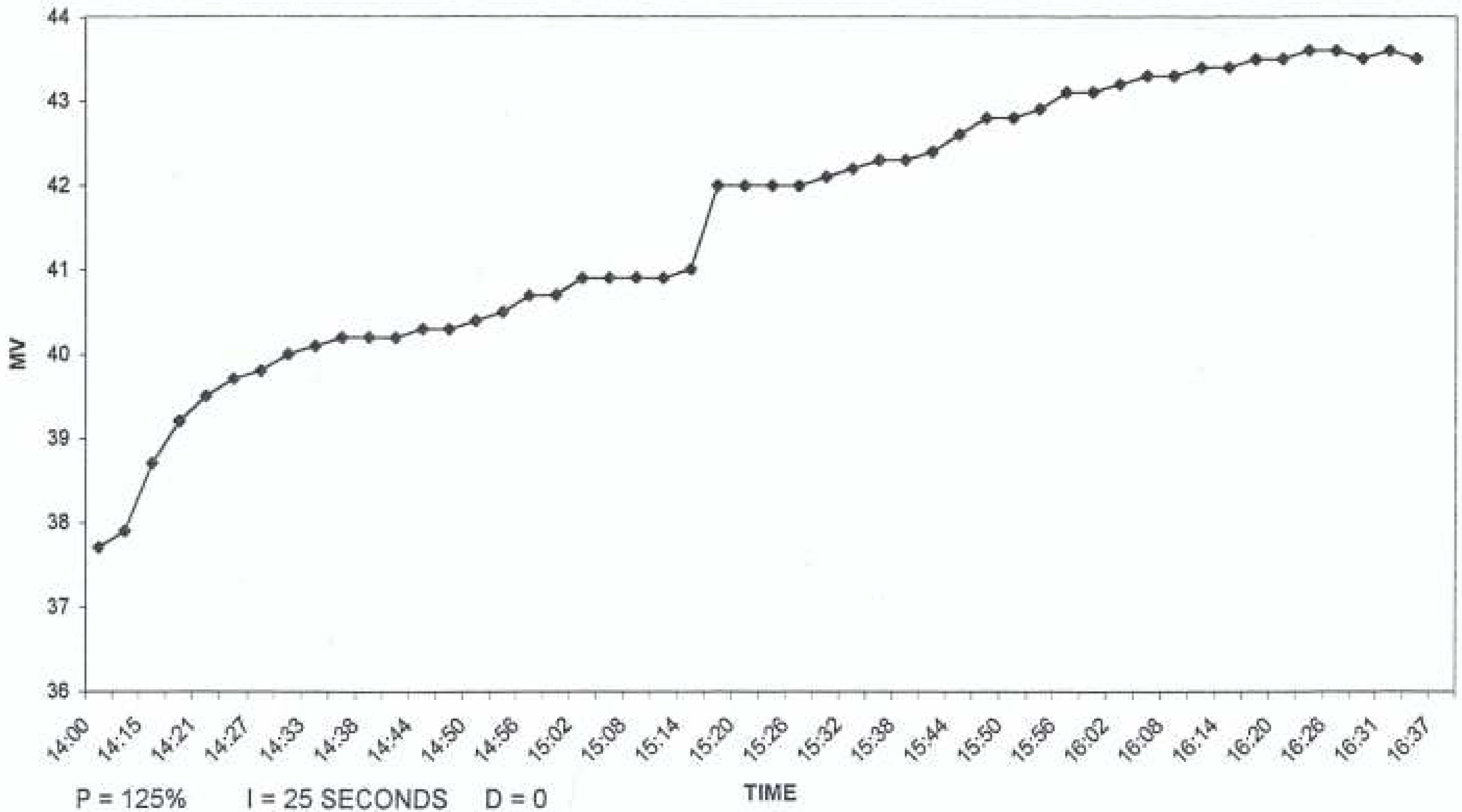
P = 10%   I = SECONDS   D = 0 SECOND

FIG. 6.1 PLANT OUTPUT RESPONSE



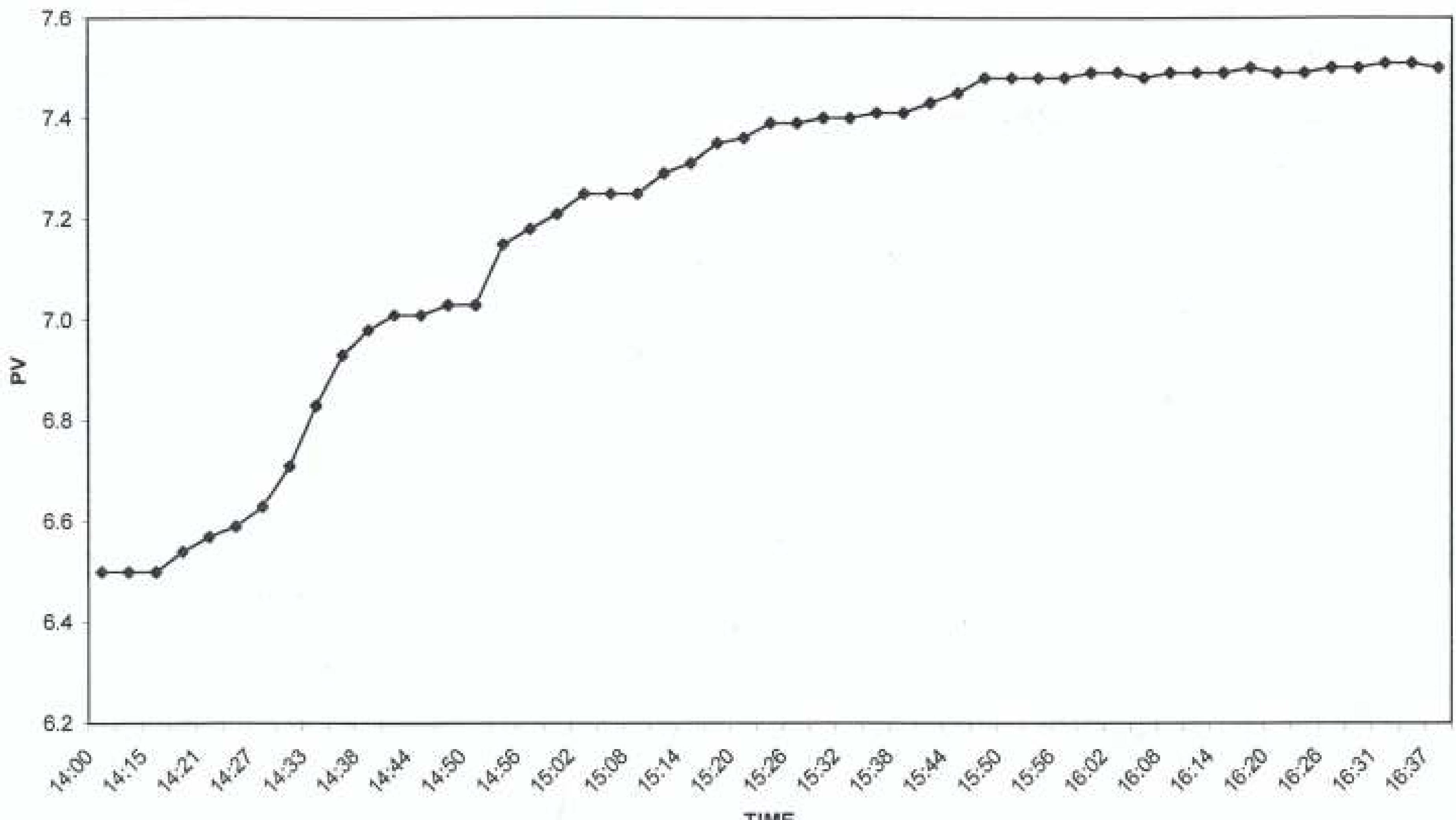
P = 10% I = 65 SECONDS D = 0 SECOND

FIG. 6.2 PLANT INPUT RESPONSE



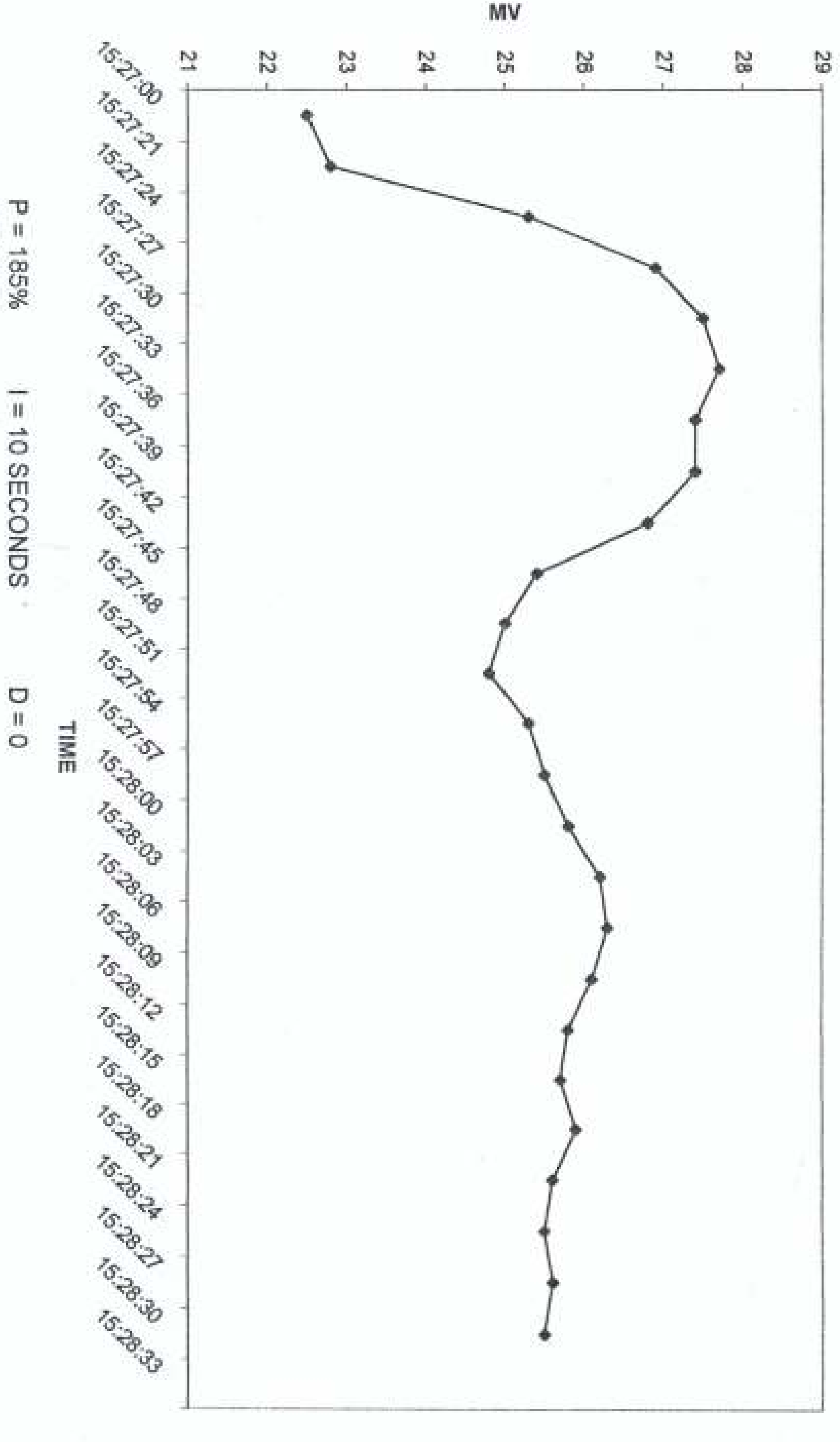
P = 125%   I = 25 SECONDS   D = 0

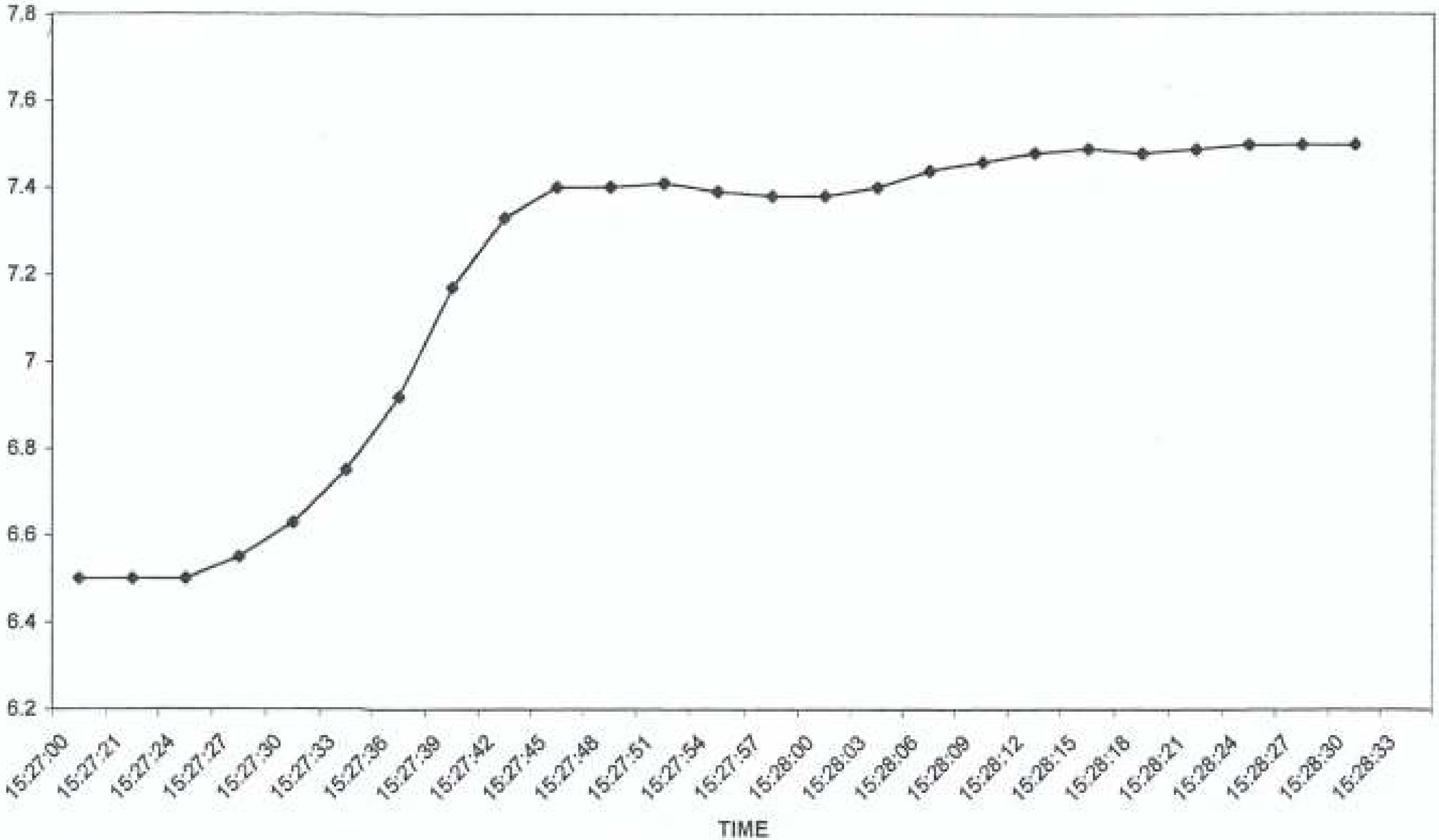
FIG. 6.3 SAMPLE ONE OUTPUT RESPONSE



P = 125%   I = 25 SECONDS   D = 0

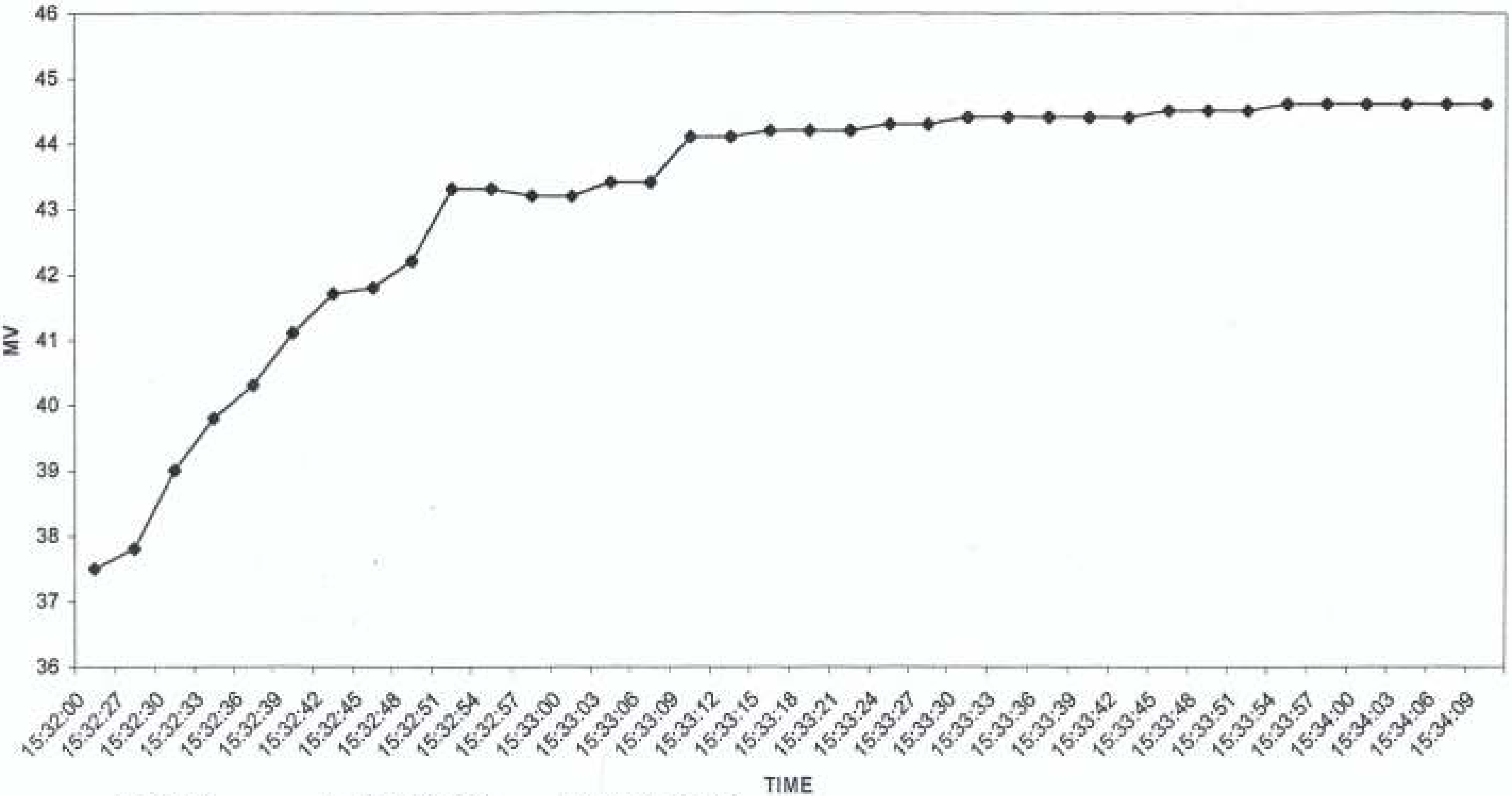
FIG.6.4 SAMPLE ONE INPUT RESPONSE


MV
21
22
23
24
25
26
27
28
29
TIME
15:27:00
15:27:21
15:27:24
15:27:27
15:27:30
15:27:33
15:27:36
15:27:39
15:27:42
15:27:45
15:27:48
15:27:51
15:27:54
15:27:57
15:28:00
15:28:03
15:28:06
15:28:09
15:28:12
15:28:15
15:28:18
15:28:21
15:28:24
15:28:27
15:28:30
15:28:33
P = 185%
I = 10 SECONDS
D = 0



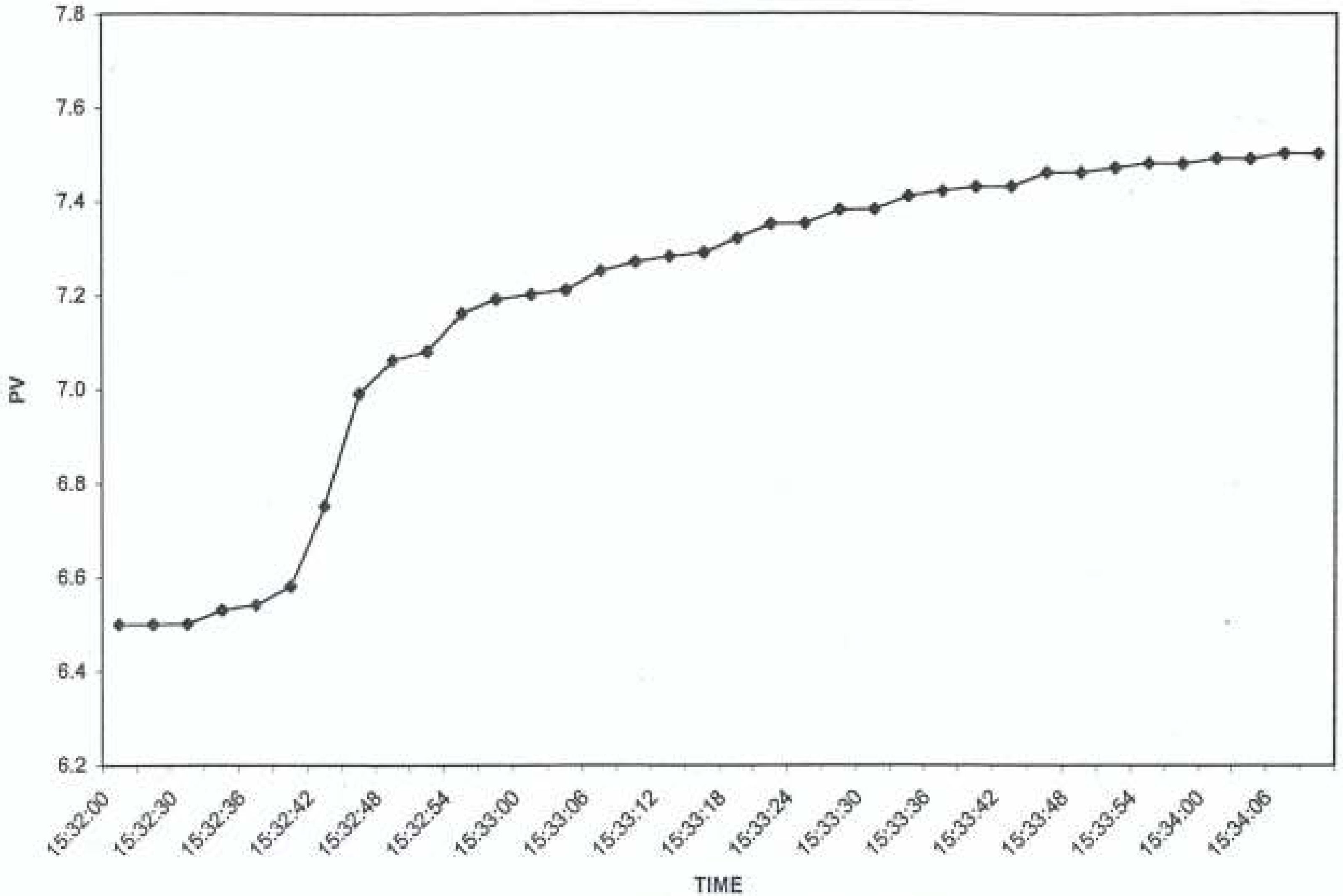
P = 185%   I = 10 SECONDS   D = 0 SECONDS

FIG.6.6 SAMPLE TWO INPUT RESPONSE



P = 220%   I = 15 SECONDS   D = 0 SECONDS

FIG. 6.7 SAMPLE THREE OUTPUT RESPONSE



P = 220%　　I = 15 SECONDS　　D = O SECOND

FIG. 6.8 SAMPLE THREE INPUT RESPONSE

CHAPTER SEVEN

CONCLUSION AND RECOMMENDATION

7.1 CONCLUSION

A technique for the optimization of a process had been presented due to its utmost importance, especially in a sensitive process like the crude distillation process. This is because, in addition to reduce manpower and low operational cost. It ensures higher quality control of the product thus obtaining high grade product.

This optimization had been achieved in a digital computer that has been configured to function as a process controller in direct digital control system. Formerly analogue computers were used as it had the ability to carry out arithmetic operations as well as its ability to simulate complex systems, deal with continuous quantities rather than discrete steps and relate directly with the process itself. But not withstanding these advantages, digital computers were used to replace the analog controllers in the systems. This arise from the fact that it has greater accuracy, increased flexibility and capability of dealing with large number of interrelated variables as well as potential low cost of technology of device.

However, the digital computer is very expensive to install. It is very complex as it require a lot of software programming and hardware development. However, optimization of the digital controller will improve the accuracy and precision of distillation in the crude distillation unit of the system.

Optimization had been carried out using discrete state-space method in a design technique which required that plant states be measured in a sampling period. A device for estimating the states of the plant from the information that is available was designed. A predictor model is the basis of many self-tuning designs: the model for casts the response as some feature time based on currently available data and the control law is choosing so that the predicted output is optimal.

The optimal design technique for optimal linear regulator with quadratic performance index was investigated by the application of the Hamiltonian function and result was applied in the derivative of the optimum control law and hence in the determination of optimum gain for the controller. Minimization was carried out by the least square techniques and optimum performance condition determined. Direct realization was adopted in the transfer function realization of the controller.

7.2 RECOMMENDATION

Two approaches for minimization of a given performance index are available in optimization methods:

i) **Minimum principle of pontryagin:** This is based on the concept of calculus of variations.

ii) **Dynamic Programming developed by Bellman:** This is based on the principle of invariant imbedding and lead to the principle of optimality that follows the basic laws of nature and does not need complex mathematical development to explain its validity. The minimum principle of Pontryagin has been used in the thesis. Another work could be carried out in future to show that whichever principle is adopted the end result will be the same.

In practical application of control theory, one of the few things engineers can assume for certain is that the signals to be encountered are not known for certain. When dealing with uncertain external influences acting on a controlled process, it is important to distinguish between noise and disturbance. Noise is an erratic, uncertain signal having no distinguishing waveform properties. The accommodation of noise in a control problem is best handled by treating the noise as a random process.

Disturbances, on the other hand, are well behaved uncertain signal having distinguish waveform properties. For accommodating such influences one should employ a method which makes use of the known waveform properties which characterizes the disturbance. The difference between the effect of noise on the system and the effect of disturbance on the system could investigated.

APPENDIX I

PLANT DATA

Pressure Loop (10PC021)

PV	=	process Variable	P	=	10%
MV	=	Manipulated Variable	I	=	65 Sec.
SV	=	Set point	D	=	0sec.

DV = Deviation or error = (PV-SV)

Time	PV (Kg/cm^2)	SV(Kg/cm^2)	DV (Kg/cm^2)	MV(%)
18: 43:01	7.98	7.48	0.5	62.5
18:43:05	7.98	7.48	0.5	66.7
18:43:11	7.98	7.48	0.5	70.0
18:43:15	7.98	7.48	0.50	74.4
18:43:20	7.98	7.84	0.50	77.5
18:43:27	7.98	7.48	0.50	79.0
18:43:30	7.97	7.48	0.49	80.0
18:43:33	7.96	7.48	0.48	82
18:43:39	7.96	7.48	0.48	82.6
18:43:42	7.96	7.48	0.48	85.4
18:43:47	7.95	7.48	0.47	86.7
18:43:50	7.83	7.48	0.35	86.0
18:44:00	7.75	7.48	0.27	82.5
18:44:04	7.73	7.48	0.25	81.0
18:44:09	7.65	7.48	0.17	73.0
18:44:15	7.63	7.48	0.15	68.8
18:44:20	7.57	7.48	0.09	67.5
18:44:27	7.50	7.48	0.02	64.0
18:44:32	7.49	7.48	0.01	65.0
18:44:50	7.49	7.48	0.01	63.2
18:45:00	7.48	7.48	0.00	62.3
18:45:07	7.48	7.48	0.00	61.2
18:45:15	7.48	7.48	0.00	58.5
18:45:30	7.47	7.48	-0.01	55.4
18:45:38	7.47	7.48	-0.01	55.0
18:45:40	7.47	7.48	-0.01	55.0
18:46:00	7.48	7.48	0.00	55.0
18:46:10	7.48	7.48	0.00	55.0
18:46:25	7.49	7.48	0.01	57.1
18:46:33	7.49	7.48	0.01	60.1
18:46:45	7.49	7.48	0.01	61.3
18:46:53	7.48	7.48	0.00	61.2%

APPENDIX II

SAMPLE ONE SIMULATION

FOR P = 125%, I = 25sec, D = 0

STEP CHANGE 7.5KG/CM^2

IN SV BY ......6.5KG/CM^2

TIME 15:14:12

TIME	PV(Kg/cm^2)	SV(Kg/cm^2)	MV(%)	OFFSET (Kg/cm)
15:14:00	6.50	6.50	37.7	0
15:14:12	6.50	7.50	37.9	-1.0
15:14:15	6.50	7.50	38.7	-1.0
15:14:18	6.54	7.50	39.2	-0.96
15:14:21	6.57	7.50	39.5	-0.93
15:14:24	6.59	7.50	39.7	-0.91
15:14:27	6.63	7.50	39.8	-0.87
15:14:30	6.71	7.50	40.0	-0.79
15:14:33	6.83	7.50	40.1	-0.67
15:14:35	6.93	7.50	40.2	0.57
15:14:38	6.98	7.50	40.2	-0.52
15:14:41	7.01	7.50	40.2	-0.49
15:14:44	7.01	7.50	40.3	-0.49
15:14:47	7.03	7.50	40.3	-0.46
15:14:50	7.03	7.50	40.4	-0.46
15:14:53	7.15	7.50	40.5	-0.35
15:14:56	7.18	7.50	40.7	-0.32
15:14:59	7.21	7.50	40.7	-0.29
15:15:02	7.25	7.50	40.9	-0.25
15:15:05	7.25	7.50	40.9	-0.25
15:15:08	7.25	7.50	40.9	-0.25
15:15:11	7.29	7.50	40.9	-0.21
15:15:14	7.31	7.50	41.0	-0.19
15:15:17	7.35	7.50	42.0	-0.15
15:15:20	7.36	7.50	4.20	-0.14
15:15:23	7.39	7.50	42.0	-0.11
15:15:26	7.39	7.50	42.0	-0.11

15:15:29	7.04	7.50	42.1	-0.15
15:15:32	7.40	7.50	42.2	-0.10
15:15:35	7.41	7.50	42.3	0.09
15:15:38	7.41	7.50	42.3	-0.09
15:15:41	7.43	7.50	42.4	-0.07
15:15:44	7.45	7.50	42.6	-0.05
15:15:47	7.48	7.50	42.8	-0.02
15:15:50	7.48	7.50	42.8	-0.02
15:15:53		7.50	42.9	
15:15:56		7.50	43.1	
15:15:59	7.49	7.50	43.1	
15:16:02		7.50	43.2	-0.01
15:16:05	7.48	7.50	43.3	
15:16:08	7.49	7.50	43.3	-0.02
15:16:11		7.50	43.3	-0.01
15:16:14		7.50	43.4	
15:16:17	7.50	7.50	43.5	
15:16:20	7.49	7.50	43.5	0.0
15:16:23	7.49	7.50	43.6	-0.01
15:16:26	7.50	7.50	43.6	-0.01
15:16:29		7.50	43.5	0.0
15:16:31	7.51	7.50	43.6	0.01
15:16:34		7.50	43.5	0.01
15:16:37	7.50	7.50		0.0

APPENDIX III

SAMPLE TWO SIMULATION

FOR P = 185%, I = 10 sec, D = 0

SAME STEP CHANGE AT TIME 15:27:21

TIME (SEC)	PV (Kg/cm^2)	SV(kg/cm^2)	MV (%)	OFFSET (kg/cm^2)
15:27:00	6.50	6.50	22.5	0.0
15:27:21	6.50	7.50	22.8	-1.0
15:27:24	6.50	7.50	25.3	-1.0
15:27:27	6.55	7.50	26.9	-0.95
15:27:30	6.63	7.50	27.5	-0.87
15:27:33	6.75	7.50	27.7	-0.75
15:27:36	6.92	7.50	27.4	-0.58
15:27:39	7.17	7.50	27.4	-0.33
15:27:42	7.33	7.50	26.8	-0.17
15:27:45	7.40	7.50	25.4	-0.10
15:27:48	7.40	7.50	25.0	-0.10
15:27:51	7.41	7.50	25.0	-0.9
15:27:54	7.39	7.50	25.3	-0.11
15:27:57	7.38	7.50	25.5	-0.12
15:28:00	7.38	7.50	25.8	-0.12
15:28:03	7.40	7.50	26.2	-0.10
15:28:06	7.44	7.50	26.3	-0.06
15:28:09	7.46	7.50	26.1	-0.04
15:28:12	7.48	7.50	25.8	-0.02
15:28:15	7.49	7.50	25.7	-0.01
15:28:18	7.48	7.50	25.7	-0.02
15:28:21	7.49	7.50	25.6	-0.01
15:28:24	7.50	7.50	25.5	0.0
15:28:27	7.50	7.50	25.6	0.0
15:28:30	7.50	7.50	25.5	0.0
15:28:33		7.50		

APPENDIX IV

SAMPLE THREE SIMULATION

FOR P = 220% I = 15sec, D = 0
SAME CHANGE AT TIME 15:32:27

TIME	PV (kg/cm^2)	SV($kgcm^2$)	MV(%)	OFFSET(kg/c^2)
15:32:00	6.50	6.50	37.5	0.0
15:32:27	6.50	7.50	37.8	-1.0
15:32:30	6.50	7.50	39.0	-1.0
15:32:33	6.53	7.50	39.8	-0.97
15:32:36	6.54	7.50	40.3	-0.96
15:32:39	6.58	7.50	41.1	-0.92
15:32:42	6.75	7.50	41.7	-0.75
15:32:45	7.99	7.50	41.8	-0.51
15:32:48	7.06	7.50	42.2	-0.44
15:32:51	7.08	7.50	42.3	-0.42
15:32:54	7.16	7.50	43.2	-0.34
15:32:57	7.19	7.50	43.2	-0.31
15:33:00	7.20	7.50	43.4	-0.30
15:33:03	7.21	7.50	43.4	-0.29
15:33:06	7.25	7.50	44.1	-0.25
15:33:09	7.27	7.50	44.1	-0.23
15:33:12	7.28	7.50	44.2	-0.22
15:33:15	7.29	7.50	44.2	-0.21
15:33:18	7.32	7.50	44.2	-0.18
15:33:21	7.35	7.50	44.3	-0.15
15:33:24	7.35	7.50	44.3	-0.15
15:33:27	7.38	7.50	44.4	-0.12
15:33:30	7.38	7.50	44.4	-0.12
15:33:33	7.41	7.50	44.4	-0.09
15:33:36	7.42	7.50	44.4	-0.08
15:33:39	7.43	7.50	44.4	-0.07
15:33:42	7.43	7.50	44.4	-0.07
15:33:45	7.46	7.50	44.5	-0.04
15:33:48	7.46	7.50	44.5	-0.04
15:33:51	7.47	7.50	44.5	-0.03
15:33:54	7.48	7.50	44.6	-0.02
15:33:57	7.49	7.50	44.6	-0.01
15:34:00	7.49	7.50	44.6	-0.01
15:34:03	7.49	7.50	44.6	-0.01
15:34:06	7.50	7.50	44.6	0.0
15:34:09	7.50	7.50	44.6	0.0

APPENDIX V

SOFT WARE

```
#include<stdio.h>
#include<stdlib.h>
#define t 1.00
#define kd 0.00
#define b1 -1.00
#define b2 0.00
#define b0 1.00

main()    {
   float a0,a1,a2,pb,ti,td,ki,kp,fp,
         a[3][3],b[3][1],a_trans[3][3],b_trans[1][3],
         c[1][3],c_tran[3][1],a_sqr[3][3],a_trn_sqr[3][3],a_b[3][1
],a_sqr_b[3][3],
         a_trn_ctn[3][3],a_t_sqct[3][3],a_tran_sqr[3][3],det_mat_a
,det_mat_b,mat_a[3][3],
         mat_b[3][3],g1,g2,g3,lambda,gc[3][3],lambda_mat[3][3],cof
_a11,
         cof_a12,cof_a13,cof_a21,cof_a22,cof_a23,cof_a31,cof_a32,c
of_a33,adj_mat_a[3][3],
         det_a,inv_a[3][3],inv_a_sq[3][3],inv_a_cb[3][3],ab1[3][3]
,ab2[3][3],ab3[3][3],mat_k[3][3],
         adj_mat_k[3][3],cof_k11,cof_k12,cof_k13,cof_k21,cof_k22,c
of_k23,cof_k31,cof_k32,
         cof_k33,det_k,inv_k[3][3],k_prime[3][3],x1[3],x2[3],x3[3]
,
         u_0[3],u_1[3],u_2[3],j_init,u_0_sqrd[3],u_1_sqrd[3],u_2_s
qrd[3],g_soln[3];
  double j_val;
  FILE *in;
  int i,j,k;
  clrscr();
  printf("              ENTER THE VARIABLES:       \n");
  printf(" PB= ");
  scanf("%f",&pb);
  printf("\n");
  printf(" Ti= ");
  scanf("%f",&ti);
  printf("\n");
  printf(" Td= ");
  scanf("%f",&td);
  printf("\n\n");

   kp=100/pb;
   ki=1/ti;
   a0=kp+(ki*t/2)+(kd/t);
   a1=-kp+(ki*t/2)-(2*kd/t);
   a2=kd/t;
```

```
a[0][0]=0.00;a[0][1]=1.00;a[0][2]=0.00;
a[1][0]=0.00;a[1][1]=0.00;a[1][2]=1.00;
a[2][0]=-b0;a[2][1]=-b1;a[2][2]=-b2;

b[0][0]=0.00;
b[1][0]=0.00;
b[2][0]=1.00;
  c[0][0]=a0;
  c[0][1]=a1;
  c[0][2]=a2;

  b_trans[0][0]=0.00;
  b_trans[0][1]=0.00;
  b_trans[0][2]=1.00;

  c_tran[0][0]=a0;
  c_tran[1][0]=a1;
  c_tran[2][0]=a2;
  a_trans[0][0]=0.00;a_trans[0][1]=0.00;a_trans[0][2]=-b0;
  a_trans[1][0]=1.00;a_trans[1][1]=0.00;a_trans[1][2]=-b1;
  a_trans[2][0]=0.00;a_trans[2][1]=1.00;a_trans[2][2]=-b2;

 for(i=0;i<=2;i++){
  for(j=0;j<=2;j++){
   a_sqr[i][j]=0.00;
   a_trn_sqr[i][j]=0.00;
   a_b[i][j]=0.00;
   a_sqr_b[i][j]=0.00;
   a_trn_ctn[i][j]=0.00;
   a_t_sqct[i][j]=0.00;
   }
  }

 for(i=0;i<=2;i++){
  for(j=0;j<=2;j++){
   for(k=0;k<=2;k++){
     a_sqr[i][j]=a_sqr[i][j]+(a[i][k]*a[k][j]);
     a_trn_sqr[i][j]=a_trn_sqr[i][j]+(a_trans[i][k]*a_trans[k][
j]);
    }
   }
  }
 for(i=0;i<=2;i++){
  for(k=0;k<=2;k++){
    a_b[i][0]=a_b[i][0]+(a[i][k]*b[k][0]);
    a_trn_ctn[i][0]=a_trn_ctn[i][0]+(a_trans[i][k]*c_tran[k][0]
);
```

```
        a_sqr_b[i][0]=a_sqr_b[i][0]+(a_sqr[i][k]*b[k][0]);
        a_t_sqct[i][0]=a_t_sqct[i][0]+(a_trn_sqr[i][k]*c_tran[k][0]
);
      }
    }

    mat_a[0][0]=0.00;mat_a[0][1]=a_b[0][0];mat_a[0][2]=a_sqr_b[0][
0];
    mat_a[1][0]=0.00;mat_a[1][1]=a_b[1][0];mat_a[1][2]=a_sqr_b[1][
0];
    mat_a[2][0]=1.00;mat_a[2][1]=a_b[2][0];mat_a[2][2]=a_sqr_b[2][
0];

    mat_b[0][0]=a0;mat_b[0][1]=a_trn_ctn[0][0];mat_b[0][2]=a_t_sqc
t[0][0];
    mat_b[1][0]=a1;mat_b[1][1]=a_trn_ctn[1][0];mat_b[1][2]=a_t_sqc
t[1][0];
    mat_b[2][0]=a2;mat_b[2][1]=a_trn_ctn[2][0];mat_b[2][2]=a_t_sqc
t[2][0];

    /*    DETERMINANTS OF "A" AND "B"  */

    det_mat_a=(mat_a[0][0]*((mat_a[1][1]*mat_a[2][2])-(mat_a[2][1]
*mat_a[1][2])))-
              (mat_a[0][1]*((mat_a[1][0]*mat_a[2][2])-(mat_a[2][0]
*mat_a[1][2])))+
              (mat_a[0][2]*((mat_a[1][0]*mat_a[2][1])-(mat_a[2][0]
*mat_a[1][1])));

    det_mat_b=(mat_b[0][0]*((mat_b[1][1]*mat_b[2][2])-(mat_b[2][1]
*mat_b[1][2])))-
              (mat_b[0][1]*((mat_b[1][0]*mat_b[2][2])-(mat_b[2][0]
*mat_b[1][2])))+
              (mat_b[0][2]*((mat_b[1][0]*mat_b[2][1])-(mat_b[2][0]
*mat_b[1][1])));

  /*      FINDINg THE VALUE OF EXPRESSION 4       */

   printf("\n Enter Value of Lambda: ");
   scanf("%f",&lambda);

   for(i=0;i<=2;i++){
    for(j=0;j<=2;j++){
     if(i==j){
       lambda_mat[i][j]=lambda; }
     else {
       lambda_mat[i][j]=0.00; }
     }
```

```
}
     g_soln[0]=(a[0][0]-lambda_mat[0][0])/c[0][0];
     g_soln[1]=(a[1][1]-lambda_mat[1][1])/c[0][1];
     if(c[0][2]==0.00)
      printf("THE VALUE OF g3 IS UNDEFINED");
     else
      g_soln[2]=(a[1][2]-lambda_mat[2][2])/c[0][2];

/*    INVERSE OF "A"    */
 cof_a11=a[0][0]*((a[1][1]*a[2][2])-(a[1][2]*a[2][1]));
 cof_a12=a[0][1]*((a[1][0]*a[2][2])-(a[1][2]*a[2][0]));
 cof_a13=a[0][2]*((a[1][0]*a[2][1])-(a[1][1]*a[2][0]));
 cof_a21=a[1][0]*((a[0][1]*a[2][2])-(a[0][2]*a[2][1]));
 cof_a22=a[1][1]*((a[0][0]*a[2][2])-(a[0][2]*a[2][0]));
 cof_a23=a[1][2]*((a[0][0]*a[2][1])-(a[0][1]*a[2][0]));
 cof_a31=a[2][0]*((a[0][1]*a[1][2])-(a[0][2]*a[1][1]));
 cof_a32=a[2][1]*((a[0][0]*a[1][2])-(a[0][2]*a[1][0]));
 cof_a33=a[2][2]*((a[0][0]*a[1][1])-(a[0][1]*a[1][0]));

 adj_mat_a[0][0]=cof_a11;adj_mat_a[0][1]=cof_a21;adj_mat_a[0][2]
=cof_a31;
 adj_mat_a[1][0]=cof_a12;adj_mat_a[1][1]=cof_a22;adj_mat_a[1][2]
=cof_a32;
 adj_mat_a[2][0]=cof_a13;adj_mat_a[2][1]=cof_a23;adj_mat_a[2][2]
=cof_a33;

  det_a=(a[0][0]*((a[1][1]*a[2][2])-(a[2][1]*a[1][2])))-(a[0][1]*
((a[1][0]*a[2][2])-(a[2][0]*a[1][2])))+
         (a[0][2]*((a[1][0]*a[2][1])-(a[2][0]*a[1][1])));

  inv_a[0][0]=adj_mat_a[0][0]/det_a;inv_a[0][1]=adj_mat_a[0][1]/d
et_a;inv_a[0][2]=adj_mat_a[0][2]/det_a;
  inv_a[1][0]=adj_mat_a[1][0]/det_a;inv_a[1][1]=adj_mat_a[1][1]/d
et_a;inv_a[1][2]=adj_mat_a[1][2]/det_a;
  inv_a[2][0]=adj_mat_a[2][0]/det_a;inv_a[2][1]=adj_mat_a[2][1]/d
et_a;inv_a[2][2]=adj_mat_a[2][2]/det_a;

   /* FINDING THE VALUE OF "k_prime" */
   for(i=0;i<2;i++){
     for(j=0;j<=2;j++){
       inv_a_sq[i][j]=0.00;
       inv_a_cb[i][j]=0.00;
      }
     }

   for(i=0;i<=2;i++){
     for(j=0;j<=2;j++){
       for(k=0;k<=2;k++)
```

```
          inv_a_sq[i][j]=inv_a_sq[i][j]+(inv_a[i][k]*inv_a[k][j]);
        }
      }

   for(i=0;i<=2;i++){
     for(j=0;j<=2;j++){
       for(k=0;k<=2;k++)
         inv_a_cb[i][j]=inv_a_cb[i][j]+(inv_a_sq[i][k]*inv_a[k][j]
);
        }
      }

   for(i=0;i<=2;i++){
        ab1[i][0]=0.00;
        ab2[i][0]=0.00;
        ab3[i][0]=0.00;
     }

   for(i=0;i<=2;i++){
       for(k=0;k<=2;k++){
         ab1[i][0]=ab1[i][0]+(inv_a[i][k]*b[k][0]);
         ab2[i][0]=ab2[i][0]+(inv_a_sq[i][k]*b[k][0]);
         ab3[i][0]=ab3[i][0]+(inv_a_cb[i][k]*b[k][0]);
        }
      }

   mat_k[0][0]=ab1[0][0];mat_k[0][1]=ab2[0][0];mat_k[0][2]=ab3[0][
0];
   mat_k[1][0]=ab1[1][0];mat_k[1][1]=ab2[1][0];mat_k[1][2]=ab3[1][
0];
   mat_k[2][0]=ab1[2][0];mat_k[2][1]=ab2[2][0];mat_k[2][2]=ab3[2][
0];

    /*  Inverse of matrix k  */
  cof_k11=ab1[0][0]*((ab2[1][0]*ab3[2][0])-(ab2[2][0]*ab3[1][0]));
  cof_k12=ab2[0][0]*((ab1[1][0]*ab3[2][0])-(ab3[1][0]*ab1[2][0]));
  cof_k13=ab3[0][0]*((ab1[1][0]*ab3[2][0])-(ab2[1][0]*ab1[2][0]));
  cof_k21=ab1[1][0]*((ab2[0][0]*ab3[2][0])-(ab3[0][0]*ab2[2][0]));
  cof_k22=ab2[1][0]*((ab1[0][0]*ab3[2][0])-(ab3[0][0]*ab1[2][0]));
  cof_k23=ab3[1][0]*((ab1[0][0]*ab2[2][0])-(ab2[0][0]*ab1[2][0]));
  cof_k31=ab1[2][0]*((ab2[0][0]*ab3[1][0])-(ab3[0][0]*ab2[1][0]));
  cof_k32=ab2[2][0]*((ab1[0][0]*ab3[1][0])-(ab3[0][0]*ab1[1][0]));
  cof_k33=ab3[2][0]*((ab1[0][0]*ab2[1][0])-(ab2[0][0]*ab1[1][0]));


   adj_mat_k[0][0]=cof_k11;adj_mat_k[0][1]=cof_k21;adj_mat_k[0][2]
=cof_k31;
   adj_mat_k[1][0]=cof_k12;adj_mat_k[1][1]=cof_k22;adj_mat_k[1][2]
```

```
=cof_k32;
   adj_mat_k[2][0]=cof_k13;adj_mat_k[2][1]=cof_k23;adj_mat_k[2][2]
=cof_k33;

     det_k=(ab1[0][0]*((ab2[1][0]*ab3[2][0])-(ab3[1][0]*ab2[2][0])
))-
                 (ab2[0][0]*((ab1[1][0]*ab3[2][0])-(ab3[1][0]*ab1[2
][0])))+
                 (ab3[0][0]*((ab1[1][0]*ab2[2][0])-(ab1[2][0]*ab2[1
][0])));

   printf("%f",det_k);
   inv_k[0][0]=cof_k11/det_k;inv_k[0][1]=cof_k21/det_k;inv_k[0][2]
=cof_k31/det_k;
   inv_k[1][0]=cof_k12/det_k;inv_k[1][1]=cof_k22/det_k;inv_k[1][2]
=cof_k32/det_k;
   inv_k[2][0]=cof_k13/det_k;inv_k[2][1]=cof_k23/det_k;inv_k[2][2]
=cof_k33/det_k;

     for(j=0;j<=2;j++)
        k_prime[0][j]=0.00;

     for(j=0;j<=2;j++){
       for(k=0;k<=2;k++)
         k_prime[0][j]=k_prime[0][j]+(b_trans[0][k]*inv_k[k][j]);
       }

    /*    SOLUTION OF EXPRESSION 5   */

    // READ THE VALUES OF THE X'S

    printf("\n");
    for(i=0;i<=2;i++){
     printf("x1( %d ): \n",i+1);
     scanf("%f",&x1[i]);printf("\n");
     printf("x2( %d ): \n",i+1);
     scanf("%f",&x2[i]);printf("\n");
     printf("x3( %d ): \n",i+1);
     scanf("%f",&x3[i]);printf("\n");
     }
   /*   U(k) values   */

   u_0[0]=-(k_prime[0][0]*x1[0]);
   u_0[1]=-(k_prime[0][1]*x1[1]);
   u_0[2]=-(k_prime[0][2]*x1[2]);

   u_1[0]=-(k_prime[0][0]*x2[0]);
   u_1[1]=-(k_prime[0][1]*x2[1]);
```

```
u_1[2]=-(k_prime[0][2]*x2[2]);

u_2[0]=-(k_prime[0][0]*x3[0]);
u_2[1]=-(k_prime[0][1]*x3[1]);
u_2[2]=-(k_prime[0][2]*x3[2]);


/*   J value  */

j_init=0.00;
 for(i=0;i<=2;i++){
   u_0_sqrd[i]=u_0[i]*u_0[i];
   u_1_sqrd[i]=u_1[i]*u_1[i];
   u_2_sqrd[i]=u_2[i]*u_2[i];
   j_init=j_init+u_0_sqrd[i]+u_1_sqrd[i]+u_2_sqrd[i];
  }
 j_val=j_init/2;

 /*   FINAL SOLUTIONS  */

 clrscr();

printf("              ***********************************************
*************\n");
printf("              *              SOLUTION   OF   COMPUTATIONS
          *\n");
printf("              ***********************************************
*************");
printf("\n");
printf("                VARIABLES                         VALUES
           ");
printf("\n");
printf(" 1.             DETERMINANT 1.                    %lf \n",det_ma
t_a);
printf(" 2.             DETERMINANT 2.                    %lf \n",det_ma
t_b);\
printf("\n");
printf(" 3.                 g  : ");
printf( "\n                                            %f ",g_soln[0]);
printf( "\n                                            %f ",g_soln[1]);
if(c[0][2]!=0)
 printf( "                                           %f  ",g_soln[2]);
else
   printf("\n                                             g3 IS UNDEFINE
D ");
printf("\n");
printf(" 4.             k1                           %f",k_prime[0][0]);
printf("\n               k2                            %f",k_prime[0][1]
```

```
);
   printf("\n              k3                           %f",k_prime[0][2]
);
   printf("\n");
   printf(" 5.          U VALUES(3)  : \n");
    for(i=0;i<=2;i++){
      printf("         U1=                             %f\n",u_0[i]);
      printf("         U2=                             %f\n",u_1[i]);
      printf("         U3=                             %f\n",u_2[i]);
       }
   printf(" 6.           J                            %f",j_val);

  /*    WRITING TO OUTPUT FILE  */

  if ((in = fopen("Output.txt", "wt"))!= NULL)  {
   fprintf(in,"              ***************************************
******************\n");
   fprintf(in,"              *            SOLUTION   OF   COMPUTATIONS
                *\n");
   fprintf(in,"              ***************************************
******************");
   fprintf(in,"\n");
   fprintf(in,"               VARIABLES                    VALUES
                ");
   fprintf(in,"\n");
   fprintf(in," 1.          DETERMINANT 1.               %lf \n",det_
mat_a);
   fprintf(in," 2.          DETERMINANT 2.               %lf \n",det_
mat_b);
   fprintf(in,"\n");
   fprintf(in," 3.           THE g MATRIX : ");
   fprintf(in,"                                    %f ",g_soln[0]);
   fprintf(in,"\n                                  %f ",g_soln[1]);
   if(c[0][2]!=0)
    fprintf(in,"                                   %f  ",g_soln[2])
;
   else
      fprintf(in,"\n                                    g3 IS UNDEFINE
D");
   fprintf(in," 4.           k1                          %f",k_prime[0][
0]);
   fprintf(in,"\n             k2                          %f",k_prime[0]
[1]);
   fprintf(in,"\n             k3                          %f",k_prime[0]
[2]);

   fprintf(in,"\n");
   fprintf(in," 5.           U VALUES(3)  : \n");
```

```
    for(i=0;i<=2;i++){
      fprintf(in,"          U1=                              %f\n",u_0[
i]);
      fprintf(in,"          U2=                              %f\n",u_1[
i]);
      fprintf(in,"          U3=                              %f\n",u_2[
i]);
      }
   fprintf(in," 6.         J                              %f",j_val)
;
}
  fclose(in);

  getch();
  return(0);
  }
```

```
        ************************************************************
***
        *              SOLUTION   OF   COMPUTATIONS
  *
        ************************************************************
***
          VARIABLES                          VALUES

 1.      DETERMINANT 1.                    -1.000000
 2.      DETERMINANT 2.                    0.527032

 3.       THE g MATRIX :
                                    -1.219512
                                    1.282051
                                    g3 IS UNDEFINED
 4.      k1                            0.000000
         k2                            0.000000
         k3                            1.000000
 5.       U VALUES(3) :
        U1=                               0.000000
        U2=                               0.000000
        U3=                              -0.000000
        U1=                               0.000000
        U2=                               0.000000
        U3=                              -0.000000
        U1=                               0.970000
        U2=                               0.150000
        U3=                              -0.000000
 6.        J                                 0.481700
```

```
          ***********************************************************
***
          *              SOLUTION   OF   COMPUTATIONS
  *
          ***********************************************************
***
            VARIABLES                          VALUES

 1.        DETERMINANT 1.                   -1.000000
 2.        DETERMINANT 2.                   1000.773376

 3.         THE g MATRIX :                                            -0.09
9923
                                  0.100077
                                  g3 IS UNDEFINED 4.          k1
                  0.000000
           k2                         0.000000
           k3                         1.000000
 5.         U VALUES(3) :
         U1=                             0.000000
         U2=                             0.000000
         U3=                             -0.000000
         U1=                             0.000000
         U2=                             0.000000
         U3=                             -0.000000
         U1=                             0.970000
         U2=                             0.150000
         U3=                             -0.000000
 6.          J                              0.481700
```

```
        *****************************************************************
***
        *              SOLUTION   OF   COMPUTATIONS
  *
        *****************************************************************
***
          VARIABLES                          VALUES

 1.      DETERMINANT 1.                    -1.000000
 2.      DETERMINANT 2.                    0.181881

 3.       THE g MATRIX :        -1.693364
                                 2.038568
                                 g3 IS UNDEFINED
 4.      k1                          0.000000
         k2                          0.000000
         k3                          1.000000
 5.       U VALUES(3) :
        U1=                             0.000000
        U2=                             0.000000
        U3=                            -0.000000
        U1=                             0.000000
        U2=                             0.000000
        U3=                            -0.000000
        U1=                             0.970000
        U2=                             0.150000
        U3=                            -0.000000
 6.        J                               0.481700
```

REFERENCE

Adiele, E & Echem, C., 1997 "Design & Construction of a Digital Controller" Unpublished A final Year Project in Electrical/Electronics Engineering Department Petroleum Training Institute Effurun, 1 pp.

Astrom, Karl J. and Bjorn Wittenmark 1983 "Computer-Controlled Systems, Theory and Design' 2nd Edition, 20 pp

Andrew S. Tanenbaum 1985 "Computer Networks" Printice Hall PTR Upper saddle River, New Jersey 0758 300 pp

Bernard Friedland, 1987 "Control System Design. An Introduction to State-space Methods' McGraw-Hill International Edition., 326 pp.

Benjamin C. Kuo, 1987 "Digital Control System" McGraw-Hill International Edition, 579 pp.

Elgred Olle I., 1967, "Control Systems Theory', 415 pp.

Eshbach. 1975: "Hand book of Engineering Fundamentals' 3rd Edition 1186-1187 pp

Franklin F.Kuo. 1966 "Network Analysis & Synthesis' Bell Telephone Laboratories Inc. John Wiley & Son. N.Y 11-12 pp

Falaki S.O., 1977, "Microprocessor Interfacing" A 600L Course work on EEE622. Unpublished.

Fink Donald, 1975: Electronic Engineering Hand book 1st Edition 24:15:16 pp

Gopal M.& Nagrath, J. J., 1993 "Control Systems Engineering" 2nd Edition, 149 pp.

Gopal M., 1989, "Digital Control Engineering" 196 pp

Grimble M.J, 1982, "Optimal Control of Linear Uncertain Multivariable Stochastic Systems' IEE Proc. Part D Vol. 129 Number 6 Nov., 263-270 pp.

Goshaidas Ray and A.K Mahalanabis, 1981 "On the state estimation of large-scale continuous time linear system' IEE Proc. Part D. Vol.128 Number 4 July, 137-141 pp.

Hesketh T. 1982 "state-space Pole-Placing Self-Tuning Regulator Using Input-Output values" IEE proc Part D Vol 129 Number 4 July, 123-127 pp.

Leang. S. Shielh and Yih T. tsay, 1981 "state-space Approach for self-Tunning Feedback Control with Pole Assignment' IEE pro. Part D. Vol. 128 Number 3, May, 93-100 pp.

Morris A S and Sterling M.J.H, 1981, 'Identification and direct Digital control of an electric arc Furnace Controller' IEE Proc. Part D. Number 3, 123-128 pp.

Norman A. Anderson: 1980 'Instrumentation for Process measurement and control chilton company Radnor, Pennsylvania 197-199 pp, 377-478 pp.

Okocha P. C and Udoh, E. E., 1986 'Investigation of compound Controller Actions in a Closed Loop Control Systems, Unpublished A Final Year project in the Electrical Engineering Department, Petroleum Training Institute, Effurun, 9 pp.

Oni J. O., 1997 'Control Engineering' A 600L Course Work on EEE 612. Unpublished.

Ogata J., 1970, 'Modern Control Engineering', 755 pp

Philips C. L and P.H Troy., N., 1990, 'Digital Control System Analysis and Design' 2nd Edition, 323 pp.

Ramsay. D.C 1981'Engineering Instrumentation & Control" ST(P) Technology Today, A series for Technicians Stanley Thernes (Publishers) ltd 199-200 pp.

Stanley M. Shinners, 1992, Modern Control System Theory and Design'
John Wiley and Sons Inc. 462 pp.

Stefano D. III, Stubberud, Williams, 1976, "Feedback & Control System's Schaum's series.

Thomas B. Kinney 1984: Process Automation Series Articles 1-6
Foxboro Company 83-84 pp.

Thomas E. Martin 1986: Industrial process control system-hand book of modern

Electronics and Electrical Engineering Edited by Charles Belove. 1181-1182 pp.

William J. Palm III, 1983, 'Modeling & Control of Dynamic Systems', 389 pp.

Yousefzadeh B, 1979, 'Basic Control Engineering', 77 pp.